

As an example, take a **gauge coupling constant** g_0 defined (taken from measurement) at some scale μ_0 . This is given by (in $d = 4 - \epsilon$ dimensions):¹⁴

$$g_0 = Zg\mu^{-\epsilon/2}.$$

Then, differentiating both sides with respect to μ ,

$$\begin{aligned} 0 &= \frac{\partial Z}{\partial \mu} g\mu^{-\epsilon/2} + Z \frac{\partial g}{\partial \mu} \mu^{-\epsilon/2} - \frac{\epsilon}{2} Zg\mu^{-\epsilon/2-1} \\ \mu \frac{\partial g}{\partial \mu} &= \frac{\epsilon}{2} g - \frac{g\mu}{Z} \frac{\partial Z}{\partial \mu} \\ \mu \frac{\partial g}{\partial \mu} &= \frac{\epsilon}{2} g - g\mu \frac{\partial}{\partial \mu} \ln Z, \end{aligned}$$

and taking the limit $\epsilon \rightarrow 0$:

$$\beta_g = \mu \frac{\partial g}{\partial \mu} = -g\gamma_g,$$

where we have defined the **anomalous dimension** of g

$$\gamma_g = \mu \frac{\partial}{\partial \mu} \ln Z. \quad (6.17)$$

The reason for keeping around the factor of μ in the definition of the β -function is that it typically changes very slowly over large differences in energy scale, so it is practical to change variable to $t = \ln \frac{\mu}{\mu_0}$, so that $\mu \frac{\partial}{\partial \mu} = \frac{\partial}{\partial t}$ and $\beta_g = \frac{\partial g}{\partial t}$.

Z can now be calculated to the required loop-order to find the β -function to that order, and in turn the running of the coupling constant with μ . By evaluating to one-loop order we can find that for our particular example of a gauge coupling constant for a generic supersymmetric model

$$\gamma_g|_{1\text{-loop}} = -\frac{1}{16\pi^2} g^2 \left(\sum_R T(R) - 3C(A) \right), \quad (6.18)$$

where the sum of Dynkin indices is over all superfields that transform under a representation R of the gauge group in question, and $C(A)$ is the **quadratic Casimir invariant** of the representation A of the vector field under the gauge group

$$C(A)\delta_{ij} = (T^a T^a)_{ij}.$$

For the adjoint representation of $U(1)$ this is 0, and for $SU(N)$ this is N . The running of the coupling constants is particularly important since it will later lead us to the concept of gauge coupling unification.

As a second relevant example, for the soft-breaking parameters M_i in (5.7) we have the one-loop β -functions

$$\beta_{M_i} \equiv \frac{d}{dt} M_i = \frac{1}{16\pi^2} g_i^2 M_i \left(2 \sum_R T(R) - 6C(A) \right). \quad (6.19)$$

¹⁴The factor $\mu^{-\epsilon/2}$ is there to ensure that the scale of g_0 is correct.

6.8 Gauge coupling unification

The one-loop β -functions for gauge couplings in a generic supersymmetric model were given in Eq. (6.18). With the MSSM field content and the gauge couplings discussed in this chapter:¹⁵

$$g_1 = \sqrt{\frac{5}{3}}g', \quad g_2 = g, \quad g_3 = g_s,$$

we arrive at

$$\beta_{g_i}|_{1\text{-loop}} = \frac{1}{16\pi^2} b_i g_i^3, \quad (6.20)$$

with in the MSSM

$$b_i^{\text{MSSM}} = \left(\frac{33}{5}, 1, -3 \right).$$

For comparison, the same result in the Standard Model is

$$b_i^{\text{SM}} = \left(\frac{41}{10}, -\frac{19}{6}, -7 \right).$$

The values of b_i for the MSSM are found from the Casimir invariant and the Dynkin index of the gauge group representations

$$C(A)_{SU(3)} = 3, \quad C(A)_{SU(2)} = 2, \quad C(A)_{U(1)} = 0,$$

and

$$T(R)_{SU(3)} = \frac{1}{2}, \quad T(R)_{SU(2)} = \frac{1}{2}, \quad T(R)_{U(1)} = \frac{3}{5}Y^2,$$

where for example $b_3 = \frac{1}{2} \cdot 12 - 3 \cdot 3 = -3$ in the MSSM, because, after careful counting, we have twelve quark/squark scalar superfields transforming under $SU(3)_C$.

