

(instead of μ and b). This gives the soft masses

$$M_i = \frac{\alpha_i}{4\pi} \Lambda N_5, \quad (\text{gaugino masses}) \quad (7.5)$$

$$m_j^2 = 2\Lambda^2 N_5 \sum C(R)_i \left(\frac{\alpha_i}{4\pi} \right)^2, \quad (\text{scalar masses}) \quad (7.6)$$

where the sum is over the quadratic Casimir invariant for the scalar superfield Φ_j that the scalar field belongs to. We clearly see that the scalar soft-masses are a two-loop effect as discussed above.

While this parameterisation looks independent of M , the messenger scale sets the starting point of the RGE running of the sparticle masses, and thus influences their magnitude. For example, the tri-linear soft-term couplings a_{ijk} are expected to be very small at the messenger scale, and are effectively set to zero, however, due to RGE running they are small, but non-zero at the electroweak scale. Since the scalar masses m_j scale as $\sqrt{N_5}$ compared to N_5 for the gauginos, we in general expect the scalars to be lighter in GMSB models. One should also notice that this parameterisation gives the same hierarchy of gaugino masses as in mSUGRA, $M_3 > M_2 > M_1$, since (7.5) is ordered in terms of the strength of the gauge couplings α_i . The origin of the hierarchy is different since in mSUGRA it comes from the running of the parameters down from the GUT scale.

7.2 Supersymmetry at lepton colliders

High-energy lepton colliders have traditionally been circular e^+e^- -colliders. These have the advantage of a well known centre-of-mass (CoM) energy given by the total energy of the electron–positron pair, clean final states, and periodic application of the accelerating gradient. The challenge is to reach high energies and high luminosities (collision rates). Since the electrons are light they radiate a lot of bremsstrahlung photons when bent in orbits. The highest energy so-far at an e^+e^- -collider was the 209 GeV CoM-energy at LEP2 in 2000. Plans are being made both for linear e^+e^- -colliders, and a muon collider where there is less bremsstrahlung because of the higher muon mass, meaning that higher energies can be reached. However, there are significant technical challenges ahead for both. The linear colliders need very long installation tunnels and a very high accelerating gradient, while the muon collider must be able to produce and store the unstable muons.

Most supersymmetry searches at lepton colliders rely on the pair production of oppositely charged sparticles with electroweak couplings from $e^+e^- \rightarrow \gamma^*/Z^* \rightarrow S\bar{S}$, where S symbolises a generic sparticle. Due to R-parity conservation these sparticles both decay to lighter sparticles and Standard Model decay products, until they, potentially after several successive decays known as a **cascade decay**, leave only the stable LSP. This LSP must be electrically (and most likely colour) neutral due to experimental constraints on massive long-lived charged particles that would bind to atomic nuclei. The neutrality of the LSP means that it escapes detectors unseen.

The search for supersymmetry thus focuses on events with the Standard Model decay products of sparticles and an imbalance in momentum conservation due to the two missing LSPs. By the measured sum of the momenta of all the visible decay products the sum of the total momenta for the invisible particles can be inferred as going in the opposite direction. This is known as a **missing energy** signature.

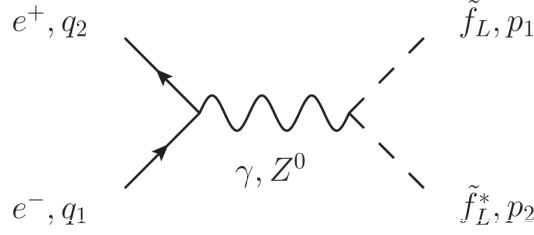


Figure 7.4: Feynman diagram for the pair production of left-handed sfermions in the s-channel at an e^+e^- -collider.

In practice such missing energy measurements are challenging experimentally due to the absence of detectors near the incoming beams in the longitudinal direction, where particles that are in principle visible may escape undetected and create an artificial momentum imbalance in the longitudinal direction. At high energies this is exacerbated by the increase of collinear bremsstrahlung from the incoming electrons at the interaction point due to interaction of the beams. This will be a particularly difficult for a future 0.5 – 3.0 TeV CoM International Linear Collider (ILC) or the Compact LInear Collider (CLIC) project.¹

We can now discuss more specifically the search for sfermions, neutralinos & charginos, and Higgs bosons at lepton colliders.

