

We show the current limits assuming degenerate wino-like states $\tilde{\chi}_2^0$ and $\tilde{\chi}_1^\pm$ in Fig. 7.17. We can see that limits up to 1050 GeV can be set for $\tilde{\chi}_2^0$ and $\tilde{\chi}_1^\pm$ when the lightest neutralino is below around 200 GeV. When the bosons in the decay go off-shell we see that the sensitivity of the searches go down significantly. If we move away from the wino-production scenario the production cross sections drop, and so does the reach of the experiments in the mass plane.

An additional important complication appears if the lightest neutralino is wino ($M_2 < M_1, |\mu|$) or higgsino ($|\mu| < M_1, M_2$). Then the lightest neutralino and chargino, $\tilde{\chi}_1^0$ and $\tilde{\chi}_1^\pm$ (for higgsinos also $\tilde{\chi}_2^0$), are degenerate, and the crucial question becomes how degenerate they are. If the mass difference is small but above around 300 MeV, then the particles are very difficult to discover at all, despite potentially having very large cross sections if they are light. The decays of the produced $\tilde{\chi}_1^\pm$ (and for higgsinos $\tilde{\chi}_2^0$) into $\tilde{\chi}_1^0$ lead to very soft decay products, either leptons in $\tilde{\chi}_1^\pm \rightarrow \ell^\pm \nu_\ell \tilde{\chi}_1^0$ (and $\tilde{\chi}_2^0 \rightarrow \ell^+ \ell^- \tilde{\chi}_1^0$), or decays to pions (the lightest hadronic states) $\tilde{\chi}_1^\pm \rightarrow \pi^\pm \tilde{\chi}_1^0$ (and $\tilde{\chi}_2^0 \rightarrow \pi^0 \tilde{\chi}_1^0$), that are unobservable above the backgrounds of a hadron collider.

If instead the mass difference becomes of the order off or less than the pion mass the width of the chargino can become so small that the state is sufficiently long lived to decay somewhere inside the detector, and not, effectively, at the interaction point. This would lead to charged tracks from the charginos that disappear somewhere inside the detector. Such disappearing track signatures are readily observable, and in fact the limits on very degenerate neutralinos and charginos are stronger than the limits on only somewhat degenerate ones.

The consequences of this complication is that it is very difficult to exclude near massless wino- or higgsino-like charginos and neutralinos at the LHC. However, since the chargino mass is constrained by the Z -width measured at LEP, there is an absolute lower limit of 45 GeV for both in this scenario. To improve significantly on this limit one may have to wait for a new lepton collider.

7.5 Precision observables

A different way to search for the signs of supersymmetric particles is their indirect effect on very accurately measured Standard Model particle properties and processes, so-called **precision observables**, mostly through loop diagrams with sparticles. We will here discuss four different types of probes: the electroweak precision observables, the value of the anomalous magnetic moment of the muon $(g-2)_\mu$, the flavour changing neutral current (FCNC) process $b \rightarrow s\gamma$ and the very rare (and FCNC) process $B_s \rightarrow \mu\mu$.

7.5.1 Electroweak precision observables

When we talk about **electroweak precision observables**, we study particle properties such as M_W (or M_Z), Γ_W , Γ_Z , m_t and $\sin\theta_W$, as well as the Higgs mass m_h and the properties of the Higgs such as its couplings to all the other particles (gauge and Yukawa couplings) and its self-coupling.

