

Since the lightest stop quark is expected to be the lightest squark in scenarios with universal soft masses at some high scale – the reasoning here is the large downward RGE running of m_{33}^Q from a common squark soft mass at some high scale due to the large top Yukawa coupling – the expected sparticle spectrum lies mostly above 1 TeV, with the possible exception of gauginos/higgsinos. This points to so-called **Split-SUSY** scenarios with heavy scalars and light gauginos, and a relatively large degree of fine-tuning. If one can live with this little hierarchy problem, it will explain why no signs of supersymmetry have been seen yet at the LHC.

If you are willing to accept fine-tuning of the stop mixing instead, or come up with a good reason for why the mixing should be just-so to give a maximal Higgs mass, you can keep fairly light stop quarks. With the addition of light higgsinos and a light gluino the model is then technically natural, these scenarios are called **Natural SUSY** and could be within the current or near future reach of the LHC. The problem with these models, as we shall see, is that the higgsinos are degenerate, and thus difficult to detect.

To do calculations with the Higgs bosons in the MSSM we need the Feynman rules that result from the relevant Lagrangian terms. Since these have been listed elsewhere we will not repeat them here, but recommend in particular the PhD-thesis of Peter Richardson [14], where they can be found in Appendix A.6, including all interactions with fermions and sfermions. These can also be found, together with all gauge and self-interactions, in the classic paper by Gunion and Haber [15]. Note that in this paper a complex Higgs singlet appears which can safely be ignored.

6.11 The gluino

The fermion partner of the Standard Model gluon g is called the **gluino** $\tilde{g}$, and as the gluon it is a colour octet Majorana fermion. We usually talk about the gluino as being one particle, however, as an adjoint representation of $SU(3)$ there are actually eight (thus octet) distinct gluons, and we write $\tilde{g}^a$ when we want to make the distinction. As a colour octet it has nothing to mix with in the MSSM – this is still true even if we add R-parity violation – and at tree level the mass is given by the soft term M_3 . Since it lives in the same superfield as the massless gluon it would otherwise had zero mass.

The one complication for the gluino is that it is strongly interacting so $M_3(\mu)$ runs quickly with energy. It is useful to instead talk about the scale-independent **pole-mass**, *i.e.* the pole of the renormalised propagator, $m_{\tilde{g}}$. Including one loop effects due to gluon exchange and squark loops, see Fig. 6.5, in the $\overline{DR}$ renormalisation scheme we get²³

$$m_{\tilde{g}} \simeq M_3(\mu) \left[1 + \frac{\alpha_s}{4\pi} \left(15 + 6 \ln \frac{\mu}{M_3} + \sum_{\text{all } \tilde{q}} A_{\tilde{q}} \right) \right],$$

where the squark loop contributions are

$$A_{\tilde{q}} = \int_0^1 dx \, x \ln \left(x \frac{m_{\tilde{q}}^2}{M_3^2} + (1-x) \frac{m_q^2}{M_3^2} - x(1-x) - i\epsilon \right).$$

Due to the 15-factor the correction can be significant (colour factor).

²³Note that the right-hand side here is μ dependent since the expression is only to finite order.

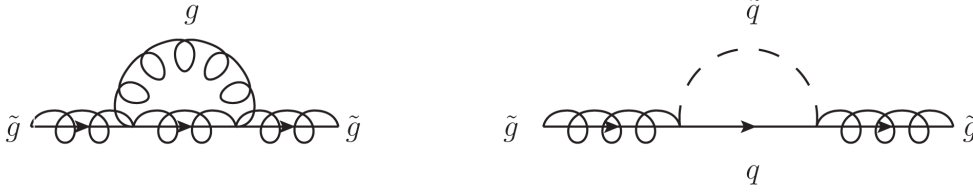


Figure 6.5: One loop contributions to the gluino mass.

Complete Feynman rules for gluinos can be found in Appendix C of the classic MSSM reference paper of Haber & Kane [16]. A more comprehensible alternative may be Appendix A.3 from the PhD-thesis of Bolz [17]. This thesis also provides a diagrammatic prescription of how to handle clashing fermion lines that can appear with Majorana fermions.

6.12 Neutralinos & Charginos

In the MSSM we have a bunch of fermion fields that can mix when electroweak symmetry is broken and we do not have to care about the $SU(2)_L \times U(1)_Y$ charges of the fields, only the charges under the remaining $U(1)_{\text{em}}$ symmetry matter. The candidates for mixing are the (Majorana) fermions from the $U(1)$ and $SU(2)$ vector superfields B^0 and W^a called the **gauginos**:

$$\tilde{B}^0 \text{ (bino)}, \quad \tilde{W}^0 \text{ (neutral wino)}, \quad \tilde{W}^\pm \text{ (charged wino)},$$

and the fermions from the Higgs superfields H_u and H_d , called **higgsinos**:

$$\tilde{H}_u^+, \quad \tilde{H}_u^0, \quad \tilde{H}_d^- \quad \text{and} \quad \tilde{H}_d^0.$$

Together these are called the **electroweakinos**.