At one-loop order we can do a neat rewrite using $\alpha_i \equiv \frac{g_i^2}{4\pi}$. Since

$$\frac{d}{dt}\alpha_i^{-1} = -2\frac{4\pi}{g_i^3}\frac{d}{dt}g_i,$$

we have:

$$\beta_{\alpha_i^{-1}} \equiv \frac{d}{dt}\alpha_i^{-1} = -\frac{8\pi}{g_i^3}\frac{1}{16\pi^2}g_i^3b_i = -\frac{b_i}{2\pi}.$$

Thus α^{-1} runs linearly with t at one loop.

By running the couplings α_i^{-1} from their values measured at the electroweak scale to high energies it is observed that in the MSSM the coupling constants intersect at a single point, which they do not naturally do in the Standard Model. See Fig. 6.3, taken from Martin [18]. The common assumption made is then that a unified gauge group, *e.g.* $SU(5)$ or $SO(10)$, is broken at that scale, called the **grand unification theory** scale or GUT-scale, down to the Standard Model gauge group. This scale is $\mu_{\text{GUT}} \approx 2 \cdot 10^{16} \text{ GeV}$, about two orders of magnitude below the Planck scale.

¹⁵The normalisation choice for g_1 may seem a bit strange, however, this is the correct numerical factor when for example breaking a unified group such as $SU(5)$ or $SO(10)$ down to the Standard Model gauge group. This factor might be different with a different unified group.

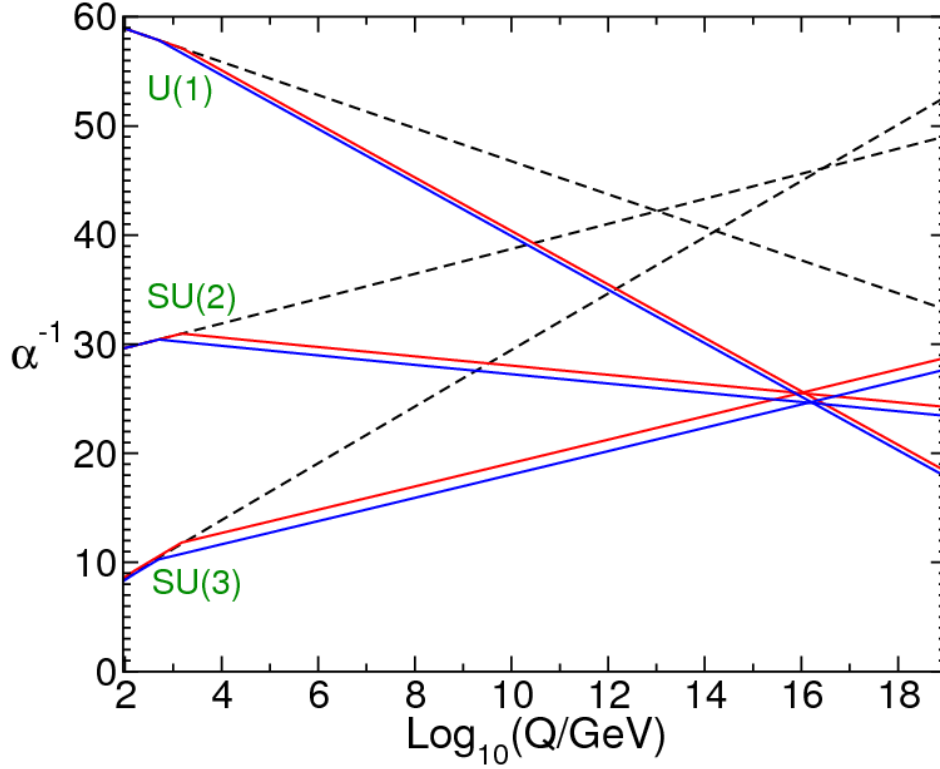


Figure 6.3: The RGE evolution of the inverse gauge couplings $\alpha_i^{-1}(Q)$ in the Standard Model (dashed lines) and the MSSM (solid lines). In the MSSM case, the sparticle mass thresholds are varied between 250 GeV and 1 TeV and $\alpha_3(m_Z)$ between 0.113 and 0.123 to create the bands shown by the red and blue lines. Two-loop effects are included.