Up to 2012 we could study all of these as *functions* of the unknown Higgs mass, looking for deviations that could be a sign of for example supersymmetry. We show a fit of the Higgs mass to all available electroweak data and direct exclusion bounds in Fig. 7.18, made by the Gfitter Collaboration in 2011 just before the LHC started taking significant amounts of data [26], a fit pretty much indicating that the most probable Standard Model Higgs mass

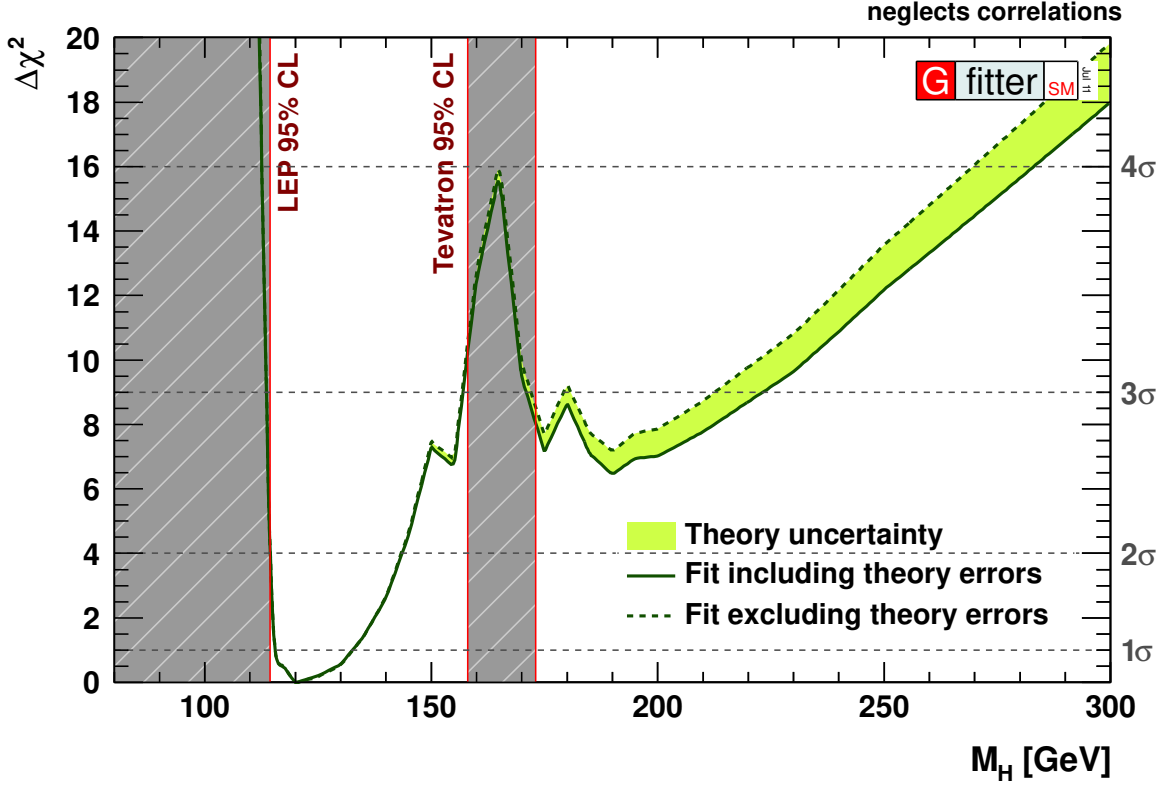


Figure 7.18: Plot of the total $\Delta\chi^2$ from all precision observable measurements and the direct exclusions bounds for the Standard Model Higgs set by LEP and the Tevatron, as a function of the Higgs mass [26].

was around 120 GeV. Here,

$$\Delta\chi^2 = \chi^2(m_h) - \min_{m_h} \chi^2(m_h),$$

is the difference in the χ^2 from the best fit point as a function of the Higgs mass, which has a standard interpretation in terms of σ s of significance for a one dimensional fit.

Figure 7.19 shows a similar plot for mSUGRA. At that time the absolute minimum of the fit, even taking into account the different number of parameters, gave a better fit for mSUGRA, $\min \chi^2_{\text{mSUGRA}} < \min \chi^2_{\text{SM}}$, but this changed quickly when the Higgs was found because of the position of the two minima.