In the Standard Model the neutral gauge field components B_μ^0 and W_μ^0 mix into the photon and the Z -boson. The neutral gauginos can mix the same way into the states

$$\tilde{\gamma} = N'_{11}\tilde{B}^0 + N'_{12}\tilde{W}^0 \quad \text{(photino)}, \quad (6.39)$$

$$\tilde{Z} = N'_{21}\tilde{B}^0 + N'_{22}\tilde{W}^0 \quad \text{(zino)}. \quad (6.40)$$

However, more generally, they also mix with the neutral higgsinos to form the four **neutralinos**²⁴

$$\tilde{\chi}_i^0 = N_{i1}\tilde{B}^0 + N_{i2}\tilde{W}^0 + N_{i3}\tilde{H}_d^0 + N_{i4}\tilde{H}_u^0, \quad i = 1, 2, 3, 4, \quad (6.41)$$

where N_{ij} indicates size of the component of each of the fields in the **gauge eigenstate basis**

$$\tilde{\psi}^{0T} = (\tilde{B}^0, \tilde{W}^0, \tilde{H}_d^0, \tilde{H}_u^0). \quad (6.42)$$

²⁴The neutral higgsinos are also Majorana fermions despite coming from scalar superfields. Unlike the (s)fermion superfields the Higgs superfields have no $\tilde{H}$ chiral partners to supply the left-right Weyl spinor combinations required for Dirac fermions. Thus the neutralinos are Majorana fermions.

In the gauge eigenstate basis the neutralino mass term can be written as

$$\mathcal{L}_{\chi-\text{mass}} = -\frac{1}{2}\tilde{\psi}^{0T} M_{\tilde{\chi}} \tilde{\psi}^0 + \text{c.c.},$$

where the mass matrix $M_{\tilde{\chi}}$ is found from the bilinear terms in the Lagrangian with gauge eigenstates to be

$$M_{\tilde{\chi}} = \begin{bmatrix} M_1 & 0 & -\frac{1}{\sqrt{2}}g'v_d & \frac{1}{\sqrt{2}}g'v_u \\ 0 & M_2 & \frac{1}{\sqrt{2}}gv_d & -\frac{1}{\sqrt{2}}gv_u \\ -\frac{1}{\sqrt{2}}g'v_d & \frac{1}{\sqrt{2}}gv_d & 0 & -\mu \\ \frac{1}{\sqrt{2}}g'v_u & -\frac{1}{\sqrt{2}}gv_u & -\mu & 0 \end{bmatrix}. \quad (6.43)$$

In this matrix, the upper left diagonal part comes from the soft terms for the $\tilde{B}^0$ and the $\tilde{W}^0$, the lower right off diagonal matrix comes from the superpotential term $\mu H_u H_d$, while the remaining entries come from Higgs-higgsino-gaugino terms from the kinetic part of the Lagrangian, *e.g.* $H_u^\dagger e^{\frac{1}{2}g\sigma W + g'B} H_u$, which become mass terms when one of the neutral Higgs component fields acquires a vev. With the Z -mass condition on the vevs (6.32) we can also write

$$\frac{1}{\sqrt{2}}g'v_d = \cos\beta \sin\theta_W m_Z, \quad (6.44)$$

$$\frac{1}{\sqrt{2}}g'v_u = \sin\beta \sin\theta_W m_Z, \quad (6.45)$$

$$\frac{1}{\sqrt{2}}gv_d = \cos\beta \cos\theta_W m_Z, \quad (6.46)$$

$$\frac{1}{\sqrt{2}}gv_u = \sin\beta \cos\theta_W m_Z. \quad (6.47)$$

The mass matrix can now be diagonalised to find the $\tilde{\chi}_i^0$ masses. If N is a unitary diagonalisation matrix for $M_{\tilde{\chi}}$, we can write

$$\mathcal{L}_{\chi-\text{mass}} = -\frac{1}{2}\tilde{\psi}^{0T} N^T N^* M_{\tilde{\chi}} N^\dagger N \tilde{\psi}^0 + \text{c.c.}, = -\frac{1}{2}\tilde{\chi}^{0T} D \tilde{\chi}^0 + \text{c.c.},$$

where $D = N^* M_{\tilde{\chi}} N^\dagger$ is diagonal and contains the neutralino masses $m_{\tilde{\chi}_i^0}$, and we see that N gives the mixing of the gauge eigenstates $\tilde{\psi}^0$ into the mass eigenstates $\tilde{\chi}^0$. We number the neutralino eigenstates in (6.41), or, equivalently, sort the mass eigenvalues after diagonalisation, so that the neutralinos are numbered from lightest to heaviest.