7.2.1 Sfermions

We can estimate the leading order amplitude of the s -channel sfermion pair production process shown in Fig. 7.4. We begin by writing down the matrix element with an intermediary γ as:

$$\mathcal{M} = \bar{v}ie\gamma^\mu u \frac{-ig_{\mu\nu}}{k^2 + i\epsilon} [-ie \cdot e_f(p_1 - p_2)^\nu], \quad (7.7)$$

which gives a squared matrix element of, assuming that the CoM s is much greater than m_Z and taking into account both the photon and the Z ,

$$|\mathcal{M}|^2 \simeq \frac{g^4 e_f^2}{8 \cos \theta_W} \frac{st + (m_{\tilde{f}}^2 - t)^2}{s^2} \times (1 + (4 \sin^2 \theta_W - 1)^2). \quad (7.8)$$

Here, we safely take $(1 + (4 \sin^2 \theta_W - 1)^2) \simeq 1$. The complete differential cross-section is then:

$$\frac{d\sigma}{dt} = \frac{1}{32\pi} \frac{1}{s^2} |\mathcal{M}|^2. \quad (7.9)$$

This cross section is relatively small due to the electroweak coupling factor g^4 and sfermion mass suppression, so sfermion production events will be rare. We show an example of the slepton pair production cross section including the Z -resonance, and the t -channel contributions from neutralinos (see below) in Fig. 7.5.

After production sfermions will decay, typically to a lighter neutralino or chargino and a Standard Model fermion f , in processes of the type $\tilde{f} \rightarrow f\tilde{\chi}_i^0$ or $\tilde{f} \rightarrow f'\tilde{\chi}_i^\pm$. Since the energy

¹For more information on these projects see the websites for the International Linear Collider <http://www.linearcollider.org/> and the Compact LInear Collider <https://clic.cern/>