Something funny happens to the gaugino soft-mass parameters M_i if we look at their running. From (6.19) the one-loop β -functions for the M_i in the MSSM are

$$\beta_{M_i}|_{1\text{-loop}} \equiv \frac{d}{dt} M_i = \frac{1}{8\pi^2} g_i^2 M_i b_i. \quad (6.21)$$

As a consequence all three ratios M_i/g_i^2 are scale independent at one loop. To see this let $R = M_i/g_i^2$, then

$$\beta_R \equiv \frac{dR}{dt} = \frac{\frac{d}{dt} M_i g_i^2 - M_i \frac{d}{dt} g_i^2}{g_i^4} = \frac{\frac{1}{8\pi^2} g_i^2 M_i b_i \cdot g_i^2 - M_i \cdot 2g_i \cdot \frac{1}{16\pi} g_i^3 b_i}{g_i^4} = 0. \quad (6.22)$$

In other words, R does not change with scale t .

If we now use that the coupling constants unify at the GUT scale to the coupling g_u , and assume that the soft-masses are the same at that scale $m_{1/2} = M_1(\mu_{\text{GUT}}) = M_2(\mu_{\text{GUT}}) = M_3(\mu_{\text{GUT}})$ ¹⁶ it follows that

$$\frac{M_1}{g_1^2} = \frac{M_2}{g_2^2} = \frac{M_3}{g_3^2} = \frac{m_{1/2}}{g_u^2}, \quad (6.23)$$

at all scales.¹⁷ This is a very powerful and predictive assumption. Because of the relationship

¹⁶Again, not unreasonable if the spontaneous symmetry breaking mechanism acts uniformly for all the gauginos.

¹⁷At one-loop level.

between the electroweak couplings and the electric charge, $e = g' \cos \theta_W = g \sin \theta_W$, it leads to the following relation

$$M_3 = \frac{\alpha_s}{\alpha} \sin^2 \theta_W M_2 = \frac{3}{5} \frac{\alpha_s}{\alpha} \cos^2 \theta_W M_1, \quad (6.24)$$

which, inserting values for the fine structure constant, the strong coupling, and the Weinberg angle, numerically predicts

$$M_3 : M_2 : M_1 \simeq 6 : 2 : 1$$

at the electroweak scale. We will return to the implications of this when discussing the gauginos in Sec. 6.12.

In Fig. 6.4, again taken from Martin [18], we show the running of the gaugino mass parameters M_i (solid black), the Higgs mass parameters $m_{H_{d/u}}^2$ (dot-dashed green), the third generation sfermion soft terms m_{d_3} , m_{Q_3} , m_{u_3} , m_{L_3} and m_{e_3} (dashed red and blue, listed from top to bottom), and the corresponding first and second generation terms (solid lines). Here we assume unified soft-mass parameters $m_{1/2}$ for the gauginos and m_0 for the Higgs and sfermions at the GUT scale.