Now all the parameters of the Standard Model – neutrinos excepted – have been determined to some precision. Thus the Standard Model is a completely constrained system. If we now do a electroweak fit the situation looks like that in Fig. 7.20, where we show the global fit to all measurements *except* the top and W masses, compared to the measured values of the W and top masses. Clearly what we are seeing here is (still?) consistent with the Standard Model.

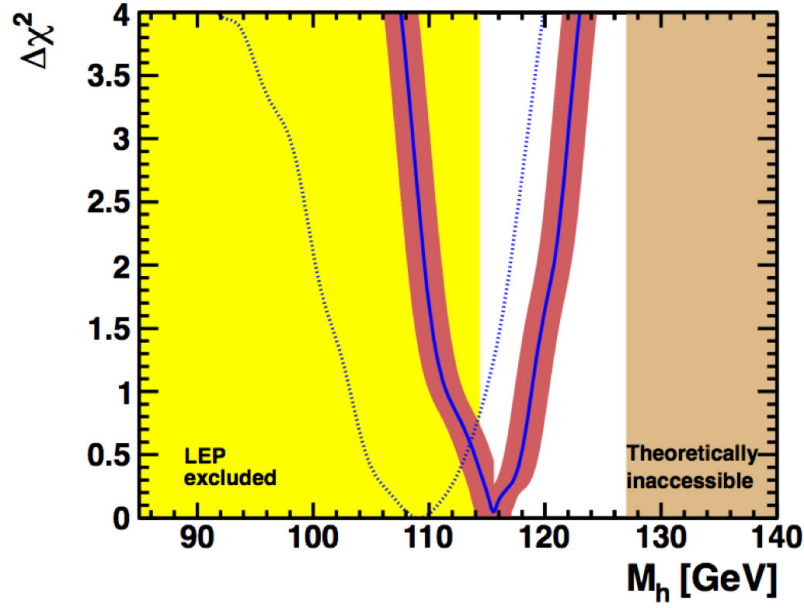


Figure 7.19: Plot of the $\Delta\chi^2$ from all precision observable measurements for mSUGRA as a function of the Higgs mass. The yellow area shows the area excluded by LEP searches (*not* included in the fit), while the brown shows the theoretically inaccessible area.

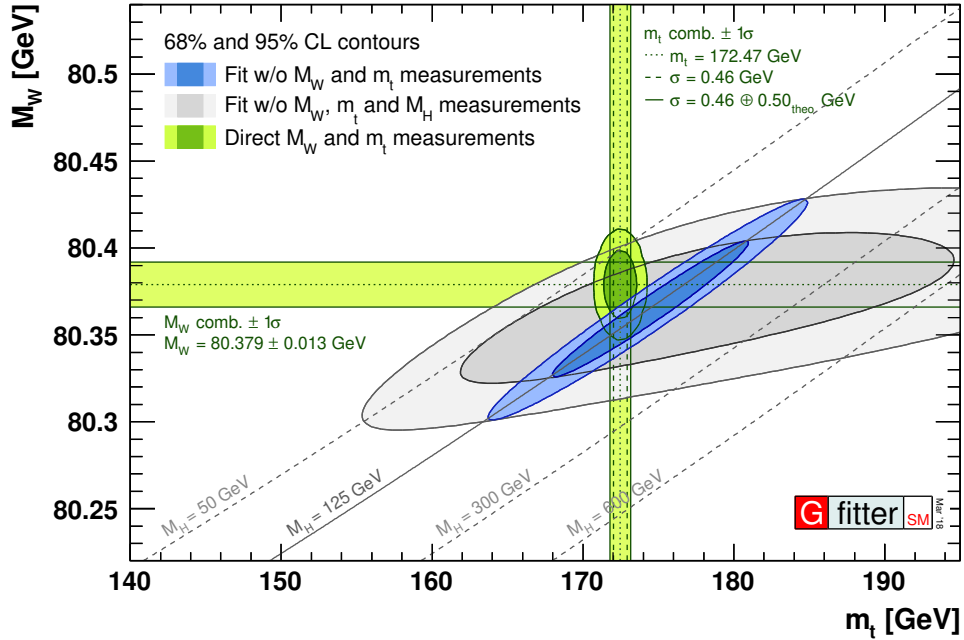


Figure 7.20: Electroweak fit excluding M_W and m_t (blue), compared to their measured values (green) [27].