The mass parameters of the neutralino mass matrix may in general be complex, leading to complex mass eigenvalues. Redefinition of fields can rotate away either the M_1 or M_2 phase, to make the parameter real and positive, but not both and not the μ -phase. This gives rise to problematic CP-violation that can easily be in contradiction with experiments. Therefore, these are often just assumed to be real in order not to violate experimental bounds. However, we are in principle perfectly happy with negative or even complex mass eigenvalues here, as this is just a phase for the corresponding mass eigenstate in (6.41).

One particularly interesting solution to the diagonalisation is in the limit where EWSB is a small effect, $m_Z \ll |\mu \pm M_1|, |\mu \pm M_2|$, and when $M_1 < M_2 \ll |\mu|$. Then the lightest neutralino

is bino-like, $\tilde{\chi}_1^0 \approx \tilde{B}^0$, the next-to-lightest is wino like, $\tilde{\chi}_2^0 \approx \tilde{W}^0$, and $\tilde{\chi}_{3,4}^0 \approx \frac{1}{\sqrt{2}}(\tilde{H}_d^0 \pm \tilde{H}_u^0)$, and the masses are to first order in $1/\mu$:

$$m_{\tilde{\chi}_1^0} = M_1 + \frac{m_Z^2 \sin^2 \theta_W \sin 2\beta}{\mu} + \dots \quad (6.48)$$

$$m_{\tilde{\chi}_2^0} = M_2 - \frac{m_W^2 \sin 2\beta}{\mu} + \dots \quad (6.49)$$

$$m_{\tilde{\chi}_{3,4}^0} = |\mu| + \frac{m_Z^2}{2\mu} (\text{sgn } \mu \mp \sin 2\beta) + \dots \quad (6.50)$$

Since the LSP is stable in R-parity conserving theories the lightest neutralino is an excellent candidate for dark matter. In particular since a neutralino with mass around 100 GeV has a natural relic density close to the measured dark matter density of the Universe. We will return to this issue in Chapter 8.

From the charged electroweakinos we can make **charginos** $\tilde{\chi}_i^\pm$ that are Dirac fermions with mass term

$$\mathcal{L}_{\chi^\pm\text{-mass}} = -\frac{1}{2} \tilde{\psi}^{\pm T} M_{\chi^\pm} \tilde{\psi}^\pm + \text{c.c.},$$

where the gauge eigenstate basis is $\tilde{\psi}^{\pm T} = (\tilde{W}^+, \tilde{H}_u^+, \tilde{W}^-, \tilde{H}_d^-)$, and the mass matrix is given by

$$M_{\tilde{\chi}^\pm} = \begin{bmatrix} 0 & 0 & M_2 & gv_d \\ 0 & 0 & gv_u & \mu \\ M_2 & gv_u & 0 & 0 \\ gv_d & \mu & 0 & 0 \end{bmatrix}.$$

Here the M_2 terms come from the soft terms for the charged winos $\tilde{W}^\pm$, the μ terms come from the superpotential as above, while the remaining terms come from the kinetic terms. We can here re-write

$$gv_d = \sqrt{2} \cos \beta m_W, \quad (6.51)$$

$$gv_u = \sqrt{2} \sin \beta m_W. \quad (6.52)$$

Diagonalising this mass matrix gives the mass eigenstates $\tilde{\chi}_i^\pm$, $i = 1, 2$. The eigenvalues are doubly degenerate, giving the same masses to the $\tilde{\chi}_i^+$ and $\tilde{\chi}_i^-$ particle and anti-particle pairs, and are explicitly given as:

$$m_{\tilde{\chi}_{1,2}^\pm} = \frac{1}{2} \left(|M_2|^2 + |\mu|^2 + 2m_W^2 \mp \sqrt{(|M_2|^2 + |\mu|^2 + 2m_W^2)^2 - 4|\mu M_2 - m_W^2 \sin^2 \beta|^2} \right).$$

In the same limit of small EWSB effects discussed above we have $\tilde{\chi}_1^\pm \approx \tilde{W}^\pm$ and $\tilde{\chi}_2^\pm \approx \tilde{H}_u^+ / \tilde{H}_d^-$ with

$$m_{\tilde{\chi}_1^\pm} = M_2 - \frac{m_W^2}{\mu} \sin 2\beta + \dots, \quad (6.53)$$

$$m_{\tilde{\chi}_2^\pm} = |\mu| + \frac{m_W^2}{\mu} \text{sgn } \mu + \dots \quad (6.54)$$

Note that in this limit $m_{\tilde{\chi}_2^0} \simeq m_{\tilde{\chi}_1^\pm}$.

