The Standard Model

Gauge theories of Electroweak and Strong Interactions

- The Standard Model of Electroweak Interactions, A. Pich & corresponding lectures
- Modern Particle Physics, Thomson 2013

Content

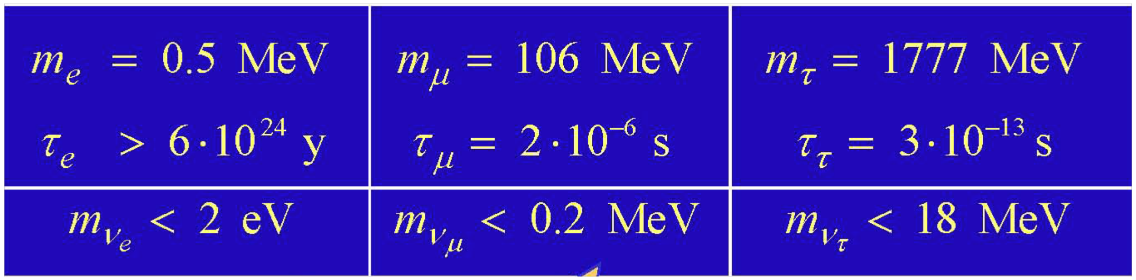
- ❑ Gauge invariance is a powerful tool to determine the dynamical forces among fundamental constituents of matter.
    - ❑ Particle content, structure and symmetries of the Standard Model Lagrangian
- ❑ Special emphasis given to phenomenological tests, established this theoretical framework as the Standard Theory of the electroweak and strong interactions:
    - ❑ electroweak precision tests, Higgs searches, quark mixing, neutrino oscillations.
- ❑ Present experimental status.

Introduction

- The Standard Model (SM) is a gauge theory, based on the symmetry group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$
  - describes strong, weak and electromagnetic interactions, via the exchange of the corresponding spin-1 gauge fields:
  - 8 massless gluons & 1 massless photon for strong and electromagnetic (EM) interactions, and 3 massive bosons, $W^\pm$ and Z^0, for the weak interaction.
- The fermionic matter content is given by the known leptons and quarks (and antiparticles), which are organized in a three-fold family structure, where each quark appears in 3 different colors:

$$\begin{bmatrix} \upsilon_e & u \\ e^- & d' \end{bmatrix}, \begin{bmatrix} \upsilon_\mu & c \\ \mu^- & s' \end{bmatrix}, \begin{bmatrix} \upsilon_\tau & t \\ \tau^- & b' \end{bmatrix}$$

$$\begin{bmatrix} \upsilon_l & q_u \\ l^- & q_d \end{bmatrix} = \begin{pmatrix} \upsilon_l \\ l^- \end{pmatrix}_L, \begin{pmatrix} q_u \\ q_d \end{pmatrix}_L, l_R^-, q_{uR}, q_{dR}$$



$m_e = 0.5\ \text{MeV}$	$m_\mu = 106\ \text{MeV}$	$m_\tau = 1777\ \text{MeV}$
$\tau_e > 6 \cdot 10^{24}\ \text{y}$	$\tau_\mu = 2 \cdot 10^{-6}\ \text{s}$	$\tau_\tau = 3 \cdot 10^{-13}\ \text{s}$
$m_{\nu_e} < 2\ \text{eV}$	$m_{\nu_\mu} < 0.2\ \text{MeV}$	$m_{\nu_\tau} < 18\ \text{MeV}$

  - The three fermionic families appear to have identical properties (gauge interactions); they differ only by their mass and their flavor quantum number.

- Gauge symmetry broken by vacuum, triggering Spontaneous Symmetry Breaking (SSB) of electroweak (EW) group to the EM subgroup:

$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \xrightarrow{SSB} SU(3)_C \otimes U(1)_{QED}$$

- SSB mechanism generates masses of weak gauge bosons, gives rise to appearance of a physical scalar particle – Higgs boson
  - Fermion masses and mixings also generated through same formalism
- SM constitutes one of the most successful achievements in modern physics
  - provides elegant theoretical framework, able to describe known experimental facts in particle physics with high precision
- To be discussed in details
  - Power of gauge principle and derivation of simpler Lagrangians of QED and QCD
  - Electroweak theoretical framework – gauge structure and SSB mechanism
  - Present phenomenological status – main precision tests performed at Z peak, tight constraints on Higgs mass from direct search
  - Flavour structure – quark mixing angles & neutrino oscillation parameters, importance of CP violation tests
  - Open questions to be investigated at future facilities
  - Useful, more technical information collected in several appendices: a minimal amount of quantum field theory concepts in Appendix A; most important algebraic properties of SU(N) matrices in App. B, short discussion on gauge anomalies in App. C

Basic Inputs from Quantum Field Theory

Wave equations – Quantum Mechanics (QM)

- ☐ Classical Hamiltonian of non-relativistic free particle $H = \frac{\vec{p}^2}{2m}$
- ☐ In QM, energy and momentum correspond to operators acting on particle wave function
  - ☐ Substitutions $H = i\hbar\frac{\partial}{\partial t}$ and $\vec{p} = i\hbar\vec{\nabla}$ lead to Schrödinger equation

$$\boxed{i\hbar\frac{\partial}{\partial t}\psi(\vec{x},t) = -\frac{\hbar^2}{2m}\vec{\nabla}^2\psi(\vec{x},t)}$$

  - ☐ relativistic covariant way: $p^\mu = i\partial^\mu \equiv i\frac{\partial}{\partial x_\mu}$
- ☐ $E^2 = \vec{p}^2 + m^2$ leads to Klein-Gordon equation, $\boxed{(\blacksquare + m^2)\phi(x) = 0}$ $\blacksquare \equiv \partial^\mu\partial_\mu = \frac{\partial^2}{\partial t^2} - \vec{\nabla}^2$
  - ☐ quadratic in time derivative because relativity puts space & time coordinates on equal footing
- ☐ Equation linear in both derivatives? Yes, Dirac equation
  - ☐ Relativistic covariance and dimensional analysis restrict its possible form to

$$\boxed{(i\gamma^\mu\partial_\mu - m)\psi(x) = 0}$$

- Dirac eq. solutions should satisfy KG relation

$$\boxed{(i\gamma^\mu\partial_\mu - m)\psi(x) = 0} \quad \Rightarrow \quad \boxed{(\blacksquare + m^2)\phi(x) = 0}$$

$$\boxed{-(i\gamma^\nu\partial_\nu + m)\big(i\gamma^\mu\partial_\mu - m\big)\psi(x) = 0 \equiv (\blacksquare + m^2)\psi(x)}$$

  - OK, provided gamma-coefficients satisfy Dirac algebra: $\{\gamma^\mu, \gamma^\nu\} \equiv \gamma^\mu\gamma^\nu + \gamma^\nu\,\gamma^\mu = 2g^{\mu\nu}$
    - Obviously components 4-vector γ^μ cannot simply be numbers.
    - 3 Pauli matrices satisfy $\{\sigma^i, \sigma^j\} = 2\,\delta^{ij}$
    - Lowest-dimensional solution to Dirac algebra: D = 4 matrices

$$\gamma^0 = \begin{pmatrix} I_2 & 0 \\ 0 & -I_2 \end{pmatrix} \;\; ; \;\; \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}$$

  - Wave function $\psi(x)$, column vector with 4 components in Dirac space
    - presence of the Pauli matrices strongly suggests it contains 2 components of spin ½
    - proper physical analysis of solutions: Dirac eq. describes simultaneously spin ½ fermion of and own antiparticle
  - Useful combinations of gamma matrices

$$\sigma^{\mu\nu} \equiv [\gamma^\mu, \gamma^\nu]$$

$$\gamma_5 \equiv \gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\,\gamma^3 = \frac{i}{4!}\varepsilon_{\mu\nu\rho\sigma}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma$$

$$\sigma^{ij} = \varepsilon^{ijk} = \begin{pmatrix} \sigma^k & 0 \\ 0 & \sigma^k \end{pmatrix} \;\; ; \;\; \sigma^{0i} = i\begin{pmatrix} 0 & \sigma^i \\ \sigma^i & 0 \end{pmatrix}. \quad ; \qquad \gamma_5 = i\begin{pmatrix} 0 & I_2 \\ I_2 & 0 \end{pmatrix}$$

    - matrix σ^{ij} is then related to the spin operator

- Some important properties of gamma-matrices $\{\gamma^\mu, \gamma^\nu\} \equiv \gamma^\mu\gamma^\nu + \gamma^\nu \gamma^\mu = 2g^{\mu\nu}$ $\quad \gamma_5 \equiv \gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3$

$$\gamma^0\gamma^\mu\gamma^0 = \gamma^{\mu\dagger}; \qquad \gamma^0\gamma^5\gamma^0 = -\gamma^{5\dagger} = -\gamma^5; \qquad \{\gamma^5, \gamma^\mu\} = 0; \qquad (\gamma_5)^2 = I_4$$

- Specially relevant for weak interactions: chirality projectors ($P_L + PR = 1$)

$$P_L \equiv \frac{1-\gamma_5}{2}; \qquad P_R \equiv \frac{1+\gamma_5}{2}; \qquad P_R^2 = P_R; \qquad P_L^2 = P_L; \qquad P_L P_R = P_R P_L = 0$$

- decompose Dirac spinor in its left-handed and right-handed chirality parts

$$\psi(x) = [P_L + P_R]\psi(x) \equiv \psi_L(x) + \psi_R(x)$$

- In massless limit, chiralities correspond to fermion helicities

$$\gamma^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}; \ \gamma^1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}; \ \gamma^2 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix}; \ \gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}; \ \gamma^5 = i\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

Lagrangian formalism

- ❑ Lagrangian formulation of physical system
  - ❑ provides compact dynamical description
  - ❑ makes it easier to discuss underlying symmetries
- ❑ Like in classical mechanics, dynamics is encoded in action $S = \int d^4x\, \mathcal{L}[\phi_i(x), \partial_\mu \phi_i(x)]$
  - ❑ Integral over four space-time coordinates preserves relativistic invariance
  - ❑ Lagrangian density $\mathcal{L}$ is a Lorentz-invariant functional of fields $\phi_i(x)$ and their derivatives
  - ❑ Space integral $\mathrm{L} = \int d^3x\, \mathcal{L}$ would correspond to usual non-relativistic Lagrangian
- ❑ Principle of stationary action
  - ❑ requires variation δS of action to be zero under small fluctuations $\delta\varphi_i$ of fields.
  - ❑ Assume $\delta\varphi_i$ differentiable & vanish outside some bounded region of space-time (allowing integration by parts), condition $\delta S = 0$ determines Euler–Lagrange (EL) equations of motion for fields

$$\frac{\partial \mathcal{L}}{\partial \phi_i} - \partial^\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial^\mu \phi_i)} \right) = 0$$

- ❑ EL equations $\frac{\partial \mathcal{L}}{\partial \phi_i} - \partial^\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial^\mu \phi_i)} \right) = 0$

❑ Appropriate Lagrangians generate KG and Dirac equations

- ❑ Should be quadratic on fields and Lorentz invariant, which determines their possible form up to irrelevant total derivatives

$$\mathcal{L} = \partial^\mu \phi^* \partial_\mu \phi - m^2 \phi^* \phi \quad \Rightarrow \quad \boxed{(\blacksquare + m^2)\phi(x) = 0}$$

$$\boxed{\mathcal{L} = \bar{\psi}\left(i\gamma^\mu \partial_\mu - m\right)\psi = 0} \quad \Rightarrow \quad \boxed{\left(i\gamma^\mu \partial_\mu - m\right)\psi(x) = 0}$$

- ❑ adjoint spinor $\bar{\psi} = \psi^\dagger \gamma^0$ closes Dirac indices
  - ❑ matrixγ^0included to guarantee proper behaviour under Lorentz transformations:
    - ▪ $\bar{\psi}\psi$ is Lorentz scalar, while $\bar{\psi}\gamma^\mu\psi$ transforms as four-vector
    - ▪ Therefore, $\mathcal{L}$ is Lorentz invariant as it should

Symmetries and conservation laws

- Assume Lagrangian of physical system
  - invariant under some set of continuous transformations
  - $\phi_i(x) \longrightarrow \phi_i'(x) = \phi_i(x) + \epsilon\delta_\epsilon\phi_i(x) + O(\epsilon^2)$ ➡ $\mathcal{L}[\phi_i(x), \partial_\mu\phi_i(x)] = \mathcal{L}[\phi_i{}'(x), \partial_\mu\phi_i{}'(x)]$
  - leading to
    $$\delta_\epsilon\mathcal{L} = 0 = \sum_i \left\{ \left[\frac{\partial\mathcal{L}}{\partial\phi_i} - \partial^\mu \left(\frac{\partial\mathcal{L}}{\partial(\partial^\mu\phi_i)} \right) \right] \delta_\epsilon\phi_i + \partial^\mu \left[\frac{\partial\mathcal{L}}{\partial(\partial^\mu\phi_i)} \delta_\epsilon\phi_i \right] \right\}$$
  - EL equation satisfied $\Longrightarrow$ system has a conserved current
    $$J_\mu = \sum_i \left[\frac{\partial\mathcal{L}}{\partial(\partial^\mu\phi_i)} \delta_\epsilon\phi_i \right]; \quad \partial^\mu J_\mu = 0$$
  - Defining conserved charge $Q \equiv \int d^3x \, J^0$
  - The condition $\partial^\mu J_\mu=0$ guarantees that $dQ/dt=0$, i.e., that Q is a constant of motion
  - *Noether's theorem* extended to general space-time transformations
  - For every continuous symmetry transformation leaving action invariant, ∃ corresponding divergenceless Noether's current and, therefore, a conserved charge.
  - Selection rules observed in Nature, where there exist several conserved quantities (E, p, L, J, Q, …), correspond to dynamical symmetries of Lagrangian

Classical electrodynamics

$$\vec{\nabla}\cdot\vec{B}=0 \qquad \vec{\nabla}\times\vec{E}+\frac{\partial\vec{B}}{\partial t}=0$$

$$\vec{\nabla}\cdot\vec{E}=\rho \qquad \vec{\nabla}\times\vec{B}-\frac{\partial\vec{E}}{\partial t}=\vec{J}$$

$$\vec{E}=-\vec{\nabla}V-\frac{\partial\vec{A}}{\partial t}; \qquad \vec{B}=\vec{\nabla}\times\vec{A}$$

- Maxwell equations
  - summarize large amount of experimental and theoretical work
  - provide unified description of electric and magnetic forces
- Very useful to rewrite equations in Lorentz covariant notation
  - Charge density ρ and EM current $\vec{J}$ transform as a four-vector $J^\mu=(\rho,\vec{J})$
  - Potentials V, $\vec{A}$ combine into $A^\mu=(V,\vec{A})$
  - Relations between potentials and fields take simple form, defining field strength tensor

$$F^{\mu\nu} \equiv \partial^\mu A^\upsilon - \partial^\upsilon A^\mu = \begin{pmatrix} 0 & -E_1 & -E_2 & -E_3 \\ E_1 & 0 & -B_3 & B_2 \\ E_2 & B_3 & 0 & -B_1 \\ E_3 & -B_2 & B_1 & 0 \end{pmatrix}; \qquad \tilde{F}^{\mu\nu}=\frac{1}{2}\epsilon^{\mu\upsilon\rho\sigma}F_{\rho\sigma}$$

  - Covariant form of Maxwell equations turns out to be very transparent $\partial_\mu\tilde{F}^{\mu\nu}=0; \quad \partial_\mu F^{\mu\nu}=J^\upsilon$

- EM dynamics clearly a relativistic phenomenon
  - but Lorentz invariance was not very explicit in original Maxwell formulation
  - Once covariant formulation adopted, equations become much simpler
  - Conservation of EM current appears now as a natural compatibility condition: $\partial_\upsilon J^\upsilon = \partial_\upsilon \partial_\mu F^{\mu\nu} = 0$
  - In terms of potential: $F^{\mu\nu} \equiv \partial^\mu A^\upsilon - \partial^\upsilon A^\mu$

$$\partial_\mu F^{\mu\nu} = J^\upsilon \quad \Longrightarrow \quad \blacksquare A^\upsilon - \partial^\upsilon(\partial_\mu A^\mu) = J^\upsilon$$