7.5.2 $(g - 2)_\mu$

The **anomalous magnetic moment** of the muon, g_μ , in effect the interaction of a muon with an external electromagnetic field shown in Fig. 7.21 a), was calculated by Dirac in relativistic quantum mechanics as $g_\mu = 2$. However, in quantum field theory we have higher order corrections from loop-process such as shown in Fig. 7.21 b), that makes g_μ deviate from two.

It was very precisely measured by the E821 experiment at BNL [28] to be:

$$g_\mu^{\text{exp}} = 2.00116592089(63),$$

or, in terms of the deviation $a_\mu = g_\mu - 2$,

$$a_\mu^{\text{exp}} = 11659208.9(6.3) \cdot 10^{-10},$$

where the parenthesis indicates the uncertainty on the last digits.

In the Standard Model we find the prediction

$$a_\mu^{\text{SM}} = 11659183.0(5.1) \cdot 10^{-10},$$

giving a difference with respect to the experimental value of

$$\Delta a_\mu \equiv a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (25.9 \pm 8.1) \cdot 10^{-10},$$

a value which is 3.2σ away from zero. Recently, the Muon g-2 Collaboration at Fermilab made a new measurement [29] confirming this discrepancy with the value

$$\Delta a_\mu = 25.1 \pm 5.9 \cdot 10^{-10},$$

corresponding to 4.2σ . The Muon g-2 Collaboration is set to make further improvements on this number, expecting to reduce the uncertainty by a factor of 2 – 4.

However, we should be aware that some of the Standard Model contributions involve hadronic loops, *e.g.* the so-called **hadronic vacuum polarisation** shown in Figure 7.21 b), where one has to rely on experimental information from low energy $e^+e^- \rightarrow \gamma^* \rightarrow \text{hadrons}$ in order to estimate a contribution of $a_\mu^{\text{HVP}} = 10.5(2.6) \cdot 10^{-10}$, which is of the same order of magnitude as the discrepancy, and may be prone to errors in the interpretation.

In the MSSM we can have contributions to a_μ at the one-loop level, either by the exchange of one of the new Higgs bosons across the muon line, or by loops containing a smuon $\tilde{\mu}$ or muon-sneutrino $\tilde{\nu}_\mu$, together with a neutralino or chargino. The diagrams for the latter two processes are shown in Figure 7.21 c) and d). These contribute opposite sign terms $a_\mu(\tilde{\chi}^0)$ and $a_\mu(\tilde{\chi}^-)$. A thorough analysis shows that we need $\mu > 0$ in order to give a positive contribution that will close the gap between the experimental value and the prediction. In order to get a sufficiently large contribution the loop masses must be less than 500 – 600 GeV for $\tan \beta = 40 - 50$ and 200 – 300 GeV for $\tan \beta \simeq 10$. However, as we saw in Sec. 7.4 this is not implausible.

7.5.3 $b \rightarrow s\gamma$

The quark level process $b \rightarrow s\gamma$ is a Flavour Changing Neutral Current (FCNC) process which must proceed through loops in the Standard Model since there are no tree-level FCNC