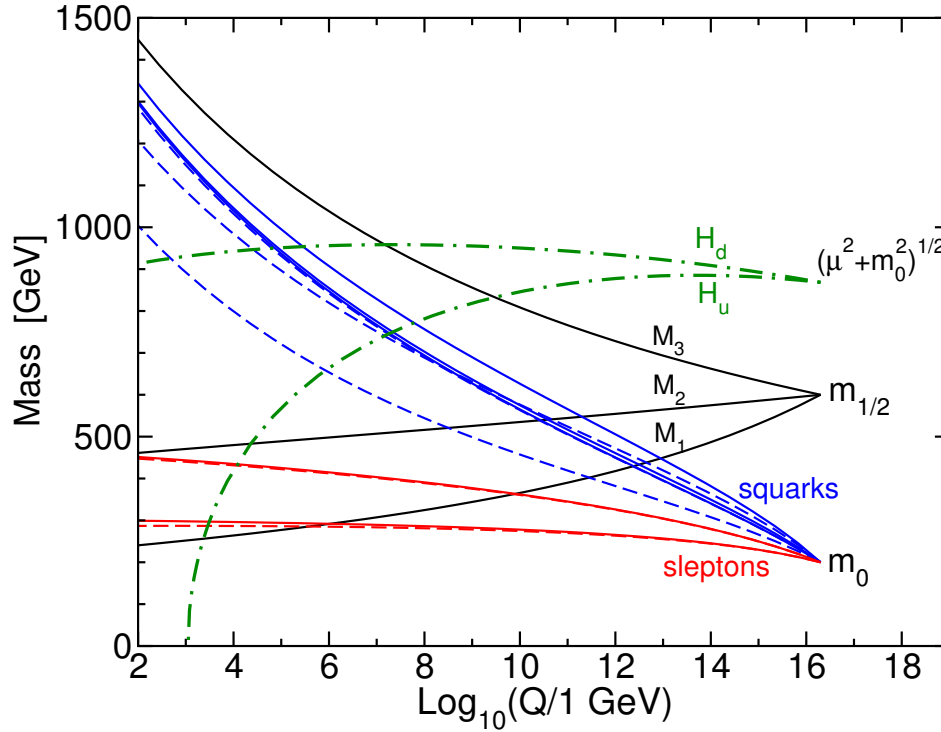


Figure 6.4: The RGE evolution of soft-mass parameters in the MSSM with typical minimal supergravity-inspired boundary conditions imposed at $2 \cdot 10^{16}$ GeV. The parameter values used for this illustration were $m_0 = 200$ GeV, $m_{1/2} = -A_0 = 600$ GeV, $\tan \beta = 10$, and $\text{sgn}(\mu) = +$.

6.9 Radiative Electroweak Symmetry Breaking

In the Standard Model both the vector bosons and the fermions are given mass spontaneously by electroweak symmetry breaking (EWSB), which is induced by the shape of the scalar

potential for the Higgs field H , which is an $SU(2)_L$ doublet of scalar fields:

$$V(H) = \mu^2 |H|^2 + \lambda |H|^4, \quad (6.25)$$

with $|H|^2 = H^\dagger H$.

Here, the requirement for successful EWSB is that $\lambda > 0$ and $\mu^2 < 0$.¹⁸ The first of these requirements ensures that the potential is **bounded from below**, *i.e.* that in the limit of large field values the potential does not turn to negative infinity. The second ensures that the minimum of the potential, the vacuum, is not given by zero field values, *i.e.* that the fields have **vacuum expectation values** (vevs).

In supersymmetry we found the following general scalar potential in Eq. (4.21) for unbroken supersymmetry,

$$V(A, A^*) = \sum_i \left| \frac{\partial W}{\partial A_i} \right|^2 + \frac{1}{2} \sum_a g^2 (A^* T^a A)^2 > 0, \quad (6.26)$$

where the first part is due to the elimination of the auxiliary F -fields in the scalar superfields, while the second part is due to the elimination of the auxiliary D -fields in the vector superfields. In addition we have to add all terms containing only relevant scalar fields from the soft breaking terms in Eq. 5.9.