  - Same dynamics described by different electromagnetic 4-potentials, giving same field strength tensor $F^{\mu\nu}$
    - Maxwell equations invariant under gauge transformations: $A^\mu \longrightarrow A'^\mu = A^\mu + \partial^\mu \Lambda$
  - Lorentz gauge $\partial_\mu A^\mu = 0 \quad \Longrightarrow \quad \blacksquare A^\upsilon = J^\upsilon$ $(= 0$ absence of an external current) $\Longrightarrow M_\gamma = 0$
  - Lorentz condition $\partial_\mu A^\mu = 0$ still allows for residual gauge invariance under transformations with restriction $\blacksquare \Lambda = 0$
    - impose second constraint on EM field A^μ, without changing $F^{\mu\nu}$
    - Since A^μ contains 4 fields ($\mu = 0, 1, 2, 3$) and there are 2 arbitrary constraints, number of physical dof = 2
    - Therefore, photon has 2 different physical polarizations

Gauge Invariance

Quantum Electro Dynamics – QED

- Lagrangian describing a free Dirac fermion $\mathcal{L}_0 = i\,\bar{\psi}(\mathrm{x})\,\gamma^{\mu}\partial_{\mu}\psi(x) - m\,\bar{\psi}(\mathrm{x})\,\psi(x)$
- $\mathcal{L}_0$ is invariant under global U(1) transformations: $\psi(x) \xrightarrow{U(1)} \psi'(x) \equiv e^{iQ\theta}\,\psi(x)$
  - where $Q\theta$ is arbitrary real constant
  - phase of $\psi(x)$ is then pure convention-dependent quantity without physical meaning
- However, free Lagrangian is no longer invariant
  - if phase transformation is space-time coordinate dependent
    - under local phase redefinitions $\theta = \theta(x)$:

$$\partial_{\mu}\psi(x) \xrightarrow{U(1)} e^{iQ\theta}\left(\partial_{\mu} + iQ\partial_{\mu}\theta\right)\psi(x)$$

  - once a given phase convention adopted at reference point x_0, same convention adopted at all space-time points
    - This looks very unnatural.
- ‘Gauge principle’ = requirement that *U(1)* phase invariance should hold locally
  - only possible if extra piece added to $\mathcal{L}_0$, transforming in such a way as to cancel $\partial_{\mu}\theta$ term
  - Introduce new spin-1 (since $\partial_{\mu}\theta$ has a Lorentz index) field $A_{\mu}(x)$, transforming as

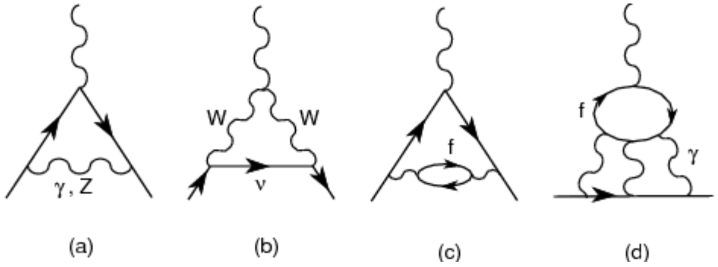
$$A_{\mu}(x) \xrightarrow{U(1)} A'_{\mu}(x) \equiv A_{\mu}(x) - \frac{1}{e}\partial_{\mu}\theta$$

- **Covariant derivative** has the required property of transforming like the field itself:

  $D_\mu\psi(x) \equiv \left(\partial_\mu + ieQA_\mu(x)\right)\,\psi(x)$ $\qquad$ $D_\mu\psi(x) \xrightarrow{U(1)} \left(D_\mu\psi\right)'(x) \equiv e^{iQ\theta}\;D_\mu\psi(x)$

- Lagrangian $\mathcal{L} \equiv i\,\bar{\psi}(\mathrm{x})\,\gamma^\mu D_\mu\psi(x) - m\,\bar{\psi}(\mathrm{x})\,\psi(x) = \mathcal{L}_0 - eQA_\mu(x)\,\bar{\psi}(\mathrm{x})\,\gamma^\mu\psi(x)$
  - is then invariant under local *U(1)* transformations
- Gauge principle generated interaction between Dirac fermion and gauge field A_μ
  - familiar vertex of Quantum Electrodynamics (QED)
  - Note: corresponding EM charge Q completely arbitrary
- A_μ as a true propagating field
  - need to add a gauge-invariant kinetic term $\mathcal{L}_{kin} \equiv -\frac{1}{4}F_{\mu\nu}\,(x)\,F^{\mu\nu}\,(x)$ $\qquad$ $F_{\mu\nu} \equiv \partial_\mu\,A_\nu - \partial_\nu\,A_\mu$
  - EM field strength remains invariant under gauge transformations

- A mass term for gauge field $\mathcal{L}_m = \frac{1}{2} m^2 A^\mu A_\mu$
  - would violate local *U(1)* gauge invariance → thus forbidden
  - → photon field predicted massless
  - Experimentally $m_\gamma < 10^{-18}$ eV
- Total Lagrangian $\mathcal{L}_0 - eQA_\mu(x)\,\bar{\psi}(x)\,\gamma^\mu\psi(x) + \mathcal{L}_{kin}$
  - gives rise to well-known Maxwell equations: $\partial_\mu F^{\mu\nu} = e\,J^\nu \equiv eQ\,\bar{\psi}\,\gamma^\nu\psi$
  - J^ν is fermion EM current.
- From simple gauge-symmetry requirement, deduced right QED Lagrangian, leading to very successful quantum field theory
  - Lepton anomalous magnetic moments
  - Feynman diagrams contributing



- Most stringent QED test
  - high-precision measurements of e and μ anomalous magnetic moments

$$a_l \equiv \frac{(g_l^\gamma - 2)}{2} \quad ; \quad \vec{\mu_l} \equiv g_l^\gamma \left(\frac{e}{2m_l}\right) \vec{S_l}$$

  - $a_e = (1\,159\,652\,180.73 \pm 0.28)\ 10^{-12}$, $a_\mu = (11\,659\,208.9 \pm 6.3)\ 10^{-10}$
- To measurable level, a_e arises entirely from virtual e´s and γ ´s
  - contributions are fully known to $O(\alpha^4)$ and (partly) $O(\alpha^5)$
  - Impressive agreement achieved between theory and experiment promoted QED to level of best theory ever built to describe Nature
  - Theoretical error dominated by uncertainty in input value of QED coupling $\alpha \equiv e^2/4\pi$
  - a_e provides most accurate determination of fine structure constant
  - $\alpha^{-1} = 137.035\,999\,084 \pm 0.000\,000\,051$

$$a_\mu^{\mathrm{th}} = \begin{cases} (11\,659\,180.2 \pm 4.9) \cdot 10^{-10} & (e^+e^- \quad \mathrm{data}) \\ (11\,659\,189.4 \pm 5.4) \cdot 10^{-10} & (\tau \quad \mathrm{data})\,. \end{cases}$$

Quantum Chromo Dynamics – QCD

Quarks and Colour

- Large number of known mesonic and baryonic states clearly signals the existence of a deeper level of elementary constituents of matter: *quarks*.
- Entire hadronic spectrum nicely classified assuming
  - mesons $M \equiv q\bar{q}$ states and baryons $B \equiv qqq$
- To satisfy Fermi–Dirac statistics, need to assume existence of a new quantum number, *colour*,
  - $N_C = 3$ different colours: $q^{\alpha}, \alpha = 1, 2, 3\ (red, green, blue)$.
- Mesons and baryons described by colour-singlet combinations

$$M = \frac{1}{\sqrt{3}} \delta^{\alpha\beta} \left| q_\alpha \bar{q}_\beta \right\rangle \qquad B = \frac{1}{\sqrt{6}} \varepsilon^{\alpha\beta\gamma} \left| q_\alpha q_\beta q_\gamma \right\rangle$$

- To avoid existence of non-observed extra states with non-zero colour,
  - Postulate: all asymptotic states are colourless, i.e., singlets under rotations in colour space
  - confinement hypothesis, implying non-observability of free quarks:
  - since quarks (and gluons) carry colour they are confined within colour-singlet bound states

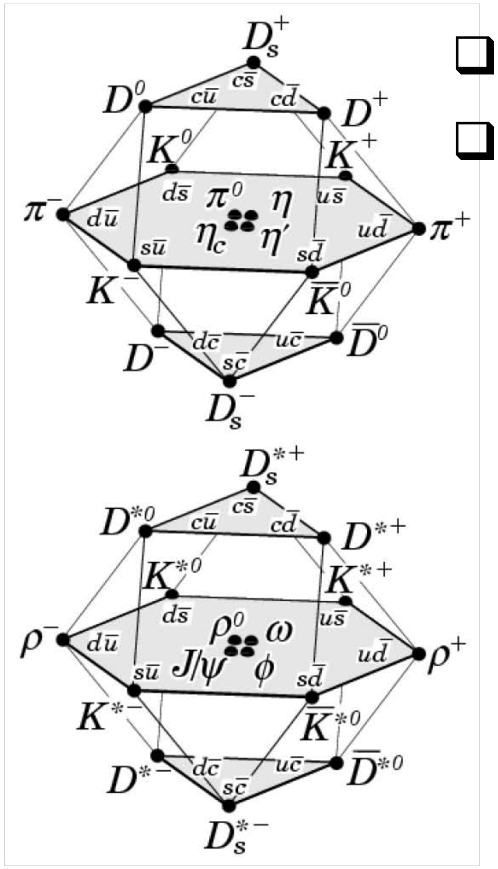
Quarks and hadrons: SU(4):u,d,s,c

Light (MeV)	$m_u \sim 5$	$m_d \sim 8$	$m_s \sim 115$
Heavy (GeV)	$M_c \sim 1.2$	$M_b \sim 4.2$	$M_t \sim 171$

❑ Meson≡ $q_1\bar{q}_2$

❑ Spin

- ❑ $\frac{1}{2}\otimes\frac{1}{2}\otimes\frac{1}{2}\longrightarrow J=0{,}1$

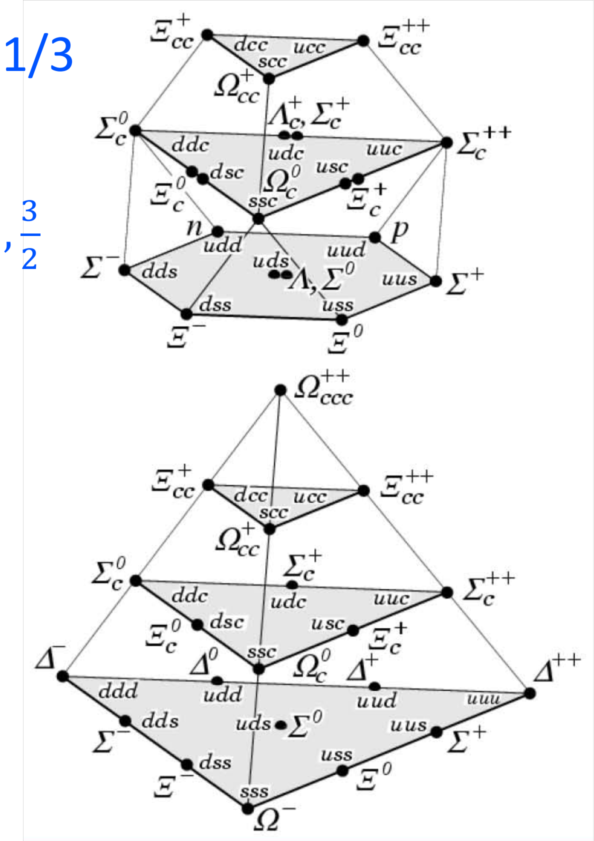


❑ Baryon≡ $q_1q_2q_3$

❑ Baryon nber: B(q)=1/3

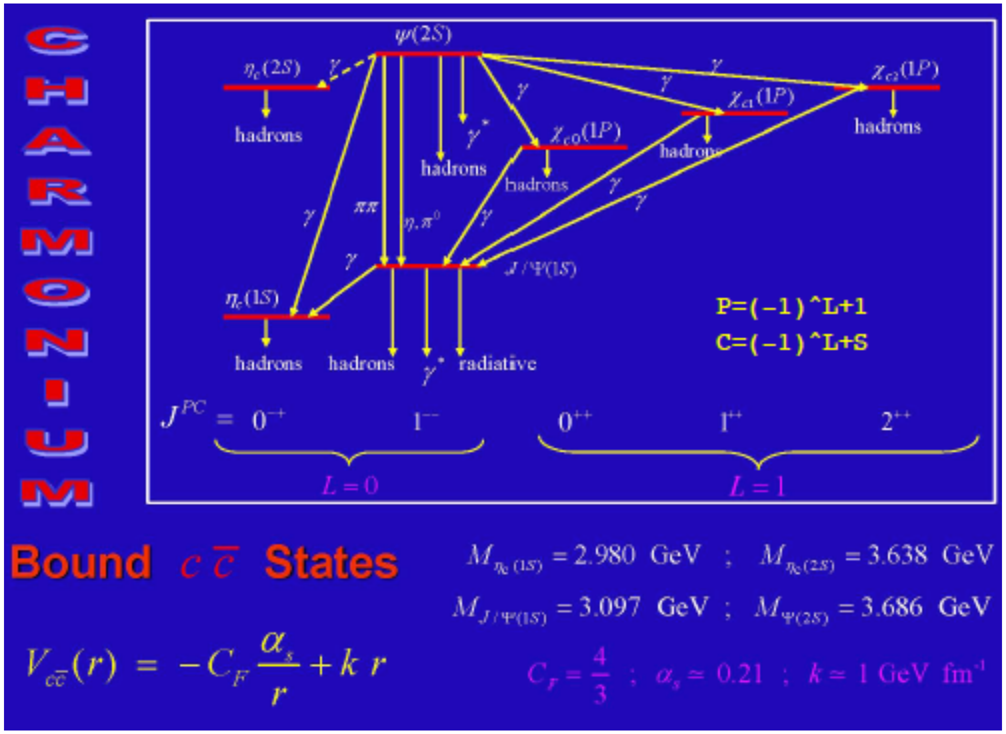
❑ Spin

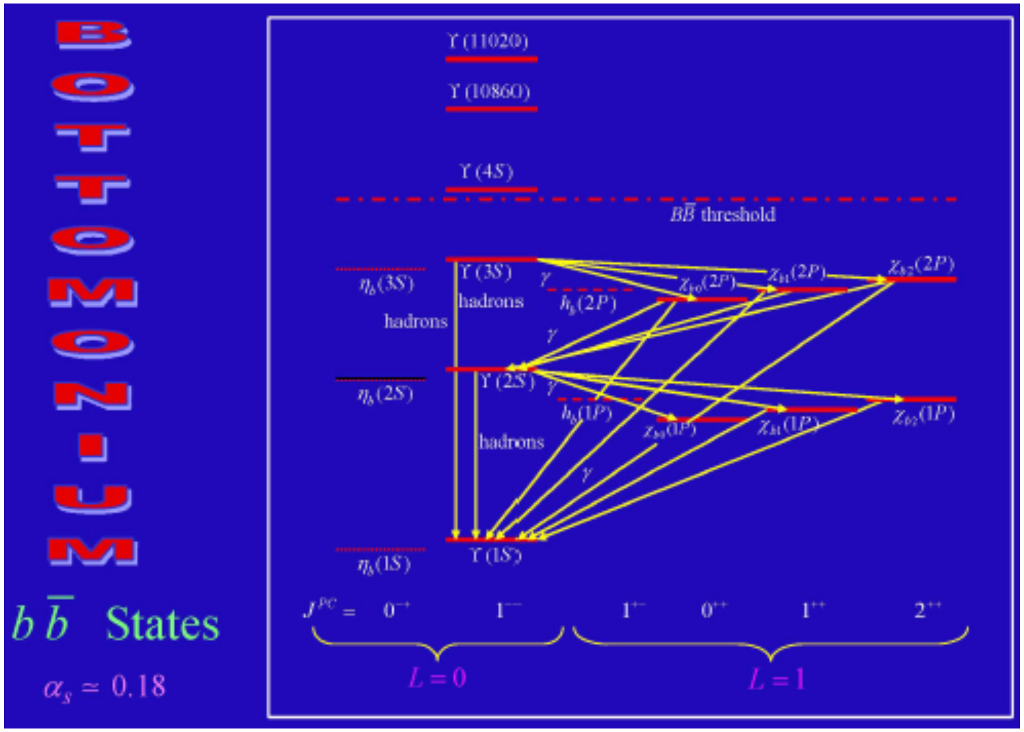
- ❑ $\frac{1}{2}\otimes\frac{1}{2}\otimes\frac{1}{2}\longrightarrow J=\frac{1}{2},\frac{3}{2}$