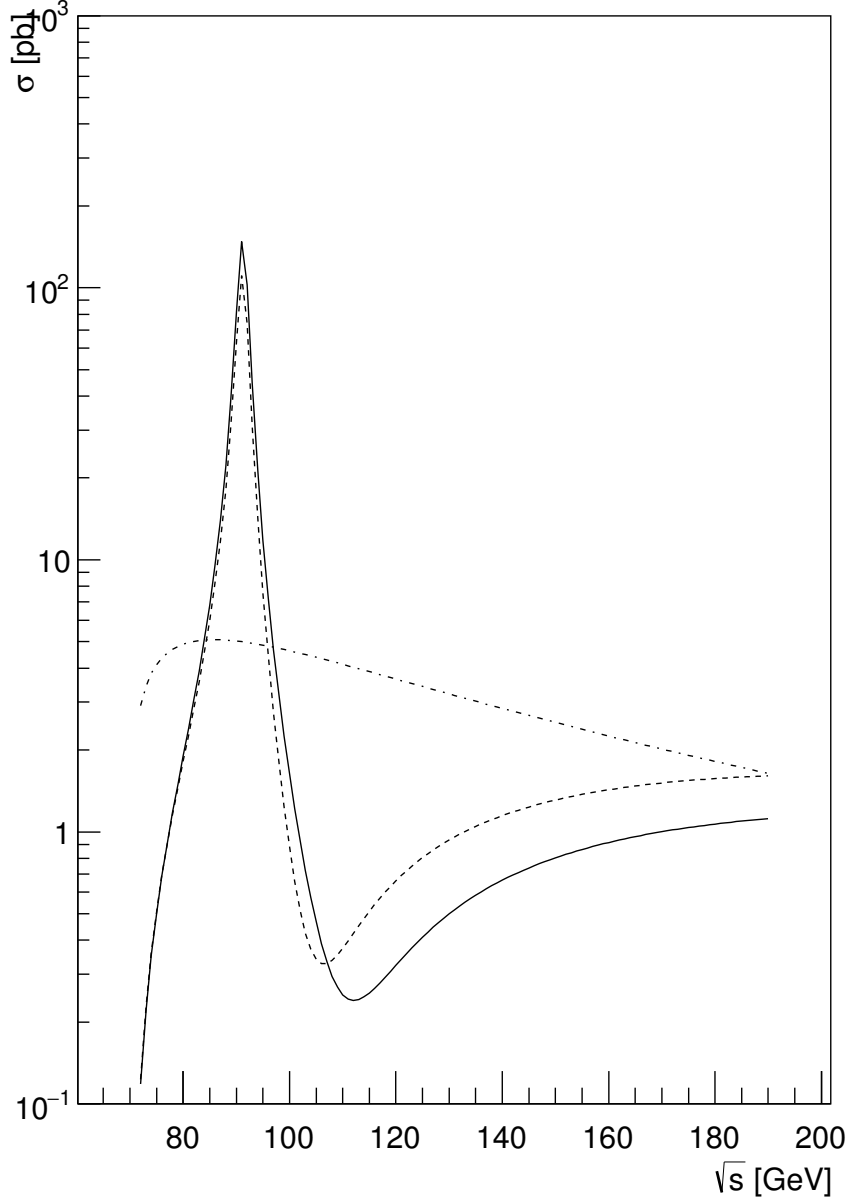


Figure 7.5: Cross sections for selectron pair production as a function of CoM energy. Cross sections for $\tilde{e}_L^* \tilde{e}_L$ (solid line), $\tilde{e}_R^* \tilde{e}_R$ (dashed line), and $\tilde{e}_L^* \tilde{e}_R$ (dashed dotted line) are shown separately. The particular model point has a common slepton mass of $m_{\tilde{e}_{L/R}} = 35$ GeV.

reach is limited, only the very lightest sfermions are likely to be producible, which are usually the sleptons, and these are likely to be near (but above) the mass of the lightest neutralino. See Fig. [6.4](#) for some expectations of the masses in an mSUGRA context. This means that

the most expected decay is $\tilde{\ell} \rightarrow \ell \tilde{\chi}_1^0$, and the total process is then $e^+e^- \rightarrow \tilde{\ell}^+\tilde{\ell}^- \rightarrow \ell^+\ell^- \tilde{\chi}_1^0\tilde{\chi}_1^0$, giving two oppositely charged Standard Model leptons and missing energy as the experimental signature.

Such a (weak) signal then needs to be discriminated from Standard Model backgrounds from for example W^+W^- pair production, which can lead to the final state $\ell^+\nu_\ell\ell^-\bar{\nu}_\ell$, featuring the same leptons and missing energy from the neutrinos. Since backgrounds are usually well under control at lepton colliders, the limit to searches is the ability to produce the sparticles at all. With a CoM energy $\sqrt{s}$ sparticles up to $\sqrt{s}/2$ are energetically possible, and a rule of thumb is that the reach approaches $\sqrt{s}/2$ from below given sufficient data, never quite getting to $\sqrt{s}/2$.

In the event of a statistically significant observation, the properties of the sparticles can be measured through the inferred cross section, and the kinematical distribution of the final state products. The ability of a lepton collider to easily change the CoM energy $\sqrt{s}$, allows for a so-called **threshold scan** of the cross section where the cross section is measured as a function of $\sqrt{s}$ around where it becomes zero. This in turn allows for a very precise measurement of the mass of the pair produced particle. As an example of a kinematic distribution, for the process discussed here, it can be shown that the energy distribution for the final state leptons is a uniform distribution between $E_{\min}$ and $E_{\max}$ where

$$E_{\max/\min} = \frac{\sqrt{s}}{4} \left(1 - \frac{m_{\tilde{\chi}_1^0}^2}{m_{\tilde{\ell}}^2} \right) \left(1 \pm \left(1 - \frac{4m_{\tilde{\ell}}^2}{s} \right)^{1/2} \right), \quad (7.10)$$

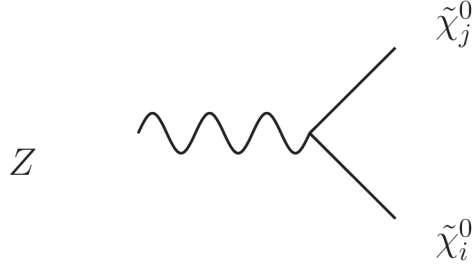
which, with a known slepton mass, also gives a handle on the LSP mass $m_{\tilde{\chi}_1^0}$ even though it is undetected.