For the scalar Higgs component fields in the MSSM this gives the potential

$$\begin{aligned} V(H_u, H_d) &= |\mu|^2 (|H_u^0|^2 + |H_u^+|^2 + |H_d^0|^2 + |H_d^-|^2) && \text{(from } F\text{-terms)} \\ &+ \frac{1}{8} (g^2 + g'^2) (|H_u^0|^2 + |H_u^+|^2 - |H_d^0|^2 - |H_d^-|^2)^2 && \text{(from } D\text{-terms)} \\ &+ \frac{1}{2} g^2 |H_u^+ H_d^{0*} + H_u^0 H_d^{-*}|^2 \\ &+ m_{H_u}^2 (|H_u^0|^2 + |H_u^+|^2) + m_{H_d}^2 (|H_d^0|^2 + |H_d^-|^2) && \text{(from soft breaking terms)} \\ &+ [b(H_u^+ H_d^- - H_u^0 H_d^0) + \text{c.c.}] && (6.27) \end{aligned}$$

This potential has 8 d.o.f. from 4 complex scalar fields H_u^+ , H_u^0 , H_d^0 and H_d^- . At the same time it has 6 parameters: the two Standard Model couplings g and g' , the magnitude of the supersymmetric parameter μ , and the three soft-breaking parameters b , $m_{H_u}^2$ and $m_{H_d}^2$.

We now want to do as in the Standard Model and break $SU(2)_L \times U(1)_Y \rightarrow U(1)_{\text{em}}$ in order to give masses to gauge bosons and SM fermions.¹⁹ To do this we need to show that (6.27) has: i) a minimum for finite, *i.e.* non-zero, field values, ii) that this minimum has a remaining $U(1)_{\text{em}}$ symmetry and iii) that the potential is bounded from below. We will here restrict our analysis to tree level, ignoring loop effects on the potential.

We start by using our $SU(2)_L$ gauge freedom, picking a gauge so that we rotate away any field value for H_u^+ at the minimum of the potential. So without loss of generality we can use that $H_u^+ = 0$ at the minimum in what follows. At the minimum we must have $\partial V / \partial H_u^+ = 0$, and by explicit differentiation of the potential one can show that $H_u^+ = 0$ then also leads to $H_d^- = 0$. This is good and proper since it guarantees our item ii), that $U(1)_{\text{em}}$ is a symmetry for the minimum of the potential, since the charged fields then have no vevs.

¹⁸This is called the **Mexican hat** or **wine bottle potential**, depending on preferences.

¹⁹You may ask why we can not use the soft-terms from the spontaneous breaking of *supersymmetry* to do this. However, the soft-terms are unable to effectively provide masses to vector bosons and fermions because they deal (mostly) with scalar fields.

We are now left with the following potential only in terms of the uncharged Higgs fields H_u^0 and H_d^0 (after gauge choice and at the minimum):

$$V(H_u^0, H_d^0) = (|\mu|^2 + m_{H_u}^2)|H_u^0|^2 + (|\mu|^2 + m_{H_d}^2)|H_d^0|^2 + \frac{1}{8}(g^2 + g'^2)(|H_u^0|^2 - |H_d^0|^2)^2 - (bH_u^0 H_d^0 + \text{c.c.}) \quad (6.28)$$

We can now absorb a complex phase in H_u^0 or H_d^0 , in order to take b to be real and positive. This does not affect other terms because they are protected by absolute values. The minimum must also have the total phase of $H_u^0 H_d^0$ real and positive, to get an as large as possible negative contribution from the b term. Thus the vevs $v_u = \langle H_u^0 \rangle$ and $v_d = \langle H_d^0 \rangle$ must have opposite phases. By the remaining $U(1)_Y$ gauge symmetry of the potential, which is effectively a phase rotation, and the fact that H_u^0 and H_d^0 have opposite hypercharge, we can transform v_u and v_d so that they are real and have the same sign. For the potential to have a negative mass term, and thus fulfill point i) above, we must then have

$$b^2 > (|\mu|^2 + m_{H_u}^2)(|\mu|^2 + m_{H_d}^2). \quad (6.29)$$

Since we have broken supersymmetry we must also check that the potential is actually bounded from below, our point iii), which was guaranteed for a supersymmetric vacuum. For large $|H_u^0|$ or $|H_d^0|$ the quartic gauge terms in (6.28) blows up to save the potential, except for $|H_u^0| = |H_d^0|$, the so-called D -flat directions. This means that we must also require

$$2b < 2|\mu|^2 + m_{H_u}^2 + m_{H_d}^2. \quad (6.30)$$

To summarise what we have learnt so far: at the minimum of the Higgs potential we know that the expectation values of the charged Higgs component fields are zero, $\langle H_u^+ \rangle = 0$ and $\langle H_d^- \rangle = 0$, and we fulfil the condition for the existence of an extremal point in the neutral Higgs component fields

$$\frac{\partial V}{\partial H_u^0} = \frac{\partial V}{\partial H_d^0} = 0. \quad (6.31)$$