We saw earlier that the soft-mass ratio

$$M_3 : M_2 : M_1 \simeq 6 : 2 : 1,$$

appears at a scale of around 1 TeV if the same soft-masses unify at the GUT-scale. From our above discussion, as long as $|\mu| \gg M_1, M_2$, this gives the very predictive mass relationships $m_{\tilde{g}} \simeq 6m_{\tilde{\chi}_1^0}$, $m_{\tilde{\chi}_2^0} \simeq m_{\tilde{\chi}_1^\pm} \simeq 2m_{\tilde{\chi}_1^0}$. However, it is important to remember that this often used relationship is based on the *conjecture* of gauge coupling unification, and the unification of gaugino soft masses!

We should mention that some authors prefer other symbols for the neutralinos and charginos. Common examples are $\tilde{N}_i$ or $\tilde{Z}_i$ for neutralinos, and $\tilde{C}_i$ or $\tilde{W}_i$ (again!) for the charginos.

Feynman rules for charginos & neutralinos can again be found in Haber & Kane [16].

6.13 Sleptons & Squarks

The **sfermions**, the scalar partners of the Standard Model fermions, consists of the **squarks** and the **sleptons**. Reading of from the MSSM Lagrangian there are multiple tree-level contributions to the sfermion masses. In the following $\tilde{F}_i$ represents a $SU(2)_L$ doublet of sfermions with generation index i , for example $\tilde{Q}_i = (\tilde{u}_{iL}, \tilde{d}_{iL})$, while $\tilde{f}_{iR}$ represents a singlet. We make the following list:

- i) Under the reasonable assumption that soft masses are (close to) diagonal^[25] the sfermions get contributions $-m_F^2 \tilde{F}_i^\dagger \tilde{F}_i$ and $-m_f^2 \tilde{f}_{iR}^* \tilde{f}_{iR}$ from the soft terms.
- ii) There are F -term contributions that come from Yukawa terms in the superpotential of the form $y_f F H \tilde{K}$. From the contribution $\sum |W_i|^2$ to the scalar potential these give Lagrangian terms $y_f^2 H^{0*} H^0 \tilde{f}_{iL}^* \tilde{f}_{iL}$ and $y_f^2 H^{0*} H^0 \tilde{f}_{iR}^* \tilde{f}_{iR}$. After EWSB we then get the mass terms $m_f^2 \tilde{f}_{iL}^* \tilde{f}_{iL}$ and $m_f^2 \tilde{f}_{iR}^* \tilde{f}_{iR}$, where $m_f = v_{u/d} y_f$. These are only significant for large Yukawa coupling y_f , and give the same mass as their Standard Model fermion partner gets from the same Yukawa terms.
- iii) There are also so-called **hyperfine** terms that come from D -terms $\sum g^2 (A^* T^a A)^2$ in the scalar potential that give Lagrangian terms of the form $(\text{sfermion})^2 (\text{Higgs})^2$ when one of the scalar fields A is a neutral Higgs field, and the other is a sfermion. Under EWSB, when the Higgs field gets a vev these become mass terms. They contribute with a mass

$$\Delta_F = (T_3 F g^2 - Y_F g'^2)(v_d^2 - v_u^2) = (T_3 F - Q_F \sin^2 \theta_W) \cos 2\beta m_Z^2,$$

where the weak isospin, T_3 , hypercharge, Y , and electric charge, Q , are for the left-handed supermultiplet F to which the sfermion belongs. However, these contributions are usually quite small.

- iv) Furthermore, there are also F -terms that combine scalars from the $\mu H_u H_d$ term and Yukawa terms $y_f F H \tilde{K}$ in the superpotential. These give Lagrangian terms $-\mu^* H^{0*} y_f \tilde{f}_L \tilde{f}_R^*$. With a Higgs vev this gives mass terms $-\mu^* v_{u/d} y_f \tilde{f}_R^* \tilde{f}_L + \text{c.c.}$

²⁵This assumption is of course made to avoid flavour changing neutral currents (FCNCs). However, it is also reasonable in that if the soft masses are diagonal, or even all the same, at a high scale, the RGE running will not create large off-diagonal terms.