Hadronisation scale $\Lambda_\chi \approx 1GeV$

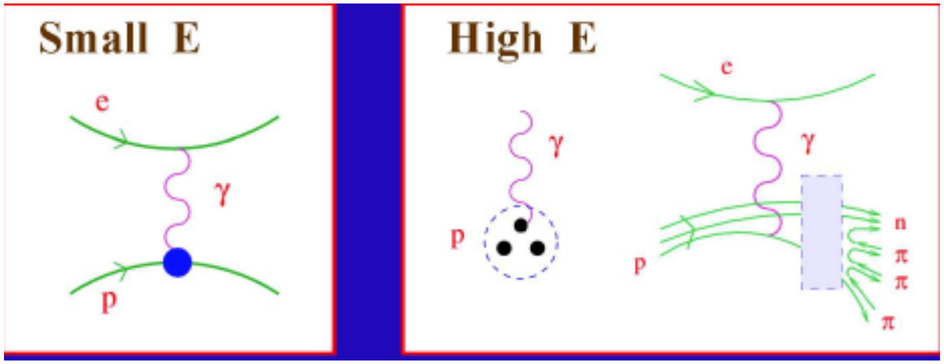
SU(2)	u,d,	$M_p = 938.3\ MeV$	$M_n = 939.6\ MeV$	$m_{u,d} \ll \Lambda_\chi$
SU(3)	u,d,s	$M_\Lambda = 1115.7\ MeV$	$M_{\Xi^0} = 1314.8\ MeV$	$m_s < \Lambda_\chi$
SU(4)	u,d,s,c	$M_{\Sigma_c} = 2453\ MeV$	$M_{\Omega_c^0} = 2697.5MeV$	$m_c \approx \Lambda_\chi$
SU(5)	u,d,s,c,b		$M_{\Lambda_b^0} = 2697.5MeV$	$m_b > \Lambda_\chi$
SU(6)	u,d,s,c,b,t	No bound states with t	$M_t = 172\ GeV$	$m_t \gg \Lambda_\chi$

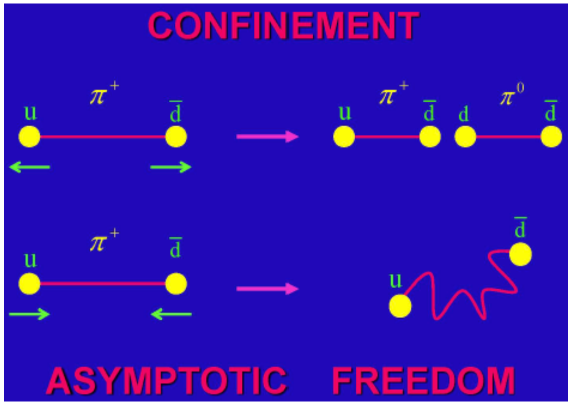


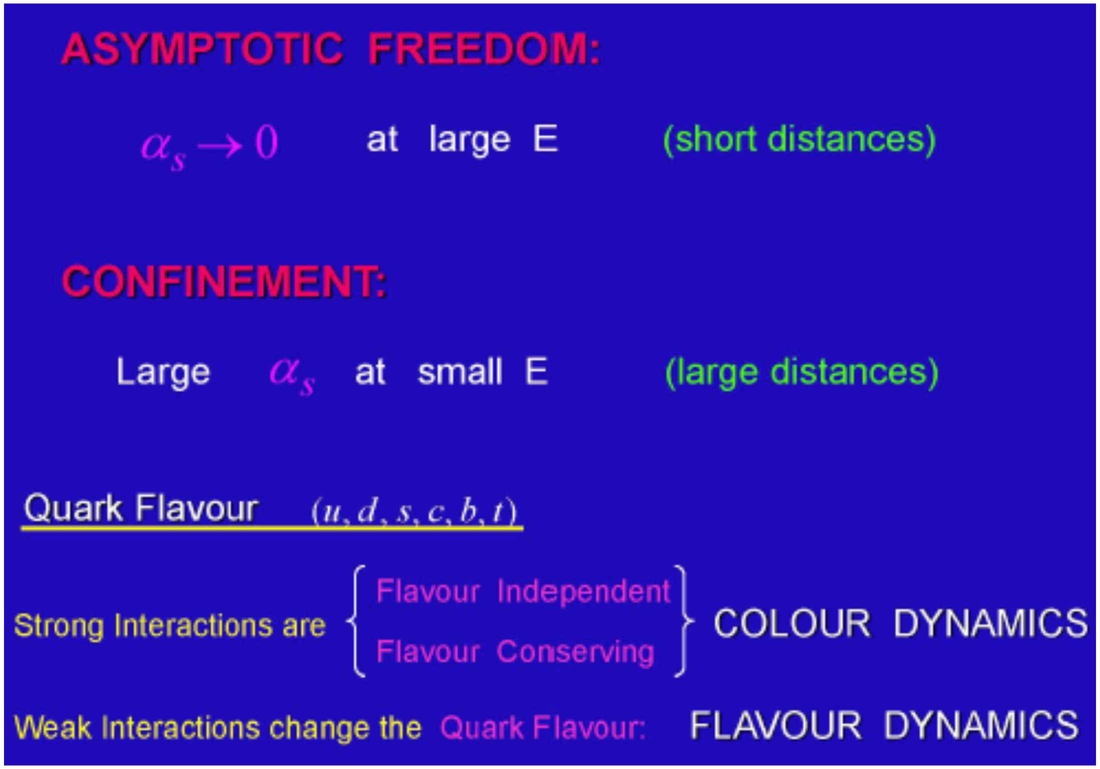


ep - scattering

- ❑ High energy hadronic processes well described through interactions of free constituent quarks
  - ❑ $\sigma(e^-p \longrightarrow e^-X) \approx \sum_q \sigma(e^-q \longrightarrow e^-q)$







ASYMPTOTIC FREEDOM:

$\alpha_s \to 0$ at large E (short distances)

CONFINEMENT:

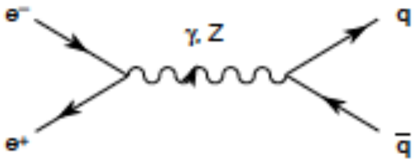
Large α_s at small E (large distances)

Quark Flavour (u, d, s, c, b, t)

$$\text{Strong Interactions are} \left\{ \begin{array}{l} \text{Flavour Independent} \\ \text{Flavour Conserving} \end{array} \right\} \text{COLOUR DYNAMICS}$$

Weak Interactions change the Quark Flavour: FLAVOUR DYNAMICS

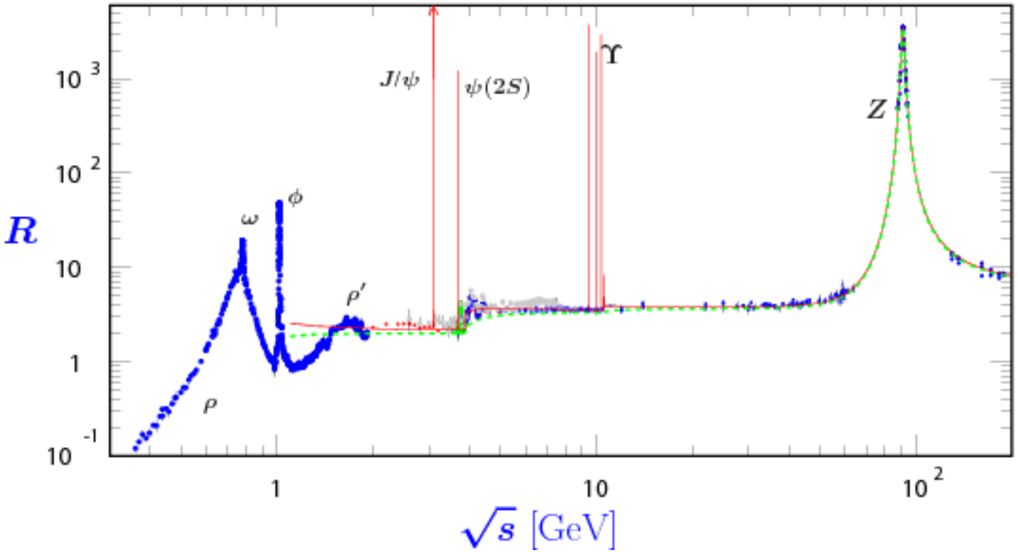
Quarks and Colour



- Direct test of colour quantum number
  - e^+e^- – annihilation into hadrons
    - quarks assumed to be confined, 100% probability to hadronise
    - summing over all possible final state quarks estimates inclusive cross-section into hadrons
  - Ratio $R_{e^+e^-} = \frac{\sigma(e^+e^- \rightarrow hadrons)}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$
  - Well below Z-resonance sum over the N_f quark flavours kinematically accessible $4m_q{}^2 < s \equiv (p^{e-} + p^{e+})^2$, weighted by N_c

$$R_{e^+e^-} \approx N_c \sum_{f=1}^{N_f} Q_f^2 = \begin{cases} \frac{2}{3} N_c = 2 & (N_f = 3 : u, d, s) \\ \frac{10}{9} N_c = \frac{10}{3} & (N_f = 4 : u, d, s, c) \\ \frac{11}{9} N_c = \frac{11}{3} & (N_f = 5 : u, d, s, c, b) \end{cases}$$

  - Discuss the figure and agreement with N_c=3!



Non-abelian Gauge theory

Exercise: derive QCD Lagrangian

- Quark field of colour α and flavour f : q_f^α
  - Vector notation in colour space: $q_f^\alpha \equiv \left(q_f^1, q_f^2, q_f^3\right)$
  - Free Lagrangian $\mathcal{L}_0$ invariant under global $SU(3)_C$ transformations in colour space

$$\mathcal{L}_0 = \sum_f \bar{q}_f(\, i\,\gamma^\mu \partial_\mu - m_f)q_f \qquad q_f^\alpha \xrightarrow{U} (\ q_f^\alpha)'\ U^\alpha{}_\beta\ q_f^\beta \quad , \qquad UU^\dagger = U^\dagger U = 1\ , \qquad detU = 1$$

- $SU(3)_C$ matrices $U = e^{i\frac{\lambda^a}{2}\theta_a}$ with $\frac{1}{2}\lambda^a, (a = 1,2,\dots,8)$ generators of fundamental representation of $SU(3)_C$ algebra, θ_a : 8 arbitrary parameters
  - λ^a : traceless matrices satisfying commutation relations $\left[\frac{\lambda^a}{2}, \frac{\lambda^b}{2}\right] = i\, f^{abc}\frac{\lambda^c}{2}$; f^{abc} structure constants
  - Require Lagrangian to be invariant under local $SU(3)_C$ transformations, $\theta_a = \theta_a(x)$
  - Need change to quark covariant derivatives: 8 independent gauge parameters → 8 gauge bosons, gluons

$$D^\mu q_f \equiv \left(\partial^\mu + ig_s\frac{\lambda^a}{2}G_a^\mu(x)\right)q_f \equiv \left(\partial^\mu + ig_s G^\mu(x)\right)q_f$$

$$D^{\mu} q_f \equiv \left(\partial^{\mu} + i g_s \frac{\lambda^a}{2} G_a^{\mu}(x) \right) q_f \equiv \left(\partial^{\mu} + i g_s G^{\mu}(x) \right) q_f$$

- Compact notation and colour identity matrix is implicit in the derivative term

$$\left[G_a^{\mu}(x) \right]_{\alpha\beta} \equiv \left(\frac{\lambda^a}{2} \right)_{\alpha\beta} G_a^{\mu}(x)$$

- Require $D^{\mu} q_f$ transform as colour quark-vector q_f, fixing transformation properties of gauge fields

$$(D^{\mu}) \longrightarrow (D^{\mu})' = U D^{\mu} U^{\dagger} \;, \qquad (G^{\mu}) \longrightarrow (G^{\mu})' = U G^{\mu} U^{\dagger} + \frac{i}{g_s} (\partial^{\mu} U) U^{\dagger}$$

- Under an infinitesimal $SU(3)_C$ transformation

$$q_f^{\alpha} \longrightarrow q_f^{\alpha} + i \left(\frac{\lambda^a}{2} \right)_{\alpha\beta} \delta\theta_a \, q_f^{\beta} \qquad G_a^{\mu} \longrightarrow \left(G_a^{\mu} \right)' = G_a^{\mu} - \frac{1}{g_s} \partial^{\mu} (\delta\theta_a) - f^{abc} \delta\theta_b G_c^{\mu}$$

- Gauge transformation of gluon fields more complicated than in QED for photon
  - Non-commutativity of $SU(3)_C$ matrices gives rise to additional term involving the gluon fields themselves.
  - For constant $\delta\theta_a$, transformation rule for gauge fields is expressed in terms of structure constants f_{abc}
  - Unique $SU(3)_C$ coupling g_s , in QED arbitrary EM charges assigned to different fermions
  - Non-linear commutation relation in QCD, no such freedom for $SU(3)_C$ as for U(1)
  - All colour-triplet quark flavours couple to gluon fields with exactly same interaction strength

- To build gauge-invariant kinetic term for gluon fields,
  - introduce corresponding field strengths:

$$G^{\mu\nu}(x) \equiv -\frac{i}{g_s}[D^\mu, D^\nu] = \partial^\mu G^\nu - \partial^\nu G^\mu + i g_s [G^\mu, G^\nu] \equiv \frac{\lambda^a}{2} G_a^{\mu\nu}(x)$$

$$G_a^{\mu\nu}(x) = \partial^\mu G_a^\nu - \partial^\nu G_a^\mu - g_s\, f^{abc} G_b^\mu\, G_c^\nu$$

  - Under a $SU(3)_C$ gauge transformation $G^{\mu\nu} \longrightarrow (G^{\mu\nu})' = U\, G^{\mu\nu}\, U^\dagger$
  - Colour trace $\mathrm{Tr}\left(G^{\mu\nu} G_{\mu\nu}\right) = \frac{1}{2} G_a^{\mu\nu}\, G_{\mu\nu}^a$ remains invariant

- SU(3)$_C$ - invariant Lagrangian of (QCD) $\mathcal{L}_{QCD} \equiv -\frac{1}{4} G_a^{\mu\nu}\, G_{\mu\nu}^a + \sum_f \bar{q}_f \left(i\, \gamma^\mu D_\mu - m_f \right) q_f$
  - *SU(3)$_C$* gauge symmetry forbids mass term for gluon fields, $\frac{1}{2} m_G^2\, G_a^\nu\, G_\nu^a$
    - not invariant under the transformation
  - QCD gauge bosons are, therefore, massless spin-1 particles

- ☐ decompose QCD Lagrangian into its different pieces (go through this and identify various pieces)

$$\mathcal{L}_{QCD} \equiv -\frac{1}{4}(\,\partial^\mu G^\nu_a - \partial^\nu G^\mu_a)(\partial_\mu G^a_\nu - \partial_\nu G^a_\mu) + \sum_f \bar{q}^\alpha_f(\,i\,\gamma^\mu \partial_\mu - m_f\,)q^\alpha_f$$

$$-g_s\, G^\mu_a \sum_f \bar{q}^\alpha_f\, \gamma_\mu \left(\frac{\lambda^a}{2}\right)_{\alpha\beta} q^\beta_f$$