In addition, for the minimum to have non-zero field values that break EWSB, and for the potential to be bounded from below, the parameters of the potential must simultaneously fulfil the inequalities

$$\begin{aligned} b^2 &> (|\mu|^2 + m_{H_u}^2)(|\mu|^2 + m_{H_d}^2), \\ 2b &< 2|\mu|^2 + m_{H_u}^2 + m_{H_d}^2. \end{aligned}$$

The resulting non-zero expectation values at the minimum for the neutral Higgs component fields are denoted v_u and v_d .

To satisfy (6.29) and (6.30), a negative value for $m_{H_u}^2$ (or $m_{H_d}^2$) can help, in particular if $|\mu|^2 + m_{H_u}^2 < 0$, as that automatically fulfils (6.29). Such a negative value is indeed perfectly allowed as a parameter in the Lagrangian. No negative particle masses will result.

In fact, if we assume that $m_{H_d} = m_{H_u}$ at some scale μ , for example the supersymmetry breaking scale, then (6.29) and (6.30) cannot be simultaneously be satisfied *at that scale*. However, to 1-loop the RGE running of these mass parameters is:

$$16\pi^2 \beta_{m_{H_u}^2} \equiv 16\pi^2 \frac{dm_{H_u}^2}{dt} = 6|y_t|^2(m_{H_u}^2 + m_{Q_3}^2 + m_{u_3}^2) + \dots$$

$$16\pi^2\beta_{m_{H_d}^2} \equiv 16\pi^2 \frac{dm_{H_d}^2}{dt} = 6|y_b|^2(m_{H_d}^2 + m_{Q_3}^2 + m_{d_3}^2) + \dots,$$

where y_t and y_b are the top and bottom quark Yukawa couplings, and $m_{Q_3} = m_{33}^Q$, $m_{u_3} = m_{33}^u$, and $m_{d_3} = m_{33}^d$, in a simplification of our previous notation for the soft-masses in Sec. 6.6. Because $y_t \gg y_b$, $m_{H_u}^2$ runs much faster with scale than $m_{H_d}^2$. If the parameters start out the same at some high scale, say from some universal supersymmetry breaking effect, as we go down to the electroweak scale $m_{H_u}^2$ becomes significantly smaller than $m_{H_d}^2$, and may become negative, fulfilling the EWSB criteria. For an illustration, see Fig. 6.4, where the running of $\mu^2 + m_{H_u}^2$ and $\mu^2 + m_{H_d}^2$ is shown. It is this property of starting out from some universal Higgs parameters at a high scale, which then by RGE effects break electroweak symmetry at lower scales, that is termed **radiative EWSB** (REWSB). Thus, in the MSSM with universal soft terms at a high scale there is an explanation why EWSB happens, it is not put in by hand in the potential as it is in the Standard Model.

Following EWSB, to get the familiar vector boson masses measured by experiment, the vevs need to satisfy the constraint from the electroweak parameters:

$$v_u^2 + v_d^2 \equiv v^2 = \frac{2m_Z^2}{g^2 + g'^2} \approx (174 \text{ GeV})^2.$$

Thus we have one free parameter coming from the two Higgs vevs in the MSSM. We can write this as

$$\tan \beta \equiv \frac{v_u}{v_d},$$

where by convention $0 < \beta < \pi/2$, so that $0 < \tan \beta < \infty$.

Using the condition for the existence of an extremal point in (6.31), the two non-SM parameters b and $|\mu|$ can be eliminated as free parameters from the model, however, not the sign of μ . Alternatively, we can choose to eliminate $m_{H_u}^2$ and $m_{H_d}^2$. You can look at this as giving away the freedom of these parameters to the vevs, and then fixing one vev by the electroweak constraint, and using $\tan \beta$ for the other.