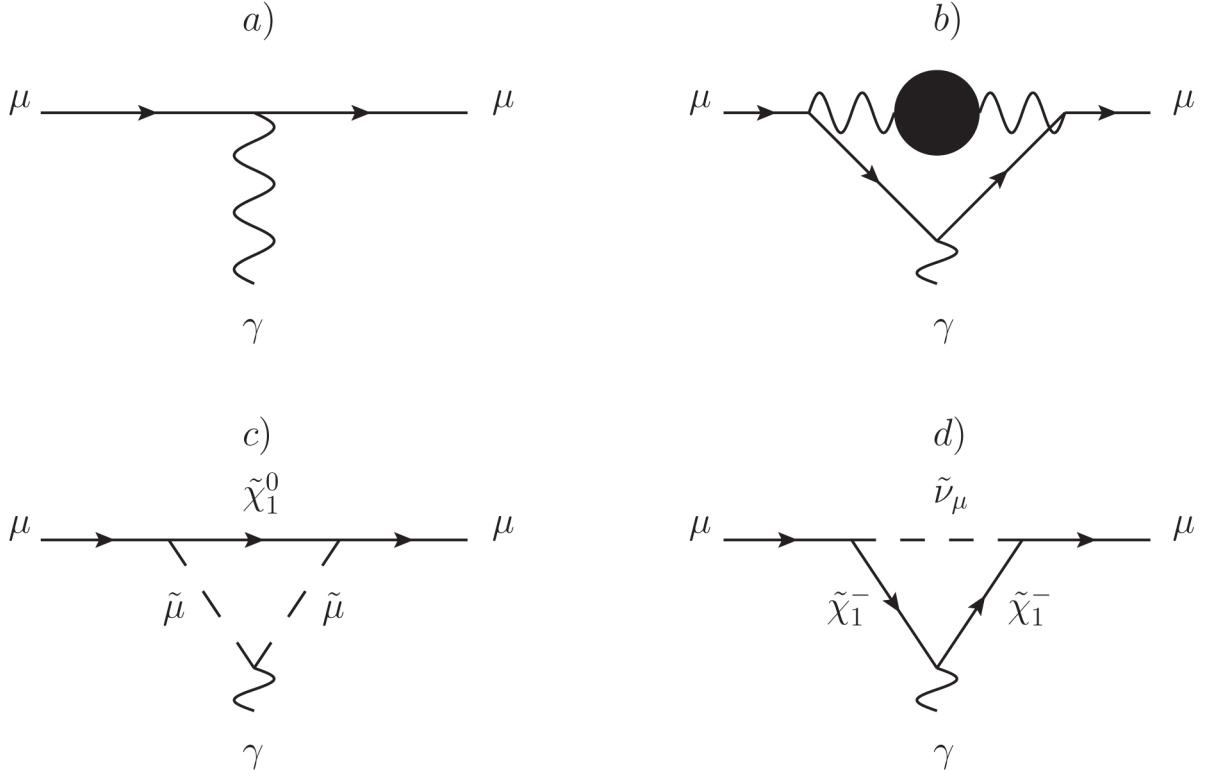


Figure 7.21: Diagrams for muon interaction with an electromagnetic field. Loop corrections to the tree level diagram a) give the value of a_μ . Diagram b) shows hadronic vacuum polarization where the blob contains QCD fields. Diagrams c) and d) show the lowest order MSSM contributions to a_μ .

interactions there. At meson level it leads to measurable decays of the type $B \rightarrow X_s \gamma$, where X_s is some meson with a strange quark, *e.g.* the decay $B \rightarrow K \gamma$. Figure 7.22 a) shows the one-loop Standard Model contribution with a virtual W and up-type quarks. This contribution is suppressed by the smallness of the CKM entries in the W -vertices which favours diagrams with a top quark in the loop, and the large masses, M_W and m_t , in the loop.

The process has been calculated at next-to-next-to-leading order (NNLO) in the Standard Model to be $\text{Br}(B \rightarrow X_s \gamma)_{\text{SM}} = (3.36 \pm 0.23) \cdot 10^{-4}$ for $E_\gamma \geq 1.6 \text{ GeV}$ [30, 31], inclusively summing over all meson final states.¹¹

Supersymmetric particles may contribute to this process, for example with off diagonal soft mass terms in diagrams such as Fig. 7.22 b) and d), where in b) a $m_{bs}^2 \tilde{b}^* \tilde{s}$ soft-breaking off-diagonal Lagrangian mass term changes a $\tilde{b}_1$ squark to a $\tilde{s}$ squark, and in d) where a $m_{tc}^2 \tilde{t}^* \tilde{c}$ term changes a $\tilde{t}_1$ to a $\tilde{c}$. Charged Higgs bosons with flavour non-diagonal couplings can also give important contributions in diagrams such as Fig. 7.22 c).