7.2.2 Neutralinos & charginos

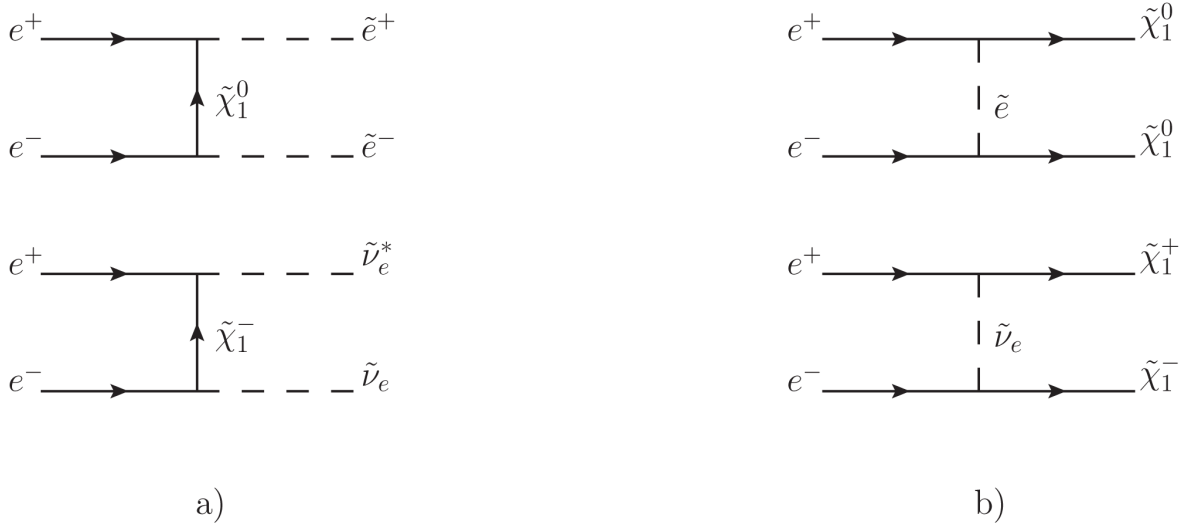
For charginos and neutralinos the production cross section depends on their wino, bino and higgsino components. You would be forgiven to think that pair production of the lightest neutralino $e^+e^- \rightarrow \tilde{\chi}_1^0\tilde{\chi}_1^0$ would be the natural sparticle to search for, however, this has some significant problems. Since it is usually the LSP it does not decay, and there is nothing in the event that can actually be measured. We cannot use the missing energy as that requires an *imbalance* in momentum. Given sufficiently hard (energetic) radiation from either the initial electron or positron, a single photon recoiling against missing energy could potentially be measured, and this, so-called **mono-photon search**, was indeed a search channel for dark matter production at LEP. However, for neutralino dark matter this does not work all that well for other reasons. The $Z\tilde{\chi}_i^0\tilde{\chi}_j^0$ vertex shown in Fig. 7.13 has the Feynman rule

$$\frac{ig}{2\cos\theta_W}\gamma^\mu \left[(N_{i3}N_{j3}^* - N_{i4}N_{j4}^*) P_L - (N_{i3}^*N_{j3} - N_{i4}^*N_{j4}) P_R \right], \quad (7.11)$$

which depends only on the higgsino components of the neutralinos, N_{i3} and N_{i4} . This can be understood from the fact that there are no ZZZ or $Z\gamma\gamma$ vertices in the Standard Model that can be supersymmetrised, only a Zhh vertex. For the photon there is no tree level coupling to the neutralinos at all since there are no direct couplings between the Higgs and the photon in the Standard Model. Thus, only neutralinos with significant higgsino components can be produced this way. To top it off, a light higgsino with a mass dominated by the μ parameter would have very similar values of N_{i3} and N_{i4} , thus canceling the coupling.

Figure 7.6: Vertex for $Z\tilde{\chi}_i^0\tilde{\chi}_j^0$.

The selectron and electron sneutrino have a special rôle for e^+e^- colliders due to the resulting t -channel diagrams. Figure 7.7 shows the t -channel diagrams that are important in pair production at a e^+e^- collider. Neutralino pair production with t -channel selectron exchange does not suffer from the same problems as neutralino pair production in the s -channel. However, the process depends on the selectron mass as $m_{\tilde{e}}^{-4}$ for large mass values and is rapidly suppressed.