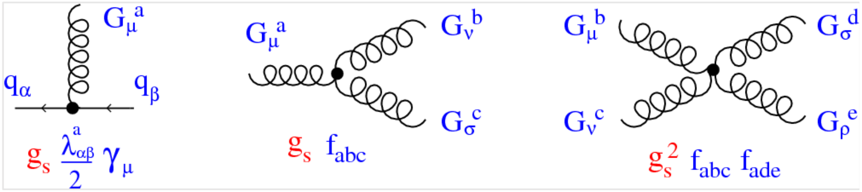
$$+\frac{g_s}{2} f^{abc}(\partial^\mu G^\nu_a - \partial^\nu G^\mu_a)\, G^b_\mu G^c_\nu \; - \frac{g_s^2}{4} f^{abc} f_{ade}\, G^\mu_b\, G^\nu_c G^d_\mu G^e_\nu$$

(i) (quadratic) kinetic terms for different fields – propagators

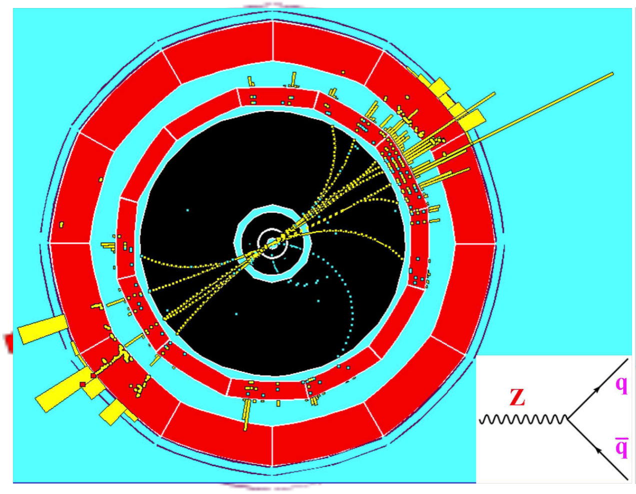
(ii) colour interaction between quarks and gluons – involves SU(3)$_C$ matrices λ_a

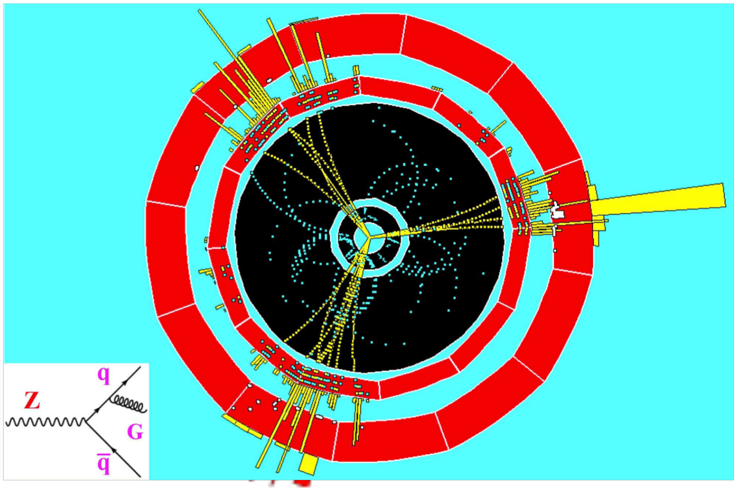
(iii) cubic and quartic gluon self-interactions – non-Abelian character of colour group

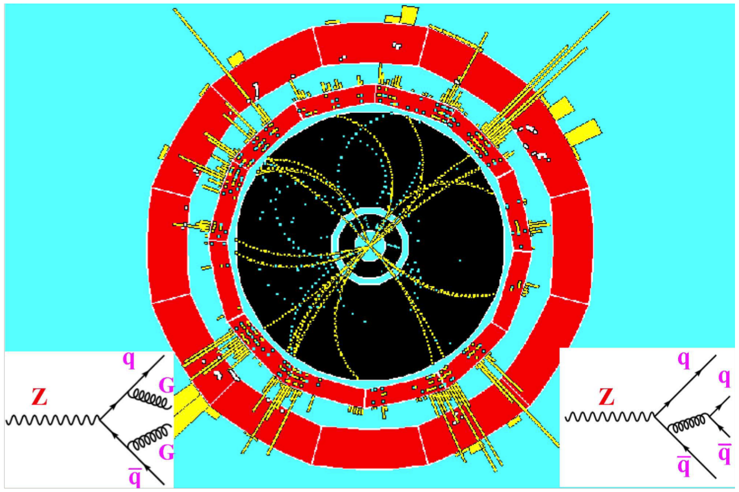
- ☐ A simple and powerful Lagrangian
  - ☐ All interactions given in terms of single universal coupling g_s
  - ☐ New feature:
    - ☐ existence of self-interactions among gauge fields
    - ☐ not present in QED
  - ☐ Expect gauge self-interactions could explain properties like
    - ☐ asymptotic freedom (strong interactions become weaker at short distances)
    - ☐ confinement (strong forces increase at large distances),
    - ☐ which do not appear in QED



- Without any detailed calculation, extract qualitative physical consequences from $\mathcal{L}_{QCD}$
  - Quarks can emit gluons. At lowest order in g_s, dominant process : emission of a single gauge boson
  - Hadronic decay of Z results in some $Z \rightarrow q\bar{q}G$ events, in addition to dominant $Z \rightarrow q\bar{q}$
    - Similar events show up in e^+e^- annihilation into hadrons
  - Ratio between 3-jet and 2-jet events provides a simple estimate of strength of strong interaction;
  - at LEP energies ($\sqrt{s} = MZ$): $\alpha_s \equiv g_s^2/4\pi \sim 0.12$.



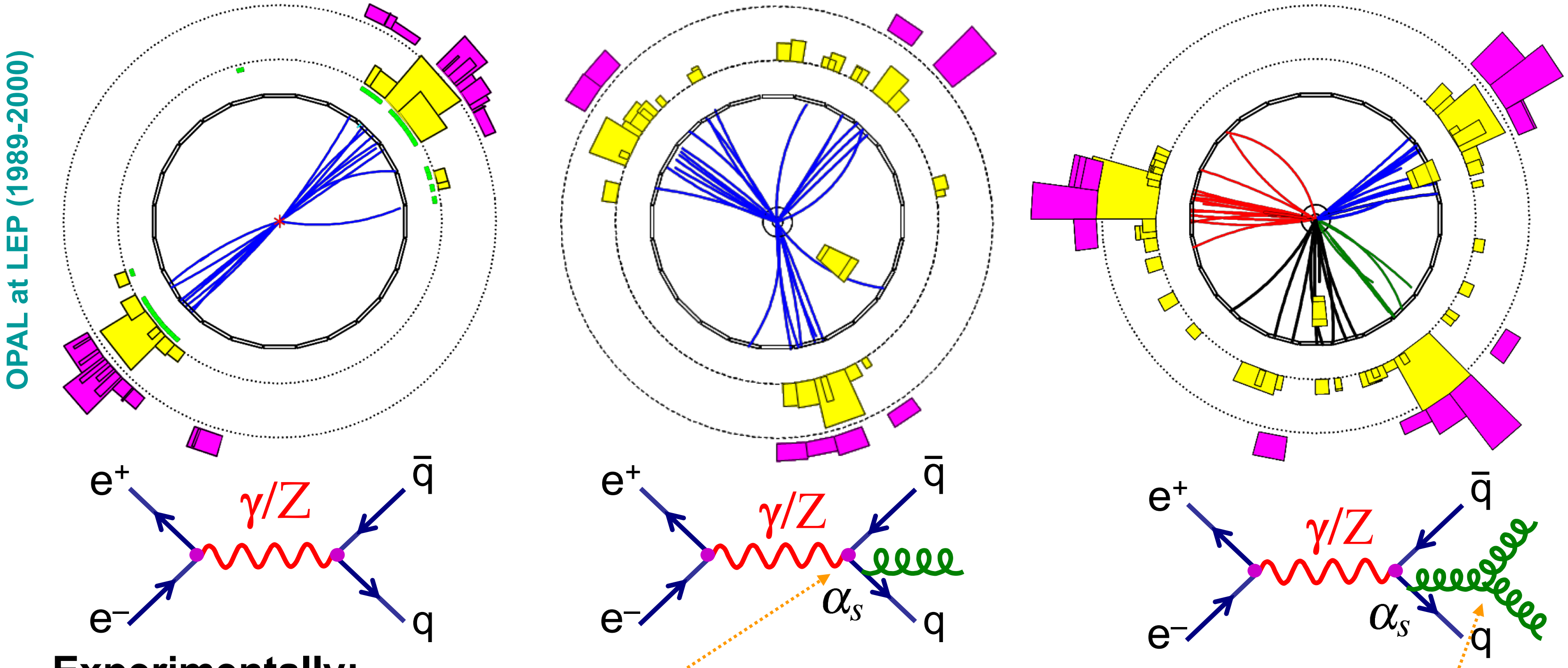




Jet production in e+e- Collisions

★ e^+e^- colliders are also a good place to study gluons

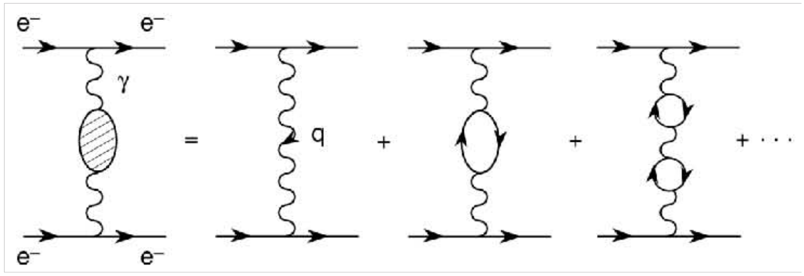
$e^+e^- \to q\bar{q} \to 2\text{jets}$ $e^+e^- \to q\bar{q}g \to 3\text{jets}$ $e^+e^- \to q\bar{q}gg \to 4\text{jets}$



Experimentally:

- Three jet rate ➡ measurement of α_s
- Angular distributions ➡ gluons are spin-1
- Four-jet rate and distributions ➡ QCD has an underlying SU(3) symmetry

Quantum corrections



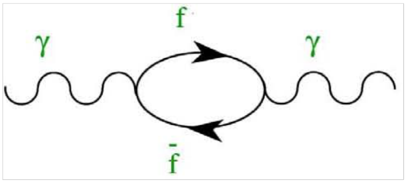
- Parameterisation of higher-order corrections
  - 2→2

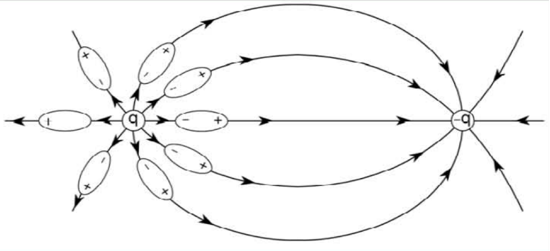
$$T(Q^2) \approx \frac{\alpha}{Q^2}\{1+\Pi(Q^2)+\Pi(Q^2)^2+\cdots\} \approx \frac{\alpha(Q^2)}{Q^2}$$

- Effective (Running) Coupling

$$\boxed{\alpha(Q^2) = \frac{\alpha}{1-\Pi(Q^2)} \approx \frac{\alpha}{1-\frac{\alpha}{3\pi}\log\left(\frac{Q^2}{m^2}\right)}}$$

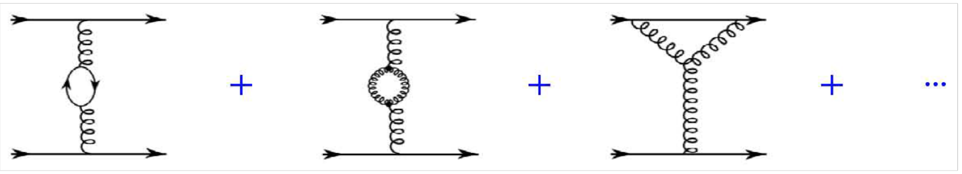
- Screening
  - $\alpha(Q^2)$ Increases with $Q^2 \equiv -q^2$ $\Longrightarrow \alpha(Q^2)$ decreases at Large Distances
- Vacuum polarisation
  - Vacuum acts as polarised dielectric medium
  - Photon couples to virtual $f\bar{f}$-pairs





$$\boxed{\frac{1}{\alpha} = \frac{1}{\alpha(m_e^2)} = 137.035999710\ (96) \qquad \frac{1}{\alpha(m_Z^2)} = 128.93 \pm 0.05}$$

QCD Running coupling constant



- ❑ Effective (Running) Coupling

$$\boxed{\alpha_s(Q^2) = \frac{\alpha_s(Q_0^2)}{1 - \beta_1 \frac{\alpha(Q_0^2)}{2\pi} \log\left(\frac{Q^2}{m^2}\right)}}$$

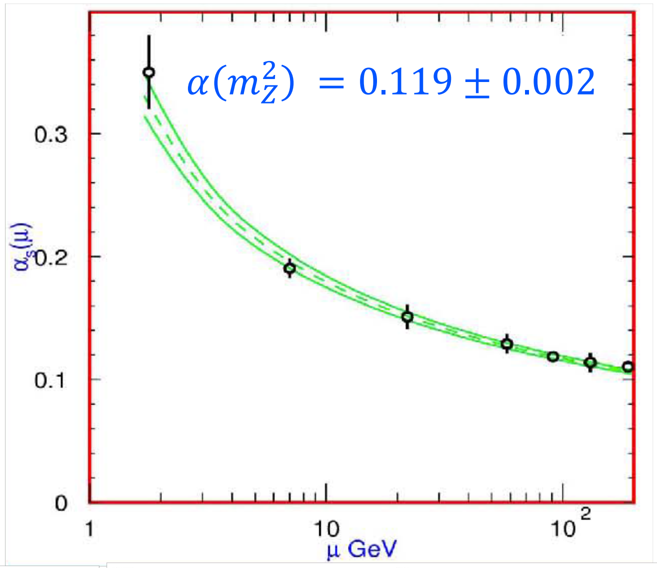
$$\beta_1 = \frac{1}{3} N_F - \frac{11}{6} N_C$$

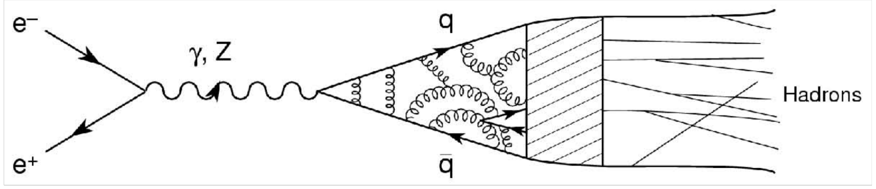
  - ❑ Contribution from Quarks AND Gluons:
  - ❑ $N_F = 6\,;\ N_C = 3 \Longrightarrow \beta_1 < 0\ ;$ ➡ $Q^2 > Q_0^2 \quad \Longrightarrow \quad \alpha_s(Q^2) < \alpha_s(Q_0^2)$

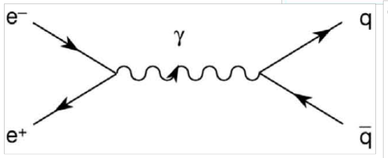
- ❑ Anti-Screening
  - ❑ $\alpha_s(Q^2)$ Decreases with $Q^2 \equiv -q^2$ $\Longrightarrow \alpha_s(Q^2)$ Decreases at SHORT Distances

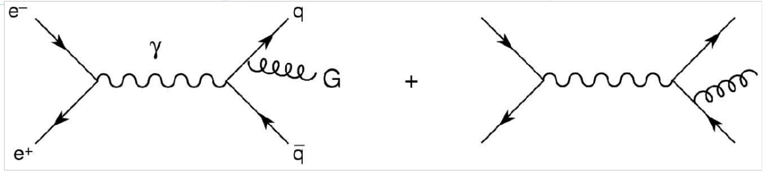
QCD Running coupling & Asymptotic Freedom