The main MSSM contributions are expected to come from the chargino–stop¹² and charged Higgs–top loops, as shown in Figs. 7.22 c) and d), respectively. However, there is little room

¹¹For the process $b \rightarrow d \gamma$ the Standard Model calculation yields $\text{Br}(B \rightarrow X_d \gamma) = 1.73_{-0.22}^{+0.12} \cdot 10^{-5}$.

¹²We usually expect higher generation off-diagonal soft terms to be larger due to RGE running controlled by Yukawa couplings.

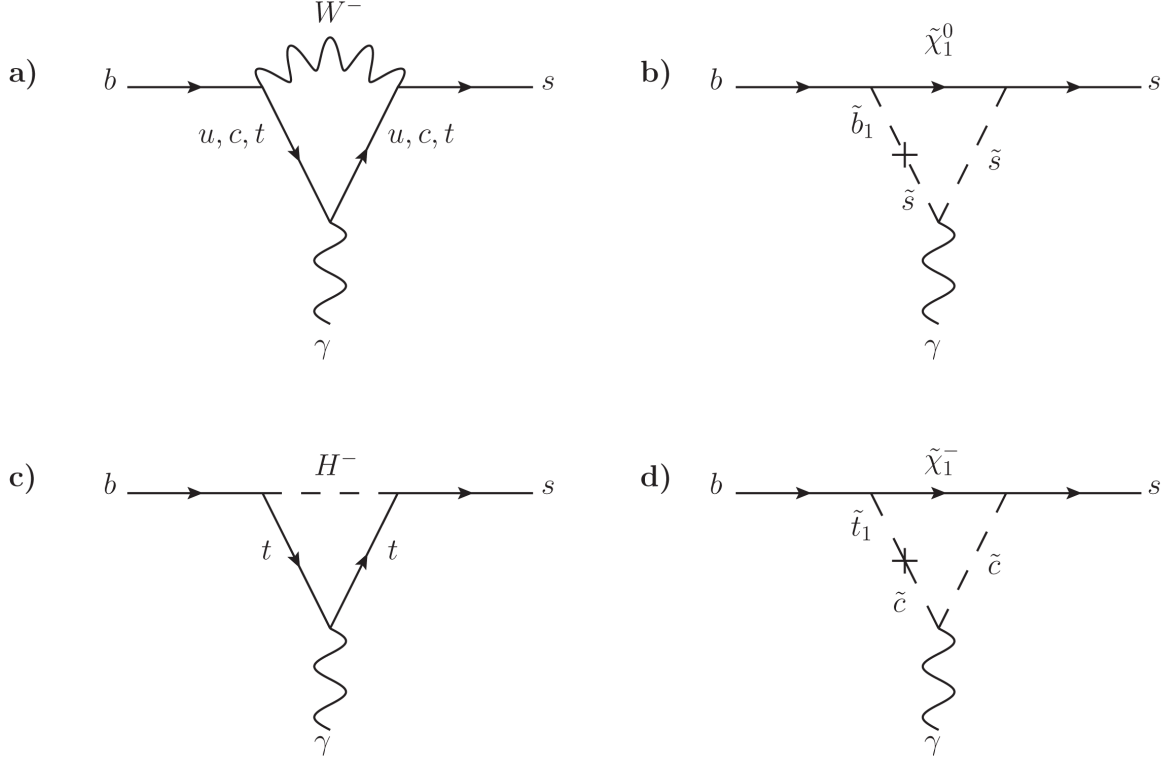


Figure 7.22: Diagrams for the process $b \rightarrow s\gamma$. a) shows the SM diagram while b), c) and d) show MSSM contributions.

for effects from supersymmetry since the current experimental world average is $\text{Br}(B \rightarrow X_s \gamma) = (3.32 \pm 0.15) \cdot 10^{-4}$ [32]. This means that either the charged Higgs is heavy enough and the stop-scharm soft mass term small enough not to contribute significantly, or that there are cancellations between the two contributions. This is both an example of a process that puts significant constraints on off-diagonal soft terms, and a process that is important in constraining the possible values of the charged Higgs mass.