Figure 7.7: The t -channel diagrams for pair production of a) selectrons and electron sneutrinos, and b) neutralinos and charginos.

The consequences of the above discussion is that it is the production of the second lightest neutralino and the lightest chargino that is typically searched for in the combinations $\tilde{\chi}_2^0\tilde{\chi}_1^0$, $\tilde{\chi}_2^0\tilde{\chi}_2^0$, and $\tilde{\chi}_1^+\tilde{\chi}_1^-$. These decay – unless there is a slepton with a mass intermediary to the produced particle and the LSP – to the LSP and a possibly off-shell vector boson, $\tilde{\chi}_2^0 \rightarrow Z^{(*)}\tilde{\chi}_1^0$ and $\tilde{\chi}_1^\pm \rightarrow W^{(*)}\tilde{\chi}_1^0$. Thus the expected experimental signature is the fermionic decay products of the vector bosons with additional missing energy from the LSPs.

7.2.3 Higgs bosons

Because of the very small Yukawa coupling of electrons, Higgs bosons are not expected to have significant direct production cross sections at a e^+e^- collider. The most realistic production channels are Higgs-strahlung where the Higgs boson is attached to a vector boson: $e^+e^- \rightarrow Zh$, $e^+e^- \rightarrow ZH$, $e^+e^- \rightarrow Ah$ and $e^+e^- \rightarrow AH$, charged Higgs pair production $e^+e^- \rightarrow H^+H^-$, and W^+W^- vector boson fusion into Higgs bosons $e^+e^- \rightarrow \nu_e \bar{\nu}_e h(H)$.

7.2.4 Current bounds at lepton colliders

The below bounds are all from the LEP (Large Electron Positron) collider, running from 1989 until 2000, which outdated all previous bounds with a top energy of $\sqrt{s} = 209$ GeV, recording an integrated luminosity (amount of data) of 233 pb^{-1} above 204 GeV. Results exist from all four LEP experiments ALEPH, DELPHI, L3 and OPAL.^[2] The numbers below are all taken from the 2014 PDG (Particle Data Group) review ^[27], but the conclusions from the LEP data has changed little since then.

While these bounds often come from pair-production of the relevant sparticles, and thus are less model dependent than the hadron collider bounds presented in the next section, there remains some model dependence in many results, which, unfortunately, is sometimes ignored in the literature. Complicating matters is a reliance by the LEP experiments on theoretical assumptions such as GUT-scale coupling and gaugino mass unification in many searches.

- **Selectron:** The strongest limit given is $m_{\tilde{e}_L} > 107$ GeV and $m_{\tilde{e}_R} > 73$ GeV (ALEPH 2002) in searches for acoplanar di-electrons.^[3] However, the limit is the result of a scan over MSSM parameter space simplified by assuming a common m_0 and $m_{1/2}$ at GUT scale. Interpreted in the even more constrained mSUGRA with $A_0 = 0$ the bounds are even stronger, 152 GeV and 95 GeV, respectively. In contrast, due to strict limits on the precisely measured Z -width, there is a fully model independent limit of $m_{\tilde{e}_{L/R}} > 40$ GeV.^[4]
- **Smuon:** $m_{\tilde{\mu}_R} > 94$ GeV (DELPHI 2003). The limit is obtained as in the MSSM scenario for the selectron.
- **Stau:** $m_{\tilde{\tau}_1} > 81.9$ GeV (DELPHI 2003) assuming exclusive $\tilde{\tau}_1 \rightarrow \tau \tilde{\chi}_1^0$ decays and $m_{\tilde{\tau}_1} - m_{\tilde{\chi}_1^0} > 15$ GeV.
- **Sneutrinos:** From LEP we have an indirect limit of $m_{\tilde{\nu}} > 94$ GeV (DELPHI 2003) in neutralino & slepton searches. This assumes $m_{\tilde{e}_R} - m_{\tilde{\chi}_1^0} > 10$ GeV. From the Z -width we can obtain the model independent limit $m_{\tilde{\nu}} > 44.7$ GeV.
- **Neutralino:** The limit $m_{\tilde{\chi}_1^0} > 46$ GeV (DELPHI 2003) has been derived from the direct searches for $\tilde{\chi}_1^0 \tilde{\chi}_2^0$ and $\tilde{\chi}_2^0 \tilde{\chi}_2^0$. This assumes gauge coupling unification and a common gaugino mass $m_{1/2}$ at GUT scale. Even in the Z -decays, the contribution to the width depends on the higgsino component of the lightest neutralino, so $m_{\tilde{\chi}_1^0} \simeq 0$ GeV is still in principle allowed ^[28].

