

Chapter 8

Supersymmetric dark matter

We have a standard model also for cosmology, the Lambda Cold Dark Matter model (Λ CDM). This models the observed universe on three main ingredients: the known Standard Model matter (dominantly the baryonic matter of the atoms, radiation and neutrinos), dark energy (Λ) and cold dark matter (CDM), where the cold signifies that this ingredient is non-relativistic. In this chapter we will look closer at candidates for dark matter (DM) in supersymmetric models.

8.1 Evidence for dark matter

The idea of dark matter goes back quite a long way. Today we have evidence for the existence of dark matter through several effects where we observe its gravitational influence on ordinary matter. We list the evidence below:

- 1) Kinematics (Zwicky 1933 [39]): The motion of galaxies (velocity dispersion) cannot be explained by the visible matter. This has also been observed on the scales of galaxies in their rotation curves (Rubin 1970 [40]).
- 2) Gravitational lensing (Tyson 1996 [41]). First observed in galactic clusters. Clusters show evidence of lensing not explained by luminous matter. Dark matter dynamics (non-interacting) are demonstrated by the Bullet cluster (Clowe 2006 [42]).
- 3) Large scale structures (clusters, superclusters, filaments and voids): The structures observed in the 2dFGRS (2-degree Field Galaxy Redshift survey Colles 2001 [43]) and the SDSS (Sloan Digital Sky Survey Tegmark 2004 [44]) imply a relative matter density of $\Omega_m \equiv \frac{\rho_m}{\rho_c} \simeq 0.29$ where $\rho_c = 1.05 \cdot 10^{-5} h^2 \text{ GeV/cm}^3$ is the critical energy density for a flat universe.¹ They also imply that the majority of DM must be **cold** (non-relativistic), because warm DM would suppress clustering.
- 4) Big-Bang Nucleosynthesis (BBN): The formation of light elements in the period $t = 1 - 1000 \text{ s}$ after the Big Bang. Measurements of Early Universe abundance of light elements, mainly D and He, points to a baryonic matter density of $\Omega_b \approx 0.04$. This gives $\Omega_{\text{leftover}} = \Omega_{\text{DM}} \approx 0.25$.

¹ h is defined through the Hubble constant H_0 as $H_0 = 100h \text{ km/Mpc/s}$.

- 5) Supernovae (Riess 1998 [45] and Perlmutter 1999 [46]): Measurements of type Ia supernovae (SNe Ia) were used as standard candles to show an accelerated expansion of the Universe. This fixes $\Omega_\Lambda - \Omega_m \simeq k$ where Ω_Λ is the energy density of dark energy/cosmological constant.
- 6) Cosmic Microwave Background (CMB) (Penzias & Wilson 1965 [47]): The temperature variation of the CMB over the sky of the order of 0.0002 K is sensitive to all cosmological parameters, and gives $\Omega_\Lambda + \Omega_m \simeq k$, where k is some constant.

The evidence above can be used to constrain the Λ CDM concordance model of cosmology, which has just a handful of ingredients such as baryonic and dark matter, radiation (photons) and dark energy. In Fig. 8.1 we show the effects of the SNe, CMB and large scale structure data (BAO) on this model in the plane of matter density Ω_m and dark energy density Ω_Λ .

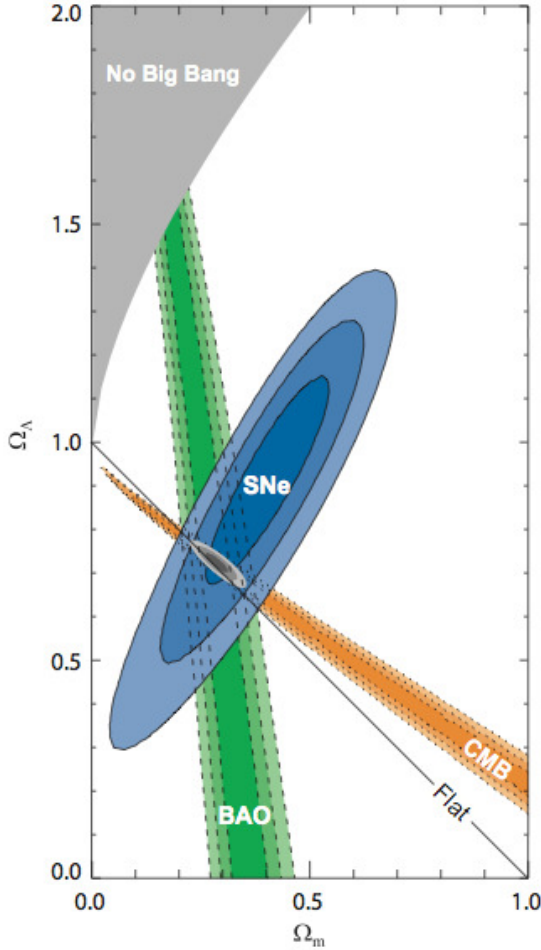


Figure 8.1: Limits from different observational sources on the dark energy density, Ω_Λ , compared to the total mass density in the universe, Ω_m .