7.5.4 $B_s \rightarrow \mu^+ \mu^-$

The process $B_s \rightarrow \mu^+ \mu^-$ is another FCNC process as either the bottom or the strange quark must change flavour in order to get a coupling to the muons. The Standard Model process is shown in Fig. 7.23 a), where the CKM factor in the W vertex allows one of the quarks to change flavour, but also suppresses the decay rate. For strange historical reasons involving darts and illegal substances this process is known as a **penguin diagram** [33].

On top of the small CKM factor, the process also suffers from what is called **helicity suppression** in the Standard Model. The Z -boson is spin-1, while the starting point meson B_s is spin-0 (pseudoscalar), meaning that the spins of the quarks in the meson are opposite. At some point in the diagram the helicity (chirality) must “flip”. This introduces an extra suppression proportional to $m_\mu^2/M_{B_s}^2$, making the expected rate extremely small and sensitive to supersymmetry contributions. We get a similarly suppressed process for B_d with a $\bar{d}$ -quark instead of the $\bar{s}$ in the initial state.

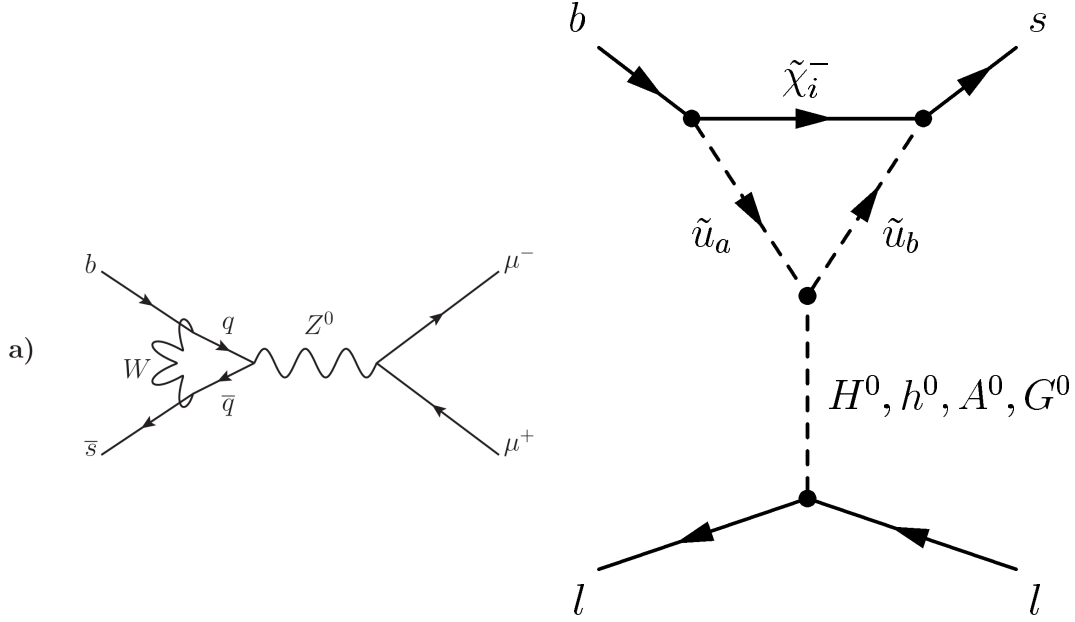


Figure 7.23: Diagrams for the process $B_s \rightarrow \mu^+ \mu^-$. Diagram a) shows one of the leading SM contributions, while b) shows one contribution from the MSSM taken from [34].