²Most of which are silly acronyms of course.

³The observant reader will notice that two electrons are always in the same plane, however, when experimentalists say acoplanar, they mean not in one plane with the beam axis.

⁴Similar model independent limits around half the Z -mass exists for all sparticles that couple to the Z .

- Here we have $m_{\tilde{\chi}_1^\pm} \geq 94$ GeV (DELPHI 2003), assuming GUT scale universality of m_0 and $m_{1/2}$ and using multiple direct search channels from production of charginos, neutralinos and sleptons. It also assumes either no third generation sfermion mixing or $m_{\tilde{\chi}_1^\pm} - m_{\tilde{\chi}_1^0} > 6$ GeV. From the Z -width we can extract a strict limit of $m_{\tilde{\chi}_1^\pm} \geq 45$ GeV.

7.3 Supersymmetry at hadron colliders

Let us first point out some more or less obvious points.⁵

- 1) Hadron colliders collide quarks and gluons. This means that we get large cross sections only for QCD charged sparticles, *i.e.* squarks and gluinos, provided their masses are low enough.
- 2) As discussed earlier, with R-parity conservation (RPC) sparticles are produced in pairs and both decay to the LSP.
- 3) Illustrated in Fig. 7.8 these sparticles can decay to the LSP in many different, and potentially complicated, cascades. The possible decays for a particular MSSM model point called SPS1a is shown in Fig. 7.9. We should realize that many of these decays are hard to distinguish from ordinary SM (background) processes, or just undetectable.
- 4) Standard Model backgrounds have much, much bigger cross sections. Figure 7.10 shows the expected backgrounds and signals produced in different channels at the 14 TeV LHC for different particle masses.
- 5) R-parity conservation gives you **missing transverse energy** $\cancel{E}_T$ at hadron colliders due to the escaping LSPs, *i.e.* an imbalance in the directional sum of all energy deposits transverse to the beam direction. There is no longitudinal energy balance in a hadron collider because the energies of the colliding partons are not known.

The consequences of the above is that we search for events with jet activity—squarks/gluinos decaying to the LSP—and missing energy from two LSPs. One simple way to do this is to define the **effective mass**

$$M_{\text{eff}} = \sum p_T^{\text{jet}} + \cancel{E}_T, \quad (7.12)$$

and search for deviations from SM expectations. Figure 7.11 shows a simulation of such a supersymmetry signal at the LHC for a benchmark MSSM model called LHC Point 2. However, there are models where this is ineffective. Imagine a scenario where only the lightest stop $\tilde{t}_1$ is copiously produced. If $m_{\tilde{t}_1} - m_{\tilde{\chi}_1^0} < m_W$ then $\tilde{t}_1 \rightarrow c\tilde{\chi}_1^0$ or $\tilde{t}_1 \rightarrow b\nu\tilde{\chi}_1^0$ decays dominate, where all final state particles have low energy (p_T), so-called **soft particles**. This is very difficult to discover with standard techniques.

One alternative to jets and lots of missing energy is to look for leptons (and some small missing energy) from gaugino pair production and decays. Searching the lepton and missing energy channels is a very effective way to isolate any production of sparticles from SM backgrounds, but for setting bounds it is bad since the only model independent production is **Drell-Yan** production, *e.g.* $q\bar{q} \rightarrow (Z/\gamma)^* \rightarrow \tilde{\chi}_1^0\tilde{\chi}_2^0, \tilde{\chi}_1^+\tilde{\chi}_1^-, \tilde{l}_L^*\tilde{l}_L, \tilde{l}_R^*\tilde{l}_R$, and $q'\bar{q} \rightarrow W^* \rightarrow \tilde{\chi}_2^0\tilde{\chi}_1^\pm$, which all have low cross sections due to the smaller electroweak coupling and the smaller

⁵You might find these very obvious, they are, however, quite important and some theory people seem oblivious to them.