A maximum likelihood fit to a selected subset of the measurements by the Planck Collaboration [48] gives the parameters for the model shown in Table 8.1. This means that the non-baryonic matter constitutes some 85% of the matter in the Universe, and that its relative energy density is quite well determined, at the percent level.

Parameter	Ω_Λ	$\Omega_m h^2$	$\Omega_b h^2$	H_0 [km/Mpc/s]	t_0 [Gy]
Value	$0.685^{+0.018}_{-0.016}$	0.1426 ± 0.0025	0.02205 ± 0.00028	67.3 ± 1.2	13.817 ± 0.048

Table 8.1: Measured values for cosmological parameters [48].

8.2 WIMP magic

The very existence of a stable Weakly² Interacting Massive Particle (WIMP) χ automatically gives an additional component to the total energy density of the Universe. WIMPs are found in a number of theories, for example the lightest neutralino of the MSSM, the lightest Kaluza-Klein particle of a theory with extra dimensions or an inert (no vev) Higgs boson.

This is due to the in equilibrium **thermal production** of the WIMP through the process $SM SM \rightarrow \chi\chi$, where SM are some Standard Model particles, and the reverse annihilation process $\chi\chi \rightarrow SM SM$, in the early hot Universe ($T \gg m_\chi$). As the temperature decreases to $T < m_\chi$ and there is not enough energy in an average collision for the production of χ to occur, only the reverse process can take place, and the comoving density³ falls with the temperature of the Universe.

The WIMPs then experience what is called a **chemical decoupling**, or loss of chemical equilibrium, due to the expansion of the Universe. This is when the WIMPs become so dilute, because of the expansion, that they in effect no longer interact inelastically, and this roughly happens when the expansion rate becomes larger than the rate of annihilation. The WIMPs then get a constant (comoving) density, we say that they experience a **freeze-out** at this temperature T_c . With weak-scale masses and couplings the freeze-out happens at $T_c \approx 0.05m_\chi$, however, this is before or at the same time as **kinetic decoupling** where the WIMPs effectively lose elastic interactions with the other matter, meaning that χ freezes-out with non-relativistic velocities and become cold dark matter.

The exact time (temperature) of freeze-out is controlled by the annihilation cross section of χ , larger cross sections keep chemical equilibrium for longer, in turn resulting in lower dark matter relic abundance. This abundance, in number density, can be found from the Boltzmann equation

$$\frac{dn_\chi}{dt} = -3Hn_\chi - \langle\sigma v\rangle(n_\chi^2 - n_\chi^{eq2}), \quad (8.1)$$

where n_χ^{eq} and n_χ are the chemical equilibrium and actual comoving number densities, H is Hubble's constant for the expansion rate, and $\langle\sigma v\rangle$ the velocity averaged annihilation cross section for $\chi\chi \rightarrow SM SM$. Solutions to this equation for increasing $\langle\sigma v\rangle$ are shown in Fig. [8.2].

In practice one must also often take into account co-annihilation with other particles with mass within 10%–20% of the χ , and numerical codes such as DarkSUSY [49] or MicrOMEGAs [50, 51] are typically used to solve the Boltzmann equation. For weak scale particles a rough approximation to the resulting dark matter density is

$$\Omega_\chi h^2 = 0.1 \times \frac{3 \times 10^{-26} \text{ cm}^3 \text{ s}^{-1}}{\langle\sigma v\rangle},$$

²Weak as in electro-weak, meaning on the same scale as the weak force.

³Taking the expansion of the universe into account by looking at the number of particles in a volume expanding at the same rate.