The predicted SM branching ratios for these processes are [35]:

$$\text{Br}(B_s \rightarrow \mu^+ \mu^-) = (3.65 \pm 0.23) \cdot 10^{-9}, \quad (7.14)$$

$$\text{Br}(B_d \rightarrow \mu^+ \mu^-) = (1.06 \pm 0.09) \cdot 10^{-10}. \quad (7.15)$$

First evidence for the $B_s \rightarrow \mu^+ \mu^-$ decay was shown by the LHCb Collaboration in 2012. The current value for the branching ratio from combined measurements at LHCb, ATLAS and CMS is [37]

$$\text{Br}(B_s \rightarrow \mu^+ \mu^-) = 2.69^{+0.37}_{-0.35} \cdot 10^{-9}, \quad (7.16)$$

$$\text{Br}(B_d \rightarrow \mu^+ \mu^-) < 1.6 \cdot 10^{-10}, \quad (7.17)$$

where the value for B_d is an upper limit at the 90% confidence level. It is interesting to observe that the measured value is lower than the Standard Model prediction at around the 2σ level, this has lead to speculations of destructive interference from new physics.

If we limit ourselves to scenarios with only diagonal soft terms the dominant contribution in the MSSM is from process such as shown in Fig. 7.23 b), with a chargino and up-type squark loop. These contributions are proportional to $\tan^6 \beta$, which makes the decay process highly sensitive to scenarios with large $\tan \beta$. To see this dependence, notice that μ couples to the mediating heavy higgses H/A^0 through the Yukawa term $y_{22}^l L_2 H_d \bar{E}_2$ in the superpotential, and the Yukawa constant in this term, $y_{22}^l = y_\mu$, is connected to the fermion mass through $m_\mu = y_\mu v \cos \beta$. Thus this vertex is proportional to $1/\cos \beta$ or $\tan \beta$, giving a factor $\tan^2 \beta$

in the amplitude squared.¹³

Furthermore, a chargino(higgsino)–stop loop can couple the strange and bottom quarks to the Higgs. These couplings are proportional to the bottom Yukawa coupling y_b , from the superpotential terms $y_{33}^d Q_3 H_d \bar{D}_3$, which appears in the stop–chargino–bottom vertex, and the $y_{32}^d Q_3 H_d \bar{D}_2$, which appears in the strange–chargino–stop vertex. Both these Yukawa couplings are proportional to y_b and thus to $1/\cos\beta$, giving a further factor of $\tan^4\beta$ in the amplitude squared. This $\tan\beta$ dependence makes $B_s \rightarrow \mu^+\mu^-$ an excellent channel for discovering supersymmetry, and puts very stringent bounds on the sparticle masses in large $\tan\beta$ scenarios.

7.6 Exercises

Exercise 7.1

From relativistic kinematics, show Eq. (7.13). *Hint:* the choice of rest frame is very important in order to simplify the calculation.

Exercise 7.2

Find the total cross section for the process $q\bar{q} \rightarrow \tilde{q}\tilde{q}^*$ via an s-channel gluon shown in Fig. 7.24.

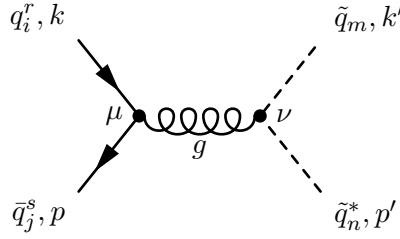


Figure 7.24: Strong production of a squark–anti-squark pair through a gluon.

¹³Remember that in the limit of large $\tan\beta$

$$\cos\beta = \pm \frac{1}{\sqrt{1 + \tan^2\beta}} = \pm \frac{1}{\tan\beta \sqrt{1 + \frac{1}{\tan^2\beta}}} \simeq \pm \frac{1}{\tan\beta}. \quad (7.18)$$