Let us make a little remark here on the parameter μ . We have what is called the μ **problem**. The soft terms all get their scale from some common mechanism at some common high energy scale, it is assumed, however, μ is a mass term in the superpotential (the only one in fact) and could *a priori* take *any* value, even M_P . Why is μ then of the order of the soft terms, which is what allows us to achieve REWSB?²⁰

6.10 Higgs boson properties

Of the eight d.o.f. in the scalar potential for the Higgs component fields three are Goldstone bosons that get eaten by Z and $W^\pm$ to give them masses. The remaining five d.o.f. form two neutral scalars h , H , two charged scalars $H^\pm$ and one neutral pseudo-scalar (CP-odd) A .²¹

²⁰This problem can be solved in extensions of the MSSM such as the Next-to-Minimal Supersymmetric Standard Model (NMSSM).

²¹In addition to the scalars, we know that the Higgs supermultiplets contain four fermions, $\tilde{H}_u^0$, $\tilde{H}_d^0$, $\tilde{H}_u^+$ and $\tilde{H}_d^-$ (higgsinos). We will see later that these mix with the fermion partners of the gauge bosons (gauginos).

At tree level one can show that these have the masses:

$$m_A^2 = \frac{2b}{\sin 2\beta} = 2|\mu|^2 + m_{H_u}^2 + m_{H_d}^2, \quad (6.32)$$

$$m_{h,H}^2 = \frac{1}{2} \left(m_A^2 + m_Z^2 \mp \sqrt{(m_A^2 - m_Z^2)^2 + 4m_Z^2 m_A^2 \sin^2 2\beta} \right), \quad (6.33)$$

$$m_{H^\pm}^2 = m_A^2 + m_W^2. \quad (6.34)$$

As a consequence m_A and $\tan \beta$ can be used to parametrise the masses of the Higgs sector (at tree level), and H , $H^\pm$ and A are in principle unbounded in mass since they grow as $b/\sin 2\beta$. However, at tree level the lightest Higgs boson is restricted to

$$m_h < m_Z |\cos 2\beta| < 91.2 \text{ GeV}. \quad (6.35)$$

In contrast we have the current best measurement of the Higgs boson mass of $m_h = 125.10 \pm 0.14 \text{ GeV}$, combining results from the LHC [13].

Fortunately, there are large loop-corrections or the MSSM would have been excluded already.²² Because of the size of the Yukawa couplings the largest corrections to the mass of the lightest Higgs comes from loops with top quarks and its supersymmetric partners, the scalar top quarks, or stops, $\tilde{t}_L$ and $\tilde{t}_R$. See Fig. 4.1 for the relevant Feynman diagrams. In the limit where the mass of the stop quarks are larger than the top, $m_{\tilde{t}_R}, m_{\tilde{t}_L} \gg m_t$, and with stop mass eigenstates close to the chiral eigenstates (more on this later), we get the dominant loop correction

$$\Delta m_h^2 = \frac{3}{4\pi^2} \cos^2 \alpha y_t^2 m_t^2 \ln \left(\frac{m_{\tilde{t}_L} m_{\tilde{t}_R}}{m_t^2} \right), \quad (6.36)$$

where α is a mixing angle for h and H with respect to the superfield component fields H_u^0 and H_d^0 , given by

$$\frac{\sin \alpha}{\sin \beta} = -\frac{m_H^2 + m_h^2}{m_H^2 - m_h^2}, \quad (6.37)$$

at tree level.

With this and other corrections the upper bound on the lightest Higgs boson mass is weaker:

$$m_h \leq 135 \text{ GeV},$$

assuming a common sparticle mass scale of around $m_{\text{SUSY}} \leq 1 \text{ TeV}$. Higher values for the sparticle masses give large fine-tuning and weaken the bound very little because of the logarithm in Eq. (6.36). The bound can be further weakened by adding extra field content to the MSSM, *e.g.* as in the NMSSM, but there is an upper perturbative limit of $m_h \approx 150 \text{ GeV}$.