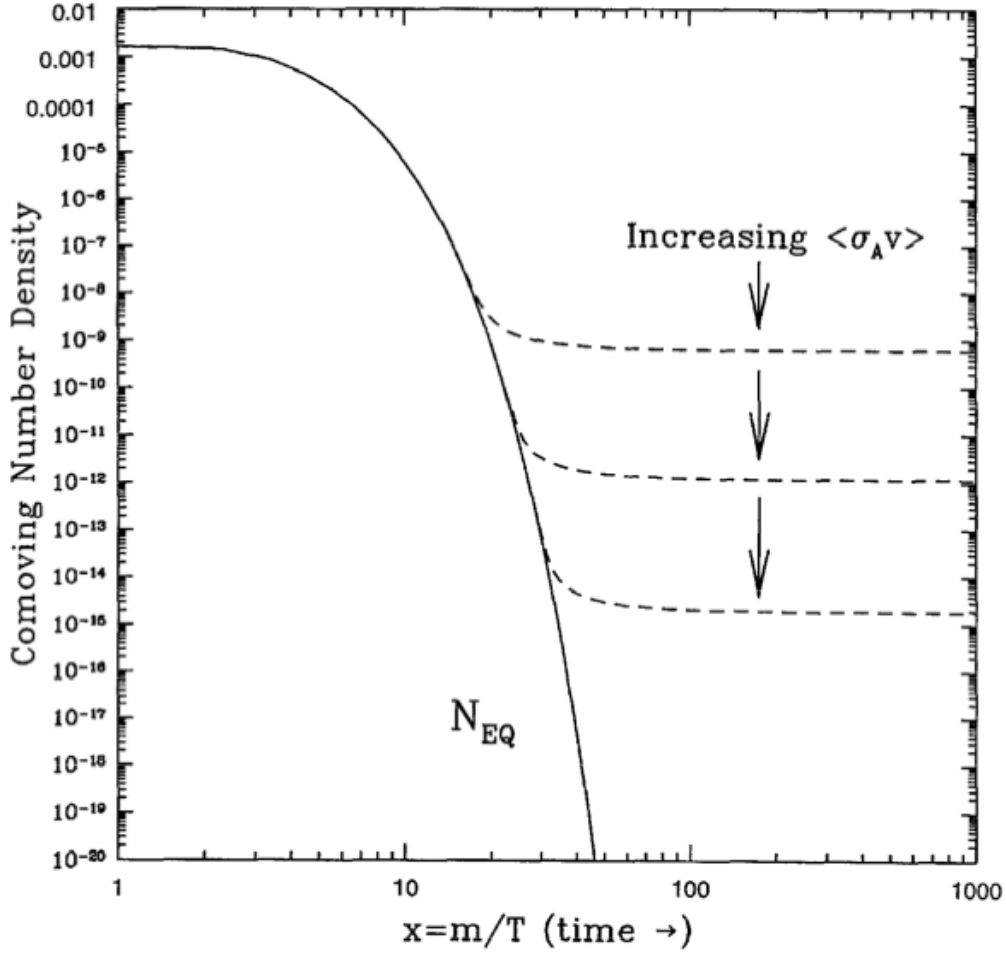


Figure 8.2: Illustration of the freeze-out of the comoving number density of a WIMP as a function of time, where the black line represents a model without chemical decoupling, and the dotted lines represent different freeze out temperatures for different velocity averaged annihilation cross sections.

and since the annihilation cross section can be shown to be

$$\langle\sigma v\rangle \approx \frac{\alpha_{weak}^2}{m_{weak}^2} \approx 10^{-25} \text{ cm}^3 \text{ s}^{-1}, \quad (8.2)$$

the predicted dark matter density is

$$\Omega_\chi h^2 \approx 0.1 \times \left(\frac{g_{weak}}{g_\chi} \right)^4 \left(\frac{m_\chi}{m_{weak}} \right)^2.$$

When compared to the values in Table [8.1](#) this is called the **WIMP-miracle** since it supplies just about the correct missing energy density for the WIMP as long as it has a weak scale mass and a weak scale coupling to the Standard Model. The strong sensitivity of the predicted dark

matter density to both the mass and the coupling, and the precisely measured experimental value, means that the WIMP scenario is very predictive and testable.

For a more detailed discussion of the WIMP miracle, see the standard cosmology book by Kolb and Turner [52].

8.3 Dark matter candidates in supersymmetry

With R-parity conservation in place we have seen that the LSP is stable and any neutral LSP can then in principle constitute all or part of the dark matter. Without R-parity only super-weakly coupling particles like gravitinos and axinos are candidates. Below we briefly discuss the various possibilities.

8.3.1 Neutralino

As soon as you have a stable neutralino LSP, you usually get into trouble trying to explain why there is so little dark matter. For example, the standard mSUGRA bino-like $\tilde{\chi}_1^0$ scenario prefers LSP masses around 100 GeV or lower not to overproduce dark matter. Due to current lower bounds on the $\tilde{\chi}_1^0$ mass and the measured Higgs mass this tends to be difficult to realise in practise. This scenario with a low mass bino-like neutralino is often called the **bulk region** scenario, which can be seen in Fig. 8.3 in the lower left corner.