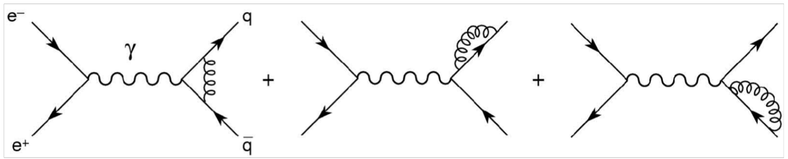
- Asymptotic Freedom
  - $\beta_1 < 0 \Longrightarrow \lim_{Q^2\to\infty} \alpha_s(Q^2) = 0$
  - $\alpha_s(Q^2)$ DECREASES at HIGH energies
- Confinement?
  - $\alpha_s(Q^2)$ INCREASES at LOW energies
  - $\alpha_s = O(1)$ at $1\ GeV$ $\Longrightarrow$ Non-Perturbative Region
  - Hadronisation Probabilty = 1







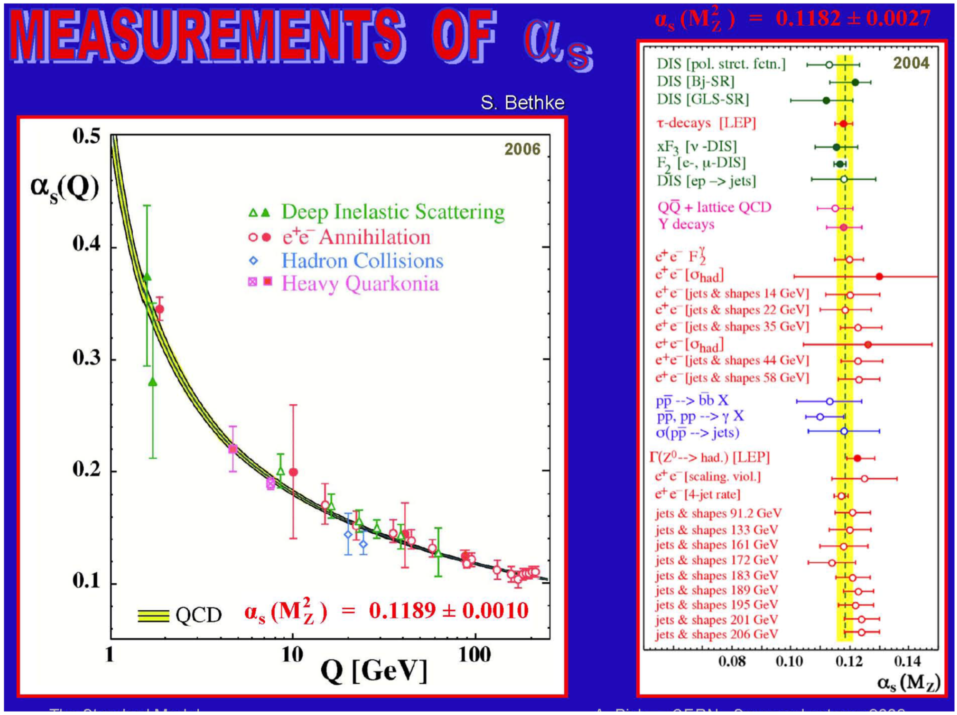




$$\sigma(e^+e^- \longrightarrow hadrons) = \sigma(e^+e^- \longrightarrow q\bar{q}) + \sigma(e^+e^- \longrightarrow q\bar{q}g) + \sigma(e^+e^- \longrightarrow q\bar{q}gg) + \sigma(e^+e^- \longrightarrow q\bar{q}q\bar{q})$$

α_s measurements

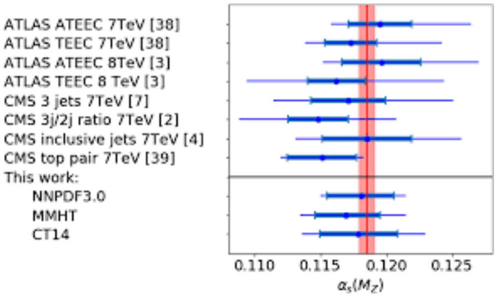
- ❑ α_s measured in various processes at different energies
  - ❑ Measurement translated to a reference energy where α_s has been measured with high precision

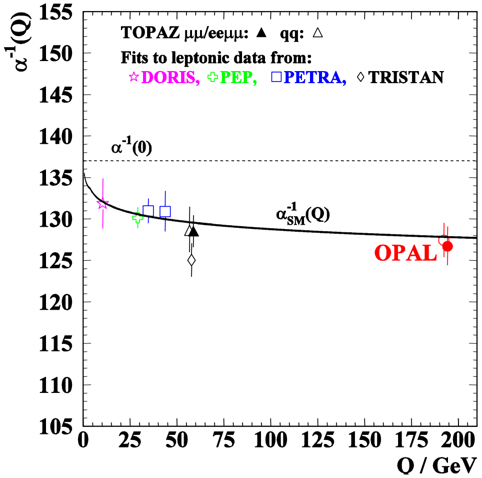


$$R_{e^+e^-} = \frac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)} = N_c \sum_{f=1}^{N_f} Q_f^2 \left\{1 + \frac{\alpha_s(s)}{\pi} + \cdots\right\}$$

$$R_Z = \frac{\Gamma(Z \to hadrons)}{\Gamma(Z \to e^+ e)} = Q_Z^{EW} N_c \left\{1 + \frac{\alpha_s(m_Z^2)}{\pi} + \cdots\right\}$$

$$R_\tau = \frac{\Gamma(\tau \to \upsilon_\tau + hadrons)}{\Gamma(\tau \to \upsilon_\tau e^- \overline{\upsilon}_e)} = N_c \left\{1 + \frac{\alpha_s(m_\tau^2)}{\pi} + \cdots\right\}$$

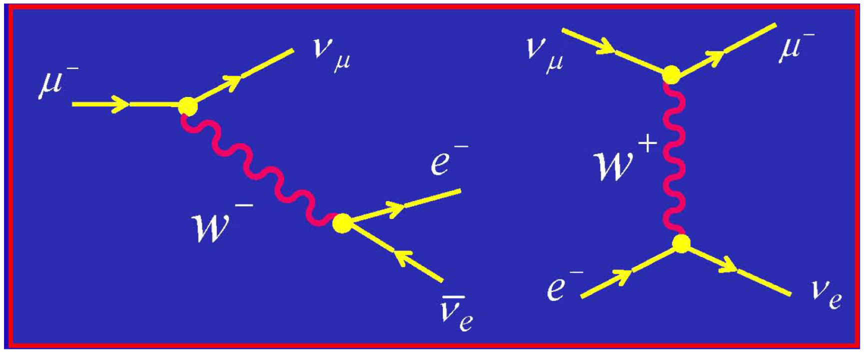




3 – Electroweak Unification

3.1 – Experimental facts

- Low-energy experiments
  - provide a large amount of information about the dynamics underlying flavour-changing processes
  - Detailed analysis of energy / angular distributions in β decays, such as $\mu^- \longrightarrow e^- \bar{\nu}_e \nu_\mu$ or $\mathrm{n} \longrightarrow p e^- \bar{\nu}_e$
    - ➔ only LH (RH) fermion (antifermion) chiralities participate in those weak transitions
    - ➔ Interaction strength universal.
  - Processes like $\pi^- \longrightarrow e^- \bar{\nu}_e$ or $\pi^- \longrightarrow \mu^- \bar{\nu}_\mu$
    - ➔ neutrinos have LH chiralities, anti-neutrinos RH



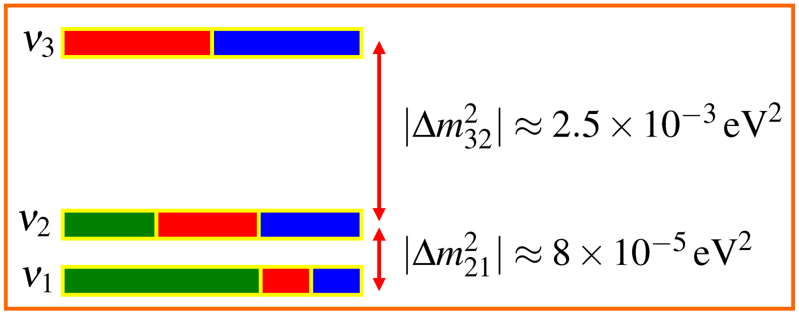
- Neutrino scattering data
  - Existence of different neutrino types ($\nu_e \neq \nu_\mu$)
  - separately conserved lepton quantum numbers ($\nu_{e,\mu} \neq \bar{\nu}_{e,\mu}$)
  - transitions observed $\bar{\nu}_e p \longrightarrow e^+ n$; $\nu_e \mathrm{n} \longrightarrow e^- p$; $\bar{\nu}_\mu p \longrightarrow \mu^+ n$; $\nu_\mu \mathrm{n} \longrightarrow \mu^- p$
  - processes not seen $\nu_e \mathrm{p} \longrightarrow e^+ n$; $\bar{\nu}_e n \longrightarrow e^- \mathrm{p}$; $\bar{\nu}_\mu p \longrightarrow e^+ n$; $\nu_\mu \mathrm{n} \longrightarrow \mu^- p$

- Together with theoretical considerations related to
  - unitarity – a proper high-energy behavior
  - absence of flavour-changing neutral-current transitions (FCNC): $\mu^- \nrightarrow e^- e^- e^+$; $s \nrightarrow d\, l^+ l^-$
- Low energy structure of modern electroweak theory good enough
  - intermediate vector bosons $W^\pm$ & Z theoretically introduced and their masses estimated before discovery
  - huge numbers of $W^\pm$ and Z decay events → much direct experimental evidence of dynamical properties
- Charged currents – interaction of quarks and leptons with $W^\pm$ bosons features:
  - Only LH fermions & RH antifermions couple to the $W^\pm$
    - 100% breaking of parity (P: left ↔ right) and charge conjugation (C: particle ↔ antiparticle).
    - However, combined transformation CP still a good symmetry.
  - $W^\pm$ bosons couple to fermionic doublets
    - electric charges of the two fermion partners differ by one unit
    - decay channels of W^- : $W^- \longrightarrow e^-\bar{\nu}_e;\ \ \mu^-\bar{\nu}_\mu;\ \tau^-\bar{\nu}_\tau; d'\bar{u}; s'\bar{c}$
    - m_t = 173 GeV > M_W = 80.4 GeV, its on-shell production through $W^- \longrightarrow b'\bar{t}$ kinematically forbidden.

- All fermion doublets couple to the W± bosons with same universal strength
- Doublet partners of *u, c, t (charge +2/3)* quarks mixtures of *d, s, b* quarks with charge −1/3

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V \begin{pmatrix} d \\ s \\ b \end{pmatrix} \; ; \qquad VV^{\dagger} = V^{\dagger}V = 1 \qquad \begin{bmatrix} \upsilon_e & u \\ e^- & d' \end{bmatrix}, \begin{bmatrix} \upsilon_\mu & c \\ \mu^- & s' \end{bmatrix}, \begin{bmatrix} \upsilon_\tau & t \\ \tau^- & b' \end{bmatrix}$$

  - weak eigenstates d ′ , s ′ , b ′ ≠ mass eigenstates d , s , b
  - related through 3X3 unitary matrix V – CKM-matrix – characterizing flavour-mixing phenomena
- Experimental evidence of neutrino oscillations
  - ν_e, ν_μ, ν_τ (flavour eigenstates) also mixtures of mass eigenstates (PMNS)
  - However, neutrino masses tiny



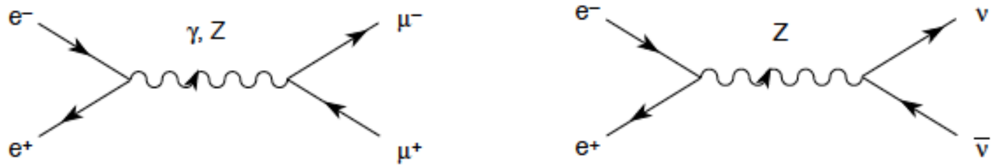
$$|\Delta m^2_{32}| \approx 2.5 \times 10^{-3}\,\mathrm{eV}^2$$

$$|\Delta m^2_{21}| \approx 8 \times 10^{-5}\,\mathrm{eV}^2$$

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Neutral currents

- neutral carriers of the EM & weak interactions have fermionic couplings with following properties:
- All interacting vertices are flavour conserving.
  - Both γ & Z couple to fermion & own antifermion, i.e., $\gamma f\bar{f}\ ; Zf\bar{f}$
  - but NO transitions of type $\mu^- \longrightarrow e^-\gamma\ ;\ \ Z\ \longrightarrow \mu^\pm e^\mp$
- Interactions depend on fermion electric charge Q_f
  - Fermions with same Q_f have exactly same universal couplings
  - Neutrinos do not have EM interactions ($Q_\nu = 0$), but have non-zero coupling to Z boson
- Photons have same interaction for both fermion chiralities,
  - but Z couplings are different for LH & RH fermions.
  - neutrino coupling to Z involves only LH chiralities.
- 3 different light neutrino species



3.2 – $SU(2)_L \otimes U(1)_Y$ theory

Exercise: derive EW Lagrangian

- ☐ Gauge invariance
  - ☐ able to determine right QED & QCD Lagrangians
  - ☐ to describe weak interactions, need more elaborated structure
    - ☐ several fermionic flavours and different properties for LH & RH fields;
    - ☐ LH fermions appear in doublets
    - ☐ massive gauge bosons $W^{\pm}$ & Z in addition to photon
- ☐ Simplest group with doublet representations: SU(2)
  - ☐ need an additional U(1) group to include also EM interactions
  - ☐ Obvious symmetry group to consider $G \equiv SU(2)_L \otimes U(1)_Y$
    - ☐ L refers to LH fields; Y: hypercharge (→ naive identification with EM does not work)
- ☐ Consider single family of quarks (valid for lepton sector)

$$\psi_1(x) = \begin{pmatrix} u \\ d \end{pmatrix}_L \;; \qquad \psi_2(x) = u_R\,; \qquad \psi_3(x) = d_R$$

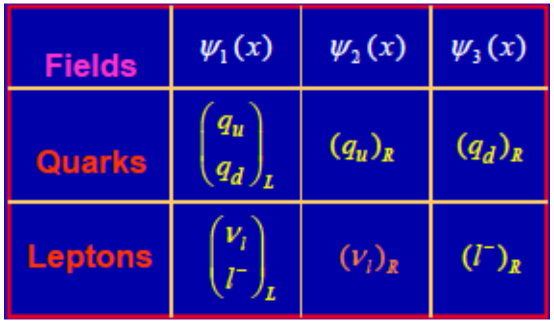
$$\psi_1(x) = \begin{pmatrix} \upsilon_e \\ e^- \end{pmatrix}_L \;; \qquad \psi_2(x) = \upsilon_{eR}; \qquad \psi_3(x) = e_R^-$$

$$\begin{bmatrix} \upsilon_e & u \\ e^- & d' \end{bmatrix}, \begin{bmatrix} \upsilon_\mu & c \\ \mu^- & s' \end{bmatrix}, \begin{bmatrix} \upsilon_\tau & t \\ \tau^- & b' \end{bmatrix}$$

- As in QED & QCD

$$\mathcal{L}_0 = i\bar{u}(x)\gamma^\mu\partial_\mu u(x) + i\bar{d}(x)\gamma^\mu\partial_\mu d(x) = \sum_{j=1}^{3} i\,\bar{\psi}_j(\mathrm{x})\,\gamma^\mu\partial_\mu\psi_j(x)$$