It is very interesting to discuss what the Higgs discovery actually implies for low-energy supersymmetry. As can be seen from the above numbers it requires rather large squark masses even in the favourable scenario with $\tan \beta \gg 1$ where the tree level mass is $m_h \sim 90 \text{ GeV}$. A naive estimate from Eq. (6.36) gives $m_{\tilde{t}} > 1 \text{ TeV}$. However, this does not take into account possible negative contributions to the Higgs mass from heavy gauginos (fermions in the vector superfields), and possible increases in the stop contribution due to tuning of the mixing of the chiral eigenstates $\tilde{t}_L$ and $\tilde{t}_R$, in the mass eigenstates $\tilde{t}_1$ and $\tilde{t}_2$.

²²It is worth pointing out here that the MSSM, despite its many parameters, is a falsifiable theory. For example, had the Higgs boson mass been $\sim 15 \text{ GeV}$ higher, which is perfectly allowed in the Standard Model, the MSSM would have been excluded.

Since the lightest stop quark is expected to be the lightest squark in scenarios with universal soft masses at some high scale – the reasoning here is the large downward RGE running of m_{33}^Q from a common squark soft mass at some high scale due to the large top Yukawa coupling – the expected sparticle spectrum lies mostly above 1 TeV, with the possible exception of gauginos/higgsinos. This points to so-called **Split-SUSY** scenarios with heavy scalars and light gauginos, and a relatively large degree of fine-tuning. If one can live with this little hierarchy problem, it will explain why no signs of supersymmetry have been seen yet at the LHC.

If you are willing to accept fine-tuning of the stop mixing instead, or come up with a good reason for why the mixing should be just-so to give a maximal Higgs mass, you can keep fairly light stop quarks. With the addition of light higgsinos and a light gluino the model is then technically natural, these scenarios are called **Natural SUSY** and could be within the current or near future reach of the LHC. The problem with these models, as we shall see, is that the higgsinos are degenerate, and thus difficult to detect.

To do calculations with the Higgs bosons in the MSSM we need the Feynman rules that result from the relevant Lagrangian terms. Since these have been listed elsewhere we will not repeat them here, but recommend in particular the PhD-thesis of Peter Richardson [14], where they can be found in Appendix A.6, including all interactions with fermions and sfermions. These can also be found, together with all gauge and self-interactions, in the classic paper by Gunion and Haber [15]. Note that in this paper a complex Higgs singlet appears which can safely be ignored.

6.11 The gluino

The **gluino** is a colour octet Majorana fermion. As such it has nothing to mix with in the MSSM (even with RPV) and at tree level the mass is given by the soft term M_3 . The one complication for the gluino is that it is strongly interacting so $M_3(\mu)$ runs quickly with energy. It is useful to instead talk about the scale-independent **pole-mass**, *i.e.* the pole of the renormalised propagator, $m_{\tilde{g}}$. Including one loop effects due to gluon exchange and squark loops, see Fig. 6.5 in the $\overline{DR}$ scheme we get:

$$m_{\tilde{g}} = M_3(\mu) \left[1 + \frac{\alpha_s}{4\pi} \left(15 + 6 \ln \frac{\mu}{M_3} + \sum_{\text{all } \tilde{q}} A_{\tilde{q}} \right) \right],$$

where the squark loop contributions are

$$A_{\tilde{q}} = \int_0^1 dx \, x \ln \left(x \frac{m_{\tilde{q}}^2}{M_3^2} + (1-x) \frac{m_q^2}{M_3^2} - x(1-x) - i\epsilon \right).$$

Due to the 15-factor the correction can be significant (colour factor).

Complete Feynman rules for gluinos can be found in Appendix C of the classic MSSM reference paper of Haber & Kane [16]. A more comprehensible alternative may be Appendix A.3 from the PhD-thesis of M. Bolz [17]. This also provides a description of how to handle clashing fermion lines that can appear with Majorana fermions.