- v) Finally, the soft Yukawa terms of the form $a_f \tilde{F} H \tilde{f}_R^*$ with a Higgs vev give mass terms $a_f v_{u/d} \tilde{f}_L \tilde{f}_R^* + \text{c.c.}$ ^[26]

For the first two generations of sfermions, terms of type ii), iv) and v) are small due to small Yukawa couplings. Then the sfermion masses are for example

$$m_{\tilde{u}_L}^2 = m_{Q_1}^2 + \Delta \tilde{u}_L, \quad (6.55)$$

$$m_{\tilde{d}_L}^2 = m_{Q_1}^2 + \Delta \tilde{d}_L, \quad (6.56)$$

$$m_{\tilde{u}_R}^2 = m_{u_1}^2 + \Delta \tilde{u}_R. \quad (6.57)$$

Mass splitting between same generation slepton/squark is then given by the hyperfine splitting

$$m_{\tilde{e}_L}^2 - m_{\tilde{\nu}_{eL}}^2 = m_{\tilde{d}_L}^2 - m_{\tilde{u}_L}^2 = -\frac{1}{2}g^2(v_d^2 - v_u^2) = -\cos 2\beta m_W^2,$$

since they have the same hypercharge, see Table 6.1. For $\tan \beta > 1$ this gives the definite prediction $m_{\tilde{e}_L}^2 > m_{\tilde{\nu}_{eL}}^2$ and $m_{\tilde{d}_L}^2 > m_{\tilde{u}_L}^2$.

The **third generation sfermions** $\tilde{t}$, $\tilde{b}$ and $\tilde{\tau}$ have a more complicated mass matrix structure, *e.g.* in the gauge eigenstate basis $(\tilde{t}_L, \tilde{t}_R)$ for stop quarks the mass term is

$$\mathcal{L}_{\text{stop}} = -(\tilde{t}_L \quad \tilde{t}_R) m_{\tilde{t}}^2 \begin{pmatrix} \tilde{t}_L \\ \tilde{t}_R \end{pmatrix},$$

where the mass matrix is given by

$$m_{\tilde{t}}^2 = \begin{bmatrix} m_{Q_3}^2 + m_t^2 + \Delta \tilde{u}_L & v(a_t^* \sin \beta - \mu y_t \cos \beta) \\ v(a_t \sin \beta - \mu^* y_t \cos \beta) & m_{u_3}^2 + m_t^2 + \Delta \tilde{u}_R \end{bmatrix}. \quad (6.58)$$

Here the diagonal elements come from i), ii) and iii), while the off-diagonal elements come from iv) and v).

To find the particle masses, we must diagonalise this matrix, writing it in terms of the mass eigenstates $\tilde{t}_1$ and $\tilde{t}_2$, acquiring also here a unitary mixing matrix for the mass eigenstates in terms of the gauge eigenstates $\tilde{t}_L$ and $\tilde{t}_R$:

$$\begin{pmatrix} \tilde{t}_1 \\ \tilde{t}_2 \end{pmatrix} = \begin{bmatrix} c_{\tilde{t}} & -s_{\tilde{t}}^* \\ s_{\tilde{t}} & c_{\tilde{t}} \end{bmatrix} \begin{pmatrix} \tilde{t}_L \\ \tilde{t}_R \end{pmatrix}, \quad (6.59)$$

where the matrix entries are related by $|c_{\tilde{t}}|^2 + |s_{\tilde{t}}|^2 = 1$ and $m_{\tilde{t}_1}^2 < m_{\tilde{t}_2}^2$ are the eigenvalues of (6.58). The suggestive form of the mixing matrix indicates that if the off-diagonal elements of the original mass matrix has only real elements, this mixing matrix can be written as an element in $SO(2)$, using sine and cosine of a mixing angle $0 \leq \theta_{\tilde{t}} < \pi$, $c_{\tilde{t}} = \cos \theta_{\tilde{t}}$ and $s_{\tilde{t}} = \sin \theta_{\tilde{t}}$. The matrices for $\tilde{b}$ and $\tilde{\tau}$ have the same structure.

Since the third generation sneutrino $\tilde{\nu}_{eL}$ does not have a corresponding right-handed state in the MSSM, there is no mixing, and it has the same mass term as the first and second generation sneutrinos.

A good source for sfermion interaction Feynman rules is the PhD-thesis of Richardson [14].

²⁶We often assume that $a_f = A_0 y_f$ in order to further reduce the FCNC, meaning that there is a global constant A_0 with unit mass relating the Yukawa couplings and the trilinear A-term couplings.