  - $\mathcal{L}_0$ is invariant under global G transformations in flavour space



Fields	$\psi_1(x)$	$\psi_2(x)$	$\psi_3(x)$
Quarks	$\begin{pmatrix} q_u \\ q_d \end{pmatrix}_L$	$(q_u)_R$	$(q_d)_R$
Leptons	$\begin{pmatrix} \nu_l \\ l^- \end{pmatrix}_L$	$(\nu_l)_R$	$(l^-)_R$

$$\psi_1(x) \overset{G}{\rightarrow} \psi_1'(x) \equiv e^{iy_1\beta}U_L\psi_1(x)$$
$$\psi_2(x) \overset{G}{\rightarrow} \psi_2'(x) \equiv e^{iy_2\beta} \quad \psi_2(x)$$
$$\psi_3(x) \overset{G}{\rightarrow} \psi_3'(x) \equiv e^{iy_3\beta} \quad \psi_3(x)$$

$$U_L = e^{i\frac{\sigma^i}{2}\alpha_i} \;; i = 1{,}2{,}3$$

    - U_L only acts on doublet field ψ_1.
    - Parameters y_i : hypercharges; $U(1)_Y$ phase transformation analogous QED.
    - Matrix transformation U_L non-Abelian as in QCD.
  - Note no mass-term – would mix the LH&RH fields – thus spoiling symmetry considerations
- Require $\mathcal{L}_0$ invariant under local $SU(2)_L \otimes U(1)_Y$ gauge transformations $\alpha_i=\alpha_i(x)$, $\beta=\beta(x)$
  - To satisfy symmetry requirement, covariant derivatives
    - $SU(2)_L$ matrix field

$$\widetilde{W}^\mu(x) \equiv \frac{\sigma^i}{2}W_i^\mu(x)$$

$$D^\mu\psi_1(x) \equiv \left(\partial^\mu + ig\widetilde{W}^\mu(x) + \; ig'y_1B^\mu(x)\;\right)\psi_1(x)$$
$$D^\mu\psi_2(x) \equiv (\partial^\mu + \; ig'y_2B^\mu(x)\;)\psi_2(x)$$
$$D^\mu\psi_3(x) \equiv (\partial^\mu + \; ig'y_3B^\mu(x)\;)\psi_3(x)$$

- 4 gauge parameters, $\alpha_i(x)$ & $\beta(x)$ → 4 different gauge bosons needed to describe $W^\pm$, Z and γ
- $D^\mu\psi_j(x)$ must transform in exactly same way as $\psi_j(x)$ fields
  - This fixes transformation properties of gauge fields

$$B^\mu(x) \overset{G}{\to} B^{\mu\prime} \equiv B^\mu(x) - \frac{1}{g'}\partial^\mu\,\beta(x)$$

$$U_L = e^{i\frac{\sigma^i}{2}\alpha_i} \qquad \widetilde{W}^\mu \overset{G}{\to} \widetilde{W}^{\mu\prime} \equiv U_L(x)\widetilde{W}^\mu U_L^\dagger(x) + \frac{i}{g}\partial^\mu U_L(x)U_L^\dagger(x)$$

  - $U(1)_L$ transformation of B^μ as in QED for photon
  - $SU(2)_L$: W^μ_i fields transform analogous to gluon fields of QCD.
  - Note:
    - ψ_j couplings to B_μ completely free as in QED, i.e., hypercharges y_j arbitrary parameters
    - $SU(2)_L$ commutation relation is non-linear → no such freedom for W^μ_i : a unique $SU(2)_L$ coupling g

- Lagrangian
  - invariant under local *G* transformations

$$\mathcal{L} = \sum_{j=1}^{3} i\,\bar{\psi}_j(\mathrm{x})\,\gamma^\mu D_\mu\psi_j(x)$$

  - To build gauge-invariant kinetic term for gauge fields, introduce corresponding field strengths

$$B_{\mu\nu} \equiv \partial_\mu\,B_\nu - \partial_\nu\,B_\mu \qquad \widetilde{W}_{\mu\upsilon} \equiv -\frac{i}{g}[D_\mu, D_\upsilon] = -\frac{i}{g}\left[\,(\partial_\mu + ig\widetilde{W}_\mu)(\partial_\upsilon + ig\widetilde{W}_\upsilon)\right] = \partial_\mu\widetilde{W}_\upsilon - \partial_\upsilon\widetilde{W}_\mu + ig[\widetilde{W}_\mu, \widetilde{W}_\upsilon]$$

$$\widetilde{W}_{\mu\upsilon} \equiv \frac{\sigma_i}{2}W^i_{\mu\upsilon}\ ; \qquad W^i_{\mu\upsilon} = \partial_\mu W^i_\upsilon - \partial_\upsilon W^i_\mu - g\epsilon^{ijk}W^j_\mu W^k_\upsilon$$

- $B_{\mu\nu}$ remains invariant under G transformations, while $\widetilde{W}_{\mu\upsilon}$ transforms covariantly

$$B_{\mu\nu} \overset{G}{\rightarrow} \equiv B_{\mu\nu}\,; \qquad \widetilde{W}_{\mu\upsilon} \overset{G}{\rightarrow} U_L \widetilde{W}_{\mu\upsilon} U_L^{\dagger}$$

- properly normalized kinetic Lagrangian

$$\mathcal{L}_{kin} = -\frac{1}{4} B_{\mu\nu}\, B^{\mu\nu} - \frac{1}{2} Tr\left[\widetilde{W}_{\mu\upsilon} \widetilde{W}^{\mu\upsilon}\right] = -\frac{1}{4} B_{\mu\nu}\, B^{\mu\nu} - \frac{1}{4} W_{\mu\nu}^{i} W_{i}^{\mu\nu}$$

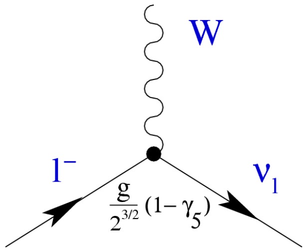
- field strength $W_{\mu\nu}^{i}$ has quadratic piece ⟶ $\mathcal{L}_{kin}$ gives rise to cubic & quartic gauge fields self-interactions
  - strength of these interactions is given by same SU(2)$_L$ coupling g of fermionic piece of Lagrangian

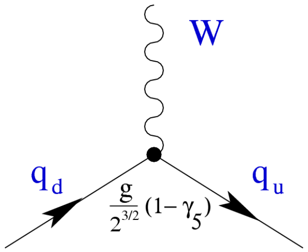
- Gauge symmetry
  - forbids gauge boson mass term
  - fermionic masses not possible –
    - would communicate LH & RH fields with ≠ properties
    - ⟶ would produce an explicit breaking gauge symmetry

$$\begin{aligned}\mathcal{L} &= i\,\bar{\psi}\,\gamma^{\mu}\partial_{\mu}\psi - m\,\bar{\psi}\psi \\ &= \bar{\psi}_L i\,\gamma^{\mu}\partial_{\mu}\psi_L + \bar{\psi}_R i\,\gamma^{\mu}\partial_{\mu}\psi_R - m(\bar{\psi}_L\psi_R + \bar{\psi}_R\,\psi_L)\end{aligned}$$

- SU(2)$_L$⊗U(1)$_Y$ Lagrangian only contains massless fields!

3.3 – Charged-current interaction





- Lagrangian contains
  - interactions of fermion fields with gauge bosons

$$\mathcal{L} \longrightarrow -g\bar{\psi}_1\gamma^{\mu}\widetilde{W}_{\mu}\psi_1 - g'B_{\mu}\sum_{j=1}^{3} y_j\,\bar{\psi}_j\,\gamma^{\mu}\psi_j$$

  - term containing SU(2)$_L$ matrix

$$\widetilde{W}_{\mu} \equiv \frac{\sigma_i}{2}W_{\mu}^{i} = \frac{1}{2}\begin{pmatrix} W_{\mu}^{3} & \sqrt{2}W_{\mu}^{\dagger} \\ \sqrt{2}W_{\mu} & -W_{\mu}^{3} \end{pmatrix}$$

    - gives rise to charged-current interactions with boson field and its complex conjugate

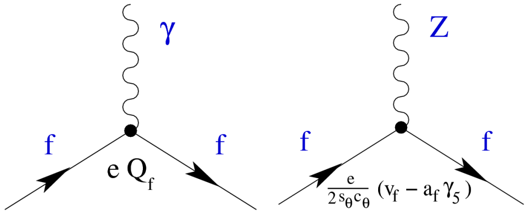
$$W_{\mu} \;= \frac{1}{\sqrt{2}}\left(W_{\mu}^{1} + iW_{\mu}^{2}\right); \qquad W_{\mu}^{\dagger} = \frac{1}{\sqrt{2}}\left(W_{\mu}^{1} - iW_{\mu}^{2}\right)$$

  - For a single family of quarks and leptons (check)

$$\mathcal{L}_{CC} = -\frac{g}{2\sqrt{2}}\left\{W_{\mu}^{\dagger}\left[\bar{u}\,\gamma^{\mu}(1-\gamma^{5})d + \bar{\nu}_e\,\gamma^{\mu}(1-\gamma^{5})e\right] + h.c\right\}$$

  - Universality of quark & lepton interactions
    - now a direct consequence of the assumed gauge symmetry.
    - BUT $\mathcal{L}_{CC}$ cannot describe observed dynamics – gauge bosons massless – long-range forces.

3.4 – Neutral-current interactions



- ❑ Lagrangian contains $\mathcal{L}_{NC} = -\, g\, W^3_\mu\ \bar{\psi}_1 \gamma^\mu \frac{\sigma_3}{2} \psi_1 \ -\ g'\, B_\mu \sum_j y_j \bar{\psi}_j \gamma^\mu \psi_j$
  - ❑ also interactions with neutral gauge fields W^3_μ and B_μ identified with the Z and the γ.
  - ❑ However, since photon has same interaction with both fermion chiralities
    - ❑ singlet gauge boson B_μ cannot be EM field, requiring $y_1=y_2=y_3$ and $g'y_j=eQ_j$ - cannot be simultaneously true
    - ❑ arbitrary combination of both neutral fields

$$\begin{pmatrix} W^3_\mu \\ B_\mu \end{pmatrix} \equiv \begin{pmatrix} cos\theta_W & sin\theta_W \\ -sin\theta_W & cos\theta_W \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}$$

  - ❑ physical Z boson massive – forbidden by local gauge symmetry.
    - ❑ possible to generate non-zero boson masses, through the spontaneous symmetry breaking (SSB) mechanism
    - ❑ neutral mass eigenstates = mixture of triplet & singlet SU(2)$_L$ fields.
  - ❑ In terms of the fields Z and γ, neutral-current Lagrangian:

$$\mathcal{L}_{NC} = -\sum_{j=1}^{3} \bar{\psi}_j\, \gamma^\mu \left\{ A_\mu \left[g\frac{\sigma_3}{2} sin\theta_W + g' y_j cos\theta_W \right] + Z_\mu \left[g\frac{\sigma_3}{2} cos\theta_W - g' y_j sin\theta_W \right] \right\} \psi_j$$

    - ❑ to get QED from A$_\mu$ piece, impose conditions (*check*): $g\, sin\theta_W = g' cos\theta_W = e\ ,\quad Y = Q - T_3 = Q - \frac{\sigma_3}{2}$

$$\mathcal{L}_{NC} = -\sum_{j=1}^{3} \bar{\psi}_j\, \gamma^\mu \left\{ A_\mu \left[g\,\frac{\sigma_3}{2} sin\theta_W + g' y_j cos\theta_W \right] + Z_\mu \left[g\,\frac{\sigma_3}{2} cos\theta_W - g' y_j sin\theta_W \right] \right\} \psi_j$$

- to get QED from A_μ piece, impose conditions: $g\ sin\theta_W = g' cos\theta_W = e$, $Y = Q - T_3 = Q - \frac{\sigma_3}{2}$
- *Q*: EM charge operator $Q_1 \equiv \begin{pmatrix} Q_{u,v} & 0 \\ 0 & Q_{d,e} \end{pmatrix}$; $Q_2 = Q_{u,v}$; $Q_3 = Q_{d,e}$

- $g\ sin\theta_W = g' cos\theta_W = e$ relates *$SU(2)_L$* and *$U(1)_Y$* couplings to *EM* coupling,
  - provides wanted unification of electroweak interactions.
- $Y = Q - T_3$ fixes the fermion hypercharges in terms of electric charge and weak isospin QNs:
  - Quarks: $y_1 = Q_u - \frac{1}{2} = Q_d + \frac{1}{2} = \frac{1}{6}$; $y_2 = Q_u = \frac{2}{3}$; $y_3 = Q_d = -\frac{1}{3}$
  - Leptons: $y_1 = Q_v - \frac{1}{2} = Q_e + \frac{1}{2} = -\frac{1}{2}$; $y_2 = Q_v = 0$; $y_3 = Q_e = -1$
- A hypothetical right-handed neutrino would have both *Q=0* and *y=0*
  - Not any kind of interaction = sterile – not considered further

- Neutral-current Lagrangian: $\mathcal{L}_{NC} = \mathcal{L}_{QED} + \mathcal{L}_{NC}^{Z}$

$$\mathcal{L}_{QED} = -eA_\mu \textstyle\sum_{j=1}^{3} \bar{\psi}_j\, \gamma^\mu Q_j \psi_j \equiv -e\, A_\mu J_{em}^\mu \qquad \mathcal{L}_{NC}^{Z} = -\frac{e}{2\, sin\theta_W\, cos\theta_W} J_Z^\mu Z_\mu$$

- Neutral fermionic current

$$J_Z^\mu \equiv \sum_{j=1}^{3} \bar{\psi}_j\, \gamma^\mu \left(\sigma_3 - 2sin^2\theta_W Q_j\right) \psi_j = J_3^\mu - 2sin^2\theta_W\, J_{em}^\mu$$

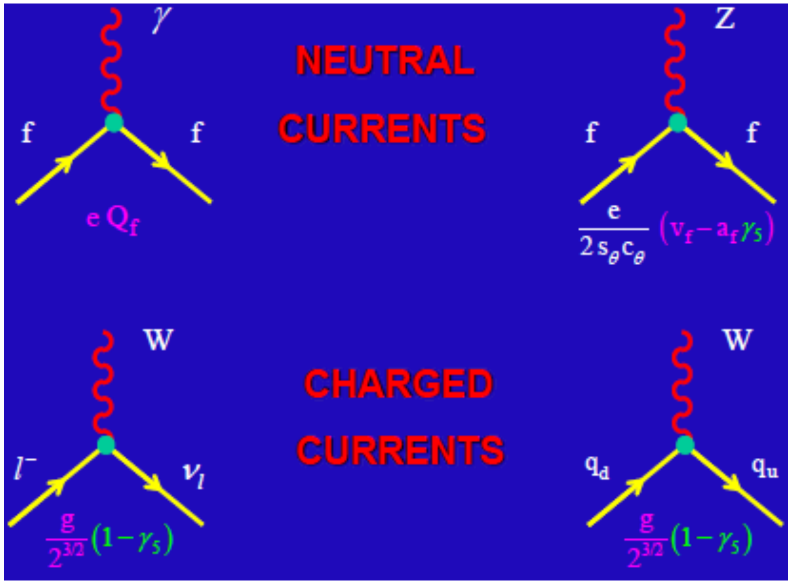
- In terms of more usual fields

$$\mathcal{L}_{NC}^{Z} = -\frac{e}{2\ sin\theta_W\ cos\theta_W} Z_\mu\ \sum_j \bar{f}\, \gamma^\mu \left(v_f - a_f\gamma_5\right) f$$

$$a_f = T_3^f; \qquad v_f = T_3^f\left(1 - 4\left|Q_f\right| sin^2\theta_W\right)$$

- Neutral-current (vector and axial-vector) couplings of different fermions

	u	d	v_e	e
$2v_f$	$1 - \frac{8}{3} sin^2\theta_W$	$-1 + \frac{4}{3} sin^2\theta_W$	1	$-1 + 4sin^2\theta_W$
$2a_f$	1	-1	1	-1



3.5 – Gauge self-interactions

$$B_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu$$

$$\widetilde{W}_{\mu\nu} = \partial_\mu W_\upsilon^i - \partial_\upsilon W_\mu^i - g\epsilon^{ijk} W_\mu^j W_\upsilon^k$$

$$W_\mu^{(\dagger)} = \frac{1}{\sqrt{2}}\left(W_\mu^1 \pm i W_\mu^2\right)$$

$$\begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} \equiv \begin{pmatrix} cos\theta_W & sin\theta_W \\ -sin\theta_W & cos\theta_W \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}$$