Alternatives to the bulk region scenario use co-annihilation or resonant annihilation to increase $\langle\sigma v\rangle$ and thus decrease the dark matter density. The **stau-coannihilation region**, where $\tilde{\tau}_1\tilde{\chi}_1^0 \rightarrow SM SM$ is an efficient annihilation process reducing the dark matter density, exists for $m_{\tilde{\tau}_1} - m_{\tilde{\chi}_1^0} \leq 10$ GeV. This makes the scenario difficult to discover at collider experiments due to the production of soft (low-energy) taus. In mSUGRA the stau-coannihilation region can be found for small m_0 , and is shown as the lower strip in Fig. 8.3 which follows the lower theoretical bound (brown) where the stau becomes the LSP. Outside of mSUGRA similar regions can also be found for smuons and selectrons, with large mass degeneracy between the slepton and the LSP.

The **stop-coannihilation region**, where $\tilde{t}_1\tilde{\chi}_1^0 \rightarrow SM SM$ is efficient typically has $m_{\tilde{t}_1} - m_{\tilde{\chi}_1^0} \leq 25$ GeV. In mSUGRA this exists for large values of $|A_0|$, small m_0 and $m_{1/2}$. Again, this is difficult to discover because of the soft decay products of the stop.

The **Higgs funnel region** is found for $2m_{\tilde{\chi}_1^0} \simeq m_{A,H}$ and large $\tan\beta$, where the neutralino has resonant annihilation through a heavy Higgs boson. For mSUGRA, this is shown in Figure 8.3 as the two diagonal structures roughly in the middle of the plot, rising as a funnel upwards.

The **chargino-coannihilation** region can be found when we have higgsino or wino LSPs, where a chargino is automatically degenerate with the LSP resulting in a lower dark matter density typically below experimental bounds. In mSUGRA this region is called the **focus point region**, and is found for large m_0 and low μ , leading to so-called split-SUSY, as the sfermion masses need to be pressed up quite a bit, and there is a large mass difference between gauginos and sfermions. The focus point region can be seen in Fig. 8.3 following the upper theoretical bound where EWSB breaks down.