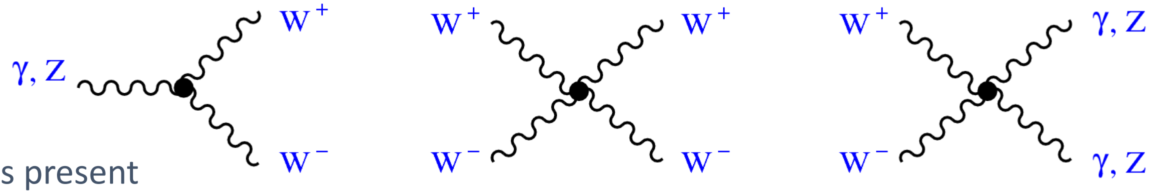
- ☐ In addition to the usual kinetic terms, $\mathcal{L}_{kin}$

$$\mathcal{L}_{kin} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} W_{\mu\nu}^i W_i^{\mu\nu}$$

- ☐ generates cubic & quartic self-interactions among gauge bosons (do it & get convinced)

$$\mathcal{L}_3 \\ = i\, e\, cot\theta_W\{(\partial^\mu W^\upsilon - \partial^\upsilon W^\mu) W_\mu^\dagger Z_\upsilon - (\partial^\mu W^{\upsilon\dagger} - \partial^\upsilon W^{\mu\dagger}) W_\mu Z_\upsilon + W_\mu W_\upsilon^\dagger (\partial^\mu Z^\upsilon - \partial^\upsilon Z^\mu)\} \\ + ie\{(\partial^\mu W^\upsilon - \partial^\upsilon W^\mu) W_\mu^\dagger A_\upsilon - (\partial^\mu W^{\upsilon\dagger} - \partial^\upsilon W^{\mu\dagger}) W_\mu A_\upsilon + W_\mu W_\upsilon^\dagger (\partial^\mu A^\upsilon - \partial^\upsilon A^\mu)\}$$

$$\mathcal{L}_4 \\ = -\frac{e^2}{2\, sin^2\theta_W}\left\{\left(W_\mu^\dagger W^\mu\right)^2 - W_\mu^\dagger W^{\mu\dagger} W_\upsilon W^\upsilon\right\} - e^2 cot^2\theta_W\{W_\mu^\dagger W^\mu Z_\upsilon Z^\upsilon - W_\mu^\dagger Z^\mu W_\upsilon Z^\upsilon\} \\ - e^2 \cot\theta_W\{2W_\mu^\dagger W^\mu Z_\upsilon A^\upsilon - W_\mu^\dagger Z^\mu W_\upsilon A^\upsilon - W_\mu^\dagger A^\mu W_\upsilon Z^\upsilon\} \\ - e^2\{W_\mu^\dagger W^\mu A_\upsilon A^\upsilon - W_\mu^\dagger A^\mu W_\upsilon A^\upsilon\}$$



- ☐ Notice
  - ☐ at least a pair of charged W bosons are always present
  - ☐ SU(2)$_L$ algebra does not generate any neutral vertex with only photons and/or Z bosons

$$\mathrm{W}_{\mu\nu} \equiv -\frac{i}{g}\left[\mathrm{D}_\mu\,,\mathrm{D}_\nu\right] \equiv \frac{\vec{\sigma}}{2}\cdot\vec{W}_{\mu\nu} \;\rightarrow\; \mathrm{U_L}\;\mathrm{W}_{\mu\nu}\;\mathrm{U_L^\dagger} \qquad ; \qquad B_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu \;\rightarrow\; B_{\mu\nu}$$

$$W^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu - g\,\varepsilon^{ijk}\,W^j_\mu W^k_\nu$$

$$\boxed{\mathcal{L}_{\mathrm{K}} = -\frac{1}{4}B_{\mu\nu}\,B^{\mu\nu} - \frac{1}{2}\mathrm{Tr}(\mathrm{W}_{\mu\nu}\,\mathrm{W}^{\mu\nu}) = -\frac{1}{4}B_{\mu\nu}\,B^{\mu\nu} - \frac{1}{4}\vec{W}_{\mu\nu}\,\vec{W}_{\mu\nu} = \mathcal{L}_{\mathrm{kin}} + \mathcal{L}_3 + \mathcal{L}_4}$$

$$\mathcal{L}_3 = i\,e\,\cot\theta_W\,\left\{(\partial^\mu W^\nu - \partial^\nu W^\mu)\,W^\dagger_\mu Z_\nu - (\partial^\mu W^{\nu\dagger} - \partial^\nu W^{\mu\dagger})\,W_\mu Z_\nu + W_\mu W^\dagger_\nu\,(\partial^\mu Z^\nu - \partial^\nu Z^\mu)\right\}$$

$$+\,i\,e\,\left\{\left(\partial^\mu W^\nu - \partial^\nu W^\mu\right)\,W^\dagger_\mu A_\nu - \left(\partial^\mu W^{\nu\dagger} - \partial^\nu W^{\mu\dagger}\right)\,W_\mu A_\nu + W_\mu W^\dagger_\nu\left(\partial^\mu A^\nu - \partial^\nu A^\mu\right)\right\}$$

$$\mathcal{L}_4 = -\frac{e^2}{2\sin^2\theta_W}\left\{\left(W^\dagger_\mu W^\mu\right)^2 - W^\dagger_\mu W^{\mu\dagger} W_\nu W^\nu\right\} - e^2\cot^2\theta_W\left\{W^\dagger_\mu W^\mu Z_\nu Z^\nu - W^\dagger_\mu Z^\mu W_\nu Z^\nu\right\}$$

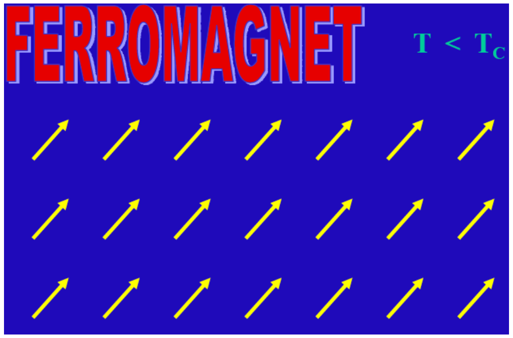
$$-e^2\cot\theta_W\left\{2W^\dagger_\mu W^\mu Z_\nu A^\nu - W^\dagger_\mu Z^\mu W_\nu A^\nu - W^\dagger_\mu A^\mu W_\nu Z^\nu\right\} - e^2\left\{W^\dagger_\mu W^\mu A_\nu A^\nu - W^\dagger_\mu A^\mu W_\nu A^\nu\right\}$$

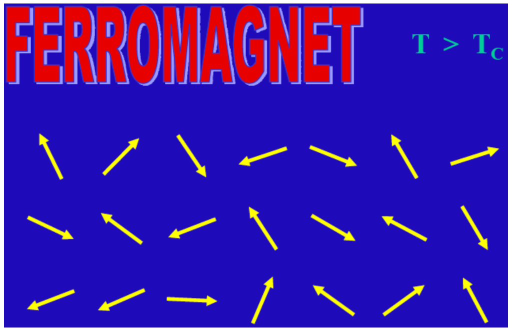
4 – Spontaneous Symmetry Breaking

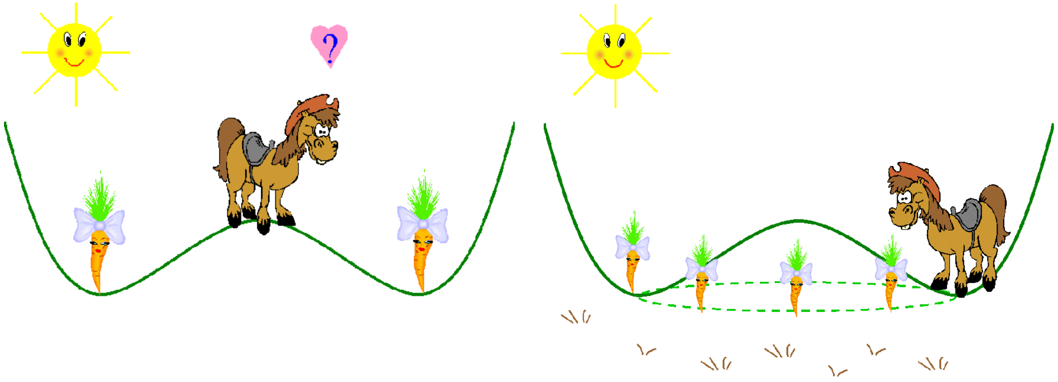
Study Thomson, MPP, ch. 17

- So far
  - able to derive charged- and neutral-current interactions of the type needed to describe weak decays
  - nicely incorporated QED into same theoretical framework
  - got additional self-interactions of gauge bosons, generated by non- Abelian structure of $SU(2)_L$ group
- Gauge symmetry guarantees a well-defined renormalizable Lagrangian.
  - However, Lagrangian makes sense only for massless gauge bosons
  - fine for photon field, not for physical $W^\pm$ and Z bosons – quite heavy objects
- To generate masses
  - need to break gauge symmetry in some way
  - While keeping fully symmetric Lagrangian to preserve renormalizability
  - Solve dilemma by possibility of getting non-symmetric results from a symmetric Lagrangian

- Consider Lagrangian
  - invariant under a group G of transformations
  - with degenerate set of states with minimal energy, transforming under G as members of a multiplet
- Symmetry spontaneously broken
  - if one of states arbitrarily selected as ground state of system
- Well-known physical example: ferromagnet
  - Hamiltonian invariant under rotations
    - ground state has electron spins aligned into some arbitrary direction
    - any higher-energy state, built from ground state by finite number of excitations, share this anisotropy
- In QFT,
  - ground state is vacuum
  - SSB mechanism will appear when there is a symmetric Lagrangian, but a non-symmetric vacuum

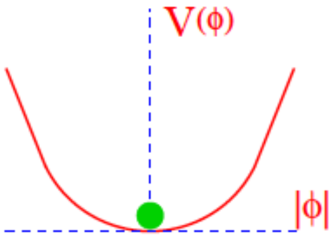


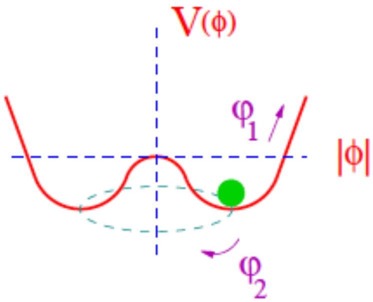




- Very simple illustration of SSB phenomenon
  - Although left and right carrots identical
    - Horse takes decision to get food
    - Not important whether he goes left or right – equivalent options – but symmetry gets broken
  - In 2 dimensions (discrete LR symmetry)
    - after 1st carrot horse makes effort – climb hill – to reach 2nd carrot
  - In 3 dimensions (continuous rotation symmetry)
    - marvelous flat circular valley for horse to move along from carrot to next without any effort.
- General property of SSB of continuous symmetries
  - Existence of flat directions connecting degenerate states of minimal energy
- In QFT
  - implies existence of massless degrees of freedom (d.o.f)

4.1 – Goldstone theorem





- consider a complex scalar field ϕ(x), with Lagrangian

$$\mathcal{L} = \partial^\mu \phi^\dagger \partial_\mu \phi - V(\phi)\,; \qquad V(\phi) = \mu^2 \phi^\dagger \phi + h\left(\phi^\dagger \phi\right)^2$$

  - $\mathcal{L}$ invariant under global phase transformations of scalar field

$$\phi\,(x) \;\longrightarrow \phi'(x) \equiv e^{i\theta}\phi(x)$$

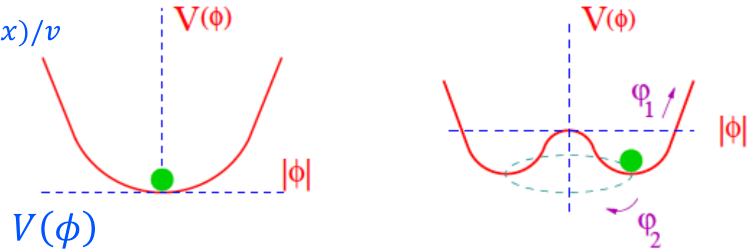
  - to have ground state, potential bounded from below, h > 0
    - $\mu^2 > 0$: potential has only trivial minimum ϕ = 0
      - massive scalar particle with mass μ and quartic coupling h.
    - $\mu^2 < 0$: minimum for field configurations satisfying

$$|\phi_0| = \sqrt{\frac{-\mu^2}{2h}} \equiv \frac{v}{\sqrt{2}}, \qquad V(\phi_0) = -\frac{h}{4}v^4$$

- U(1) phase invariance ($\mu^2 < 0$)
  - degenerate states of minimum energy, $\phi_0(x) = v/\sqrt{2}\, e^{i\theta}$
  - choose particular solution, θ = 0, as ground state ⟶ symmetry spontaneously broken
  - parametrize excitations over ground state as $\phi\,(x) \equiv \frac{1}{\sqrt{2}}[v + \varphi_1(x) + i\varphi_2(x)]$

$$V(\phi) = -\frac{h}{4}v^4 - \mu^2\varphi_1^2 + hv\varphi_1(\varphi_1^2 + \varphi_2^2) + \frac{h}{4}(\varphi_1^2 + \varphi_2^2)^2$$

  - φ_1 describes massive state $m_{\varphi_1}^2 = -2\mu^2$, while φ_2 is massless



- To 1st order in the fields $\phi\,(x) \equiv \frac{1}{\sqrt{2}}[v+\varphi_1(x)]e^{i\varphi_2(x)/v}$
- V and $\mathcal{L}$ take form $V(\phi) = V(\phi_0) + \frac{1}{2}m_{\varphi_1}^2\varphi_1^2 + hv\,\varphi_1^3 + \frac{h}{4}\varphi_1^4$

$$\mathcal{L}(\phi) = \frac{1}{2}\partial_\mu\varphi_1\partial^\mu\varphi_1 + \frac{1}{2}\left(1+\frac{\varphi_1}{v}\right)^2\partial_\mu\varphi_2\partial^\mu\varphi_2 - V(\phi)$$

$$\mathcal{L}(\phi) = \frac{1}{2}\partial_\mu\varphi_1\partial^\mu\varphi_1 - \frac{1}{2}m_{\varphi_1}^2\varphi_1^2 - hv\,\varphi_1^3 - \frac{h}{4}\varphi_1^4 + \frac{1}{2}\partial_\mu\varphi_2\partial^\mu\varphi_2 + \frac{\varphi_1}{v}\partial_\mu\varphi_2\partial^\mu\varphi_2 + \frac{1}{4}\left(\frac{\varphi_1}{v}\right)^2\partial_\mu\varphi_2\partial^\mu\varphi_2 - \frac{h}{4}v^4$$

  - φ_1: massive state $m_{\varphi_1}^2 = -2\mu^2$; φ_2: massless (Goldstone) boson
- $\mu^2 < 0$ – Case with Spontaneous Symmetry Breaking – appearance of massless particle
  - field φ2 describes excitations around a flat direction in V – into states with same energy as chosen ground state
  - excitations do not cost any energy – correspond to a massless state

- Existence of massless excitations associated with SSB mechanism
  - completely general result – **Goldstone theorem**
  - if Lagrangian invariant under continuous symmetry group G, but vacuum only invariant under subgroup H ⊂ G, then there must exist as many massless spin-0 particles (Nambu–Goldstone bosons) as broken generators (i.e., generators of G which do not belong to H)