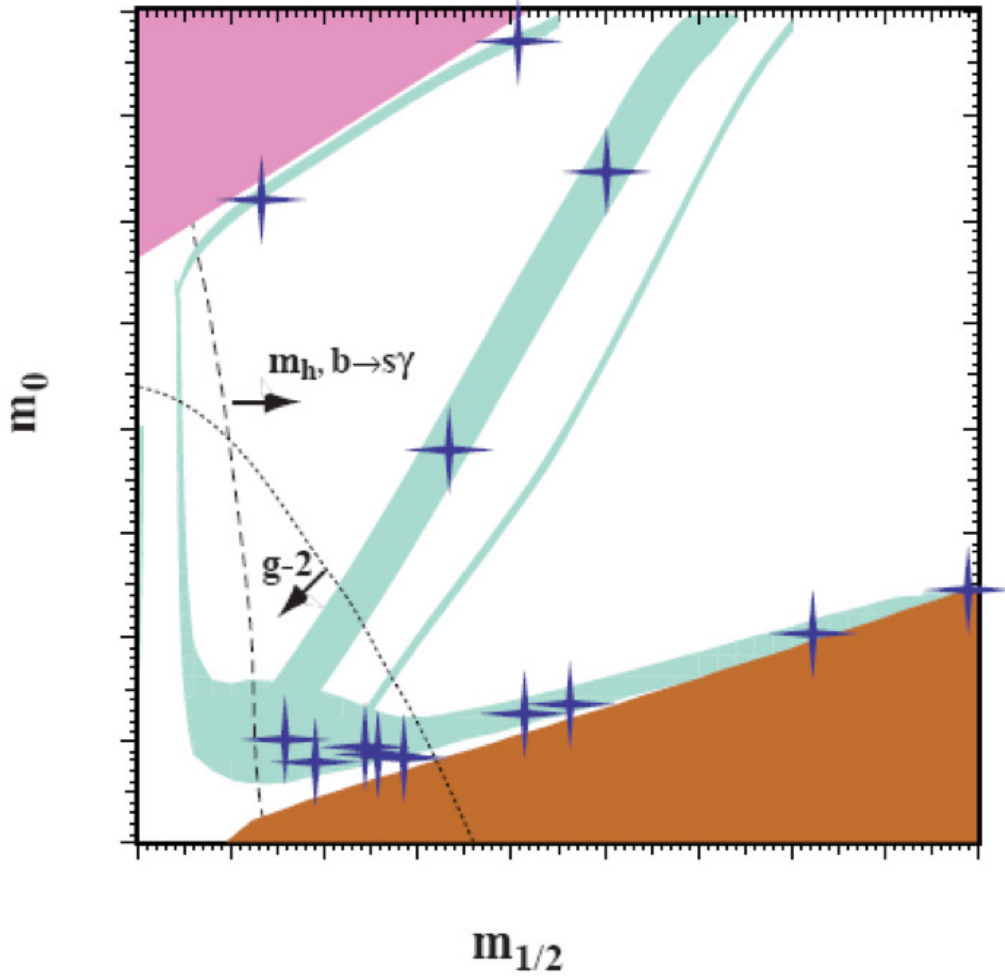


Figure 8.3: Generic illustration of the allowed neutralino dark matter regions (puke green) in the $(m_0, m_{1/2})$ -plane for mSUGRA. Except for the low m_0 and $m_{1/2}$ regions the area outside of the allowed green region gives too much dark matter. The dashed line shows the Higgs mass limit which pushes towards larger values of $m_{1/2}$, while the dotted line represents the limit from the anomalous magnetic moment of the muon.

8.3.2 Sneutrinos

The left handed sneutrino $\tilde{\nu}_L$ is happily excluded as a potential dark matter candidate due to the large cross section for $\tilde{\nu}_L q \rightarrow \tilde{\nu}_L q$ via Z -exchange.⁴ The large cross means that it should already have been seen by direct detection experiments, see Sec 8.4. It is also problematic to get $m_{\tilde{\nu}_L} < m_{\tilde{t}_L}$ due to hyperfine-splitting. However, the right-handed sneutrino $\tilde{\nu}_R$ couples very weakly and is still a viable candidate. This would necessitate adding right-handed

⁴For the neutralinos this problem only exists for a higgsino $\tilde{\chi}_1^0$ LSP, as the wino and bino do not couple to the Z .

neutrinos and sneutrino superfields to the MSSM.

8.3.3 Gravitino

The **gravitino** is the massive spin-3/2 supersymmetric partner of the graviton in models with supergravity. With only gravitational strength interactions, the gravitino is *not* a WIMP as it is never in chemical equilibrium. If it is the LSP it can be created from the decays of the next-to-lightest supersymmetric particle (NLSP), giving, in R-parity conserving scenarios,

$$\Omega_{\tilde{G}} = \frac{m_{\tilde{G}}}{m_{\text{NLSP}}} \Omega_{\text{NLSP}}.$$

However, these scenarios are problematic because the NLSP is long-lived due to the extremely weak interaction and creates potential trouble in BBN by injecting energy that changes the production of light elements.

Alternatively, the gravitino can be created in non-thermal production processes in the period of reheating after inflation. One potential process $gg \rightarrow \tilde{g}\tilde{G}$ is shown in Fig. 8.4. The reverse process $\tilde{g}\tilde{G} \rightarrow gg$ is not efficient as the density of gravitinos is never high enough given the small cross section. This type of dark matter creation process is often called **freeze-in**. For the gravitino this gives a new magic formula:

$$\Omega_{\tilde{G}} h^2 \approx 0.5 \cdot \left(\frac{T_R}{10^{10} \text{ GeV}} \right) \left(\frac{100 \text{ GeV}}{m_{\tilde{G}}} \right) \left(\frac{m_{\tilde{g}}}{1 \text{ TeV}} \right)^2, \quad (8.3)$$

where T_R is the unknown and very weakly constrained reheating temperature. This production mechanism is also valid for models with R-parity violation. There the small gravitino coupling $\propto \frac{1}{M_P}$ makes the gravitinos very long-lived, with lifetimes longer than the age of the Universe, but not absolutely stable because of the R-parity violating operators.

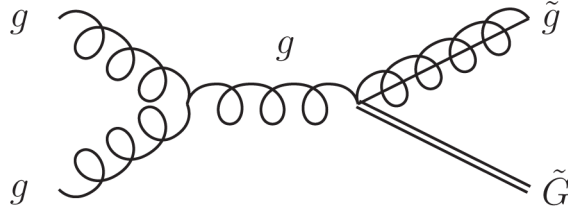


Figure 8.4: One possible diagram for the non-thermal production of gravitinos from the scattering of gluons.

8.3.4 Others

One could even imagine colour charged supersymmetric particles as dark matter, in particular the gluino, which, if stable, after hadronisation with Standard Model quarks and/or gluons would form so-called **R-hadrons**. These have very strict limits from searches, but these limits are somewhat obfuscated by complications in modelling R-hadron scattering.