Brout-Englert-Higgs (BEH) mechanism

- SSB of a complex scalar field with a potential $V(\phi) = \mu^2\phi^2 + \lambda\phi^4$
  - embedded in a theory with a local gauge symmetry
  - Example of U(1) local gauge symmetry used to introduce main ideas
  - Lagrangian $\mathcal{L} = (\partial_\mu\phi)^*(\partial^\mu\phi) - V(\phi)$ with $V(\phi) = \mu^2(\phi^*\phi) + \lambda(\phi^*\phi)^2$
    - Not invariant under local transformations

$$\phi(x) \to \phi'(x) = e^{ig\chi(x)}\phi(x)$$

    - Ok if

$$\partial_\mu \to D_\mu = \partial_\mu + igB_\mu \qquad B_\mu \to B'_\mu = B_\mu - \partial_\mu\chi(x) \qquad \mathcal{L} = (D_\mu\phi)^*(D^\mu\phi) - V(\phi^2)$$

  - combined $\mathcal{L}$ for complex scalar field ϕ and (massless) gauge field B

$$\mathcal{L} = -\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} + (D_\mu\phi)^*(D^\mu\phi) - \mu^2\phi^2 - \lambda\phi^4$$

  - term involving the covariant derivatives

$$\begin{aligned}(D_\mu\phi)^*(D^\mu\phi) &= (\partial_\mu - igB_\mu)\phi^*(\partial^\mu + igB^\mu)\phi \\ &= (\partial_\mu\phi)^*(\partial^\mu\phi) - igB_\mu\phi^*(\partial^\mu\phi) + ig(\partial_\mu\phi^*)B^\mu\phi + g^2B_\mu B^\mu\phi^*\phi\end{aligned}$$

- full expression for Lagrangian

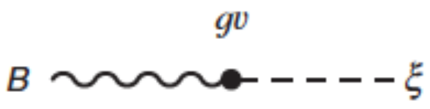
$$\mathcal{L} = -\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu} + (\partial_\mu\phi)^*(\partial^\mu\phi) - \mu^2\phi^2 - \lambda\phi^4$$
$$- igB_\mu\phi^*(\partial^\mu\phi) + ig(\partial_\mu\phi^*)B^\mu\phi + g^2B_\mu B^\mu\phi^*\phi$$

- Break symmetry, expand complex scalar field ϕ about vacuum state

$$\phi(x) = \tfrac{1}{\sqrt{2}}(v + \eta(x) + i\xi(x))$$

$$\mathcal{L} = \underbrace{\tfrac{1}{2}(\partial_\mu\eta)(\partial^\mu\eta) - \lambda v^2\eta^2}_{\text{massive } \eta} + \underbrace{\tfrac{1}{2}(\partial_\mu\xi)(\partial^\mu\xi)}_{\text{massless } \xi} - \underbrace{\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}g^2v^2B_\mu B^\mu}_{\text{massive gauge field}} - V_{int} + gvB_\mu(\partial^\mu\xi)$$

- $V_{int}(\eta,\xi,B)$ contains the 3- and 4-point interaction terms of fields η, ξ and B
- SSB produces a massive scalar field η and a massless Goldstone boson ξ.
- In addition, previously massless gauge field B has acquired a mass term $1/2\ g^2v^2B_\mu B^\mu$
- Pbs: $gvB_\mu(\partial^\mu\xi)$ term – direct coupling between Goldstone field ξ and gauge field B
  - Associated with longitudiunal polarisation state of B
  - Additional degree of freedom; Non-physical fields?



- Appropriate gauge transformation → eliminate ξ field from $\mathcal{L}$

$$\tfrac{1}{2}(\partial_\mu\xi)(\partial^\mu\xi) + gvB_\mu(\partial^\mu\xi) + \tfrac{1}{2}g^2v^2B_\mu B^\mu = \tfrac{1}{2}g^2v^2\left[B_\mu + \frac{1}{gv}(\partial_\mu\xi)\right]^2$$

$$\tfrac{1}{2}(\partial_\mu\xi)(\partial^\mu\xi) + gvB_\mu(\partial^\mu\xi) + \tfrac{1}{2}g^2v^2B_\mu B^\mu = \tfrac{1}{2}g^2v^2\left[B_\mu + \frac{1}{gv}(\partial_\mu\xi)\right]^2$$

- make the gauge transformation $B_\mu(x) \to B'_\mu(x) = B_\mu(x) + \frac{1}{gv}\partial_\mu\xi(x)$
- ➔ $$\mathcal{L} = \underbrace{\tfrac{1}{2}(\partial^\mu\eta)(\partial_\mu\eta) - \lambda v^2\eta^2}_{\text{massive }\eta} + \underbrace{- \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}g^2v^2B^{\mu\prime}B'_\mu}_{\text{massive gauge field}} - V_{int}$$
- choice of gauge corresponds to taking $\chi(x) = -\xi(x)/gv$ in $B_\mu \to B'_\mu = B_\mu - \partial_\mu\chi(x)$.
- corresponding gauge transformation of original complex scalar field $\phi(x)$ is

$$\phi(x) \to \phi'(x) = e^{-ig\frac{\xi(x)}{gv}}\phi(x) = e^{-i\xi(x)/v}\phi(x) \qquad \phi(x) = \frac{1}{\sqrt{2}}(v + \eta(x) + i\xi(x)) \;\sim\; \phi(x) \approx \frac{1}{\sqrt{2}}\left[v + \eta(x)\right]e^{i\xi(x)/v}$$

$$\phi(x) \to \phi'(x) = \frac{1}{\sqrt{2}}e^{-i\xi(x)/v}\left[v + \eta(x)\right]e^{i\xi(x)/v} = \frac{1}{\sqrt{2}}(v + \eta(x))$$

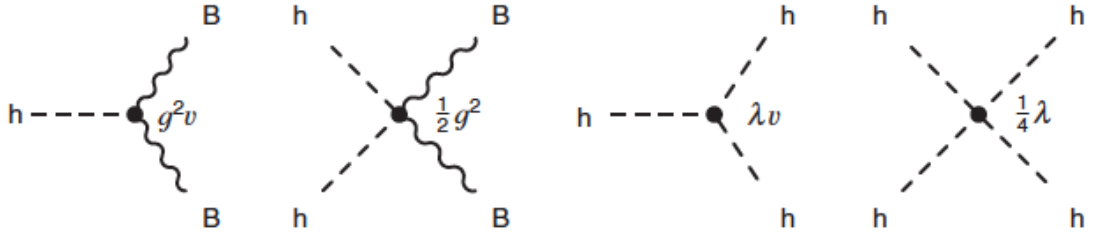
- ➔ Unitary Gauge eliminates Goldstone field $\xi(x)$ from $\mathcal{L}$
  - corresponds to choosing complex scalar field $\phi(x)$ to be entirely real

$$\phi(x) = \frac{1}{\sqrt{2}}(v + \eta(x)) \equiv \frac{1}{\sqrt{2}}(v + h(x))$$

- Important to remember
  - Physical predictions of theory do not depend on choice of gauge,
  - In unitary gauge, fields appearing in $\mathcal{L}$ correspond to physical particles – no “mixing” terms $B_\mu(\partial^\mu\xi)$
  - D.o.f. corresponding to Goldstone field $\xi(x)$ no longer appears in $\mathcal{L}$;
    - replaced longitudinal polarisation state of massive gauge field B
    - Goldstone boson has been “eaten” by the gauge field

$$\begin{aligned}\mathcal{L} &= (D_\mu\phi)^*(D^\mu\phi) - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} - \mu^2\phi^2 - \lambda\phi^4 \\ &= \tfrac{1}{2}(\partial_\mu - igB_\mu)(v+h)(\partial^\mu + igB^\mu)(v+h) - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} - \tfrac{1}{2}\mu^2(v+h)^2 - \tfrac{1}{4}\lambda(v+h)^4 \\ &= \tfrac{1}{2}(\partial_\mu h)(\partial^\mu h) + \tfrac{1}{2}g^2B_\mu B^\mu(v+h)^2 - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} - \lambda v^2h^2 - \lambda vh^3 - \tfrac{1}{4}\lambda h^4 + \tfrac{1}{4}\lambda v^4.\end{aligned}$$

$$\begin{aligned}\mathcal{L} = &\underbrace{\tfrac{1}{2}(\partial_\mu h)(\partial^\mu h) - \lambda v^2h^2}_{\text{massive } h \text{ scalar}} - \underbrace{\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}g^2v^2B_\mu B^\mu}_{\text{massive gauge boson}} \\ &+ \underbrace{g^2vB_\mu B^\mu h + \tfrac{1}{2}g^2B_\mu B^\mu h^2}_{h,B \text{ interactions}} - \underbrace{\lambda vh^3 - \tfrac{1}{4}\lambda h^4}_{h \text{ self-interactions}}.\end{aligned}$$



$$m_B = g\,v,$$

$$m_H = \sqrt{2\lambda}\,v.$$

  - Vacuum expectation value v sets the scale for the masses of both the gauge boson and the Higgs boson

4.2 – Massive gauge bosons

- Goldstone theorem a priori worsens mass problem by adding massless scalars …
- What about local gauge symmetry
  - consider $SU(2)_L$ doublet of complex scalar fields

$$\phi\,(x) \equiv \begin{pmatrix} \phi^{(+)}(x) \\ \phi^{(0)}(x) \end{pmatrix}$$

  - Gauge scalar Lagrangian $\mathcal{L}_S$ invariant under local $SU(2)_L \otimes U(1)_Y$ transformations

$$\mathcal{L}_S = D^\mu \phi^\dagger D_\mu \phi - \mu^2 \phi^\dagger \phi - h\left(\phi^\dagger \phi\right)^2 \qquad h > 0\,; \quad \mu^2 < 0$$

$$D^\mu \phi = \left(\partial^\mu + ig\widetilde{W}^\mu + \, ig' y_\phi B^\mu\,\right)\phi \qquad y_\phi = Q_\phi - T_3 = \frac{1}{2}$$

  - Value of scalar hypercharge fixed by requiring correct couplings between $\phi(x)$ and $A^\mu(x)$
  - i.e., photon does not couple to $\phi^{(0)}$, and $\phi^{(+)}$ has right electric charge
- The potential very similar to Goldstone model one
  - infinite set of degenerate states with minimum energy satisfying $\left|\langle 0|\phi^{(0)}|0\rangle\right| = \sqrt{\frac{-\mu^2}{2h}} \equiv \frac{v}{\sqrt{2}}$
    - association classical ground state with quantum vacuum more explicit
    - Since electric charge conserved, only neutral scalar field can acquire a vacuum expectation value (VEV).
    - Once particular ground state chosen , $SU(2)_L \otimes U(1)_Y$ symmetry spontaneously broken to EM subgroup $U(1)_{QED}$,
    - by construction $U(1)_{QED}$ still remains true symmetry of vacuum
    - According to Goldstone theorem 3 massless states should appear

- Parametrize scalar doublet in general form $\phi\ (x) = e^{i\frac{\sigma_i}{2}\theta^i(x)}\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + H\ (x) \end{pmatrix}$
  - Local SU(2)$_L$ invariance ➔ rotate away any dependence on $\vec{\theta}(x)$
    - 3 fields $\vec{\theta}(x)$: would-be massless Goldstone bosons associated with SSB mechanism
    - Unitary Gauge ➔

$$\phi\ (x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + H\ (x) \end{pmatrix}$$

  - Covariant derivative couples scalar multiplet to SU(2)$_L$ ⊗ U(1)$_Y$ gauge bosons

$$D^\mu\phi = \left(\partial^\mu + ig\widetilde{W}^\mu + \ ig'y_\phi B^\mu\ \right)\phi$$

    - physical (unitary) gauge $\vec{\theta}(x) = 0 \implies$ kinetic part of $\mathcal{L}_S$

$$(D^\mu\phi)^\dagger D^\mu\phi \ \overset{\theta_i}{\rightarrow}\ \frac{1}{2}\ \partial_\mu H\partial^\mu H + (v + h)^2\left\{\frac{g^2}{4}W_\mu^\dagger W^\mu + \frac{g^2}{8\cos^2\theta_W}Z_\mu\ Z^\mu\right\}$$

  - VEV of neutral scalar v generated quadratic term for the W± & Z , which acquire mass

$$M_Z\cos\theta_W\ = M_W\ = \frac{1}{2}vg$$

  - Clever way to give masses to intermediate carriers of weak force:
    - Add $\mathcal{L}_S$ to SU(2)$_L$ ⊗ U(1)$_Y$ model
  - Total Lagrangian invariant under gauge transformations
    - This guarantees renormalizability of the associated QFT

- Details leading to $\Longrightarrow$ kinetic part of $\mathcal{L}_S$ in previous slide

$$D_\mu\phi = \frac{1}{2\sqrt{2}}\begin{pmatrix} 2\partial_\mu + ig_W W_\mu^{(3)} + ig'B_\mu & ig_W[W_\mu^{(1)} - iW_\mu^{(2)}] \\ ig_W[W_\mu^{(1)} + iW_\mu^{(2)}] & 2\partial_\mu - ig_W W_\mu^{(3)} + ig'B_\mu \end{pmatrix}\begin{pmatrix} 0 \\ v+h \end{pmatrix}$$

$$= \frac{1}{2\sqrt{2}}\begin{pmatrix} ig_W(W_\mu^{(1)} - iW_\mu^{(2)})(v+h) \\ (2\partial_\mu - ig_W W_\mu^{(3)} + ig'B_\mu)(v+h) \end{pmatrix}.$$

$$(D_\mu\phi)^\dagger(D^\mu\phi) = \tfrac{1}{2}(\partial_\mu h)(\partial^\mu h) + \tfrac{1}{8}g_W^2(W_\mu^{(1)} + iW_\mu^{(2)})(W^{(1)\mu} - iW^{(2)\mu})(v+h)^2$$

$$+ \tfrac{1}{8}(g_W W_\mu^{(3)} - g'B_\mu)(g_W W^{(3)\mu} - g'B^\mu)(v+h)^2$$

- Gauge bosons masses determined by terms in $(D_\mu\phi)^\dagger(D^\mu\phi)$ quadratic in gauge boson fields

$$\tfrac{1}{8}v^2g_W^2\left(W_\mu^{(1)}W^{(1)\mu} + W_\mu^{(2)}W^{(2)\mu}\right) + \tfrac{1}{8}v^2\left(g_W W_\mu^{(3)} - g'B_\mu\right)\left(g_W W^{(3)\mu} - g'B^\mu\right)$$

- In $\mathcal{L}$, mass terms for the W(1) and W(2) spin-1 fields will appear as

$$\tfrac{1}{2}m_W^2 W_\mu^{(1)}W^{(1)\mu} \quad \text{and} \quad \tfrac{1}{2}m_W^2 W_\mu^{(2)}W^{(2)\mu}$$

$$m_W = \tfrac{1}{2}g_W v.$$

- Terms in $\mathcal{L}$ quadratic in the neutral $W^{(3)}$ and B fields

$$\tfrac{v^2}{8}\left(g_W W^{(3)}_\mu - g' B_\mu\right)\left(g_W W^{(3)\mu} - g' B^\mu\right) = \tfrac{v^2}{8}\begin{pmatrix} W^{(3)}_\mu & B_\mu \end{pmatrix}\begin{pmatrix} g_W^2 & -g_W g' \\ -g_W g' & g'^2 \end{pmatrix}\begin{pmatrix} W^{(3)\mu} \\ B^\mu \end{pmatrix} = \tfrac{v^2}{8}\begin{pmatrix} W^{(3)}_\mu & B_\mu \end{pmatrix}\mathbf{M}\begin{pmatrix} W^{(3)\mu} \\ B^\mu \end{pmatrix}$$

- Off-diagonal elements of M couple $W^{(3)}$ and B fields → mixing
- PDiagonalise **M** to get physical boson fields – independent eigenstates of free Hamiltonian

$$\det(\mathbf{M} - \lambda I) = 0 \qquad (g_W^2 - \lambda)(g'^2 - \lambda) - g_W^2 g'^2 = 0 \qquad \lambda = 0 \quad \text{or} \quad \lambda = g_W^2 + g'^2$$

$$\frac{1}{8}v^2\begin{pmatrix} A_\mu & Z_\mu \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 0 & g_W^2 + g'^2 \end{pmatrix}\begin{pmatrix} A^\mu \\ Z^\mu \end{pmatrix} \qquad \tfrac{1}{2}\begin{pmatrix} A_\mu & Z_\mu \end{pmatrix}\begin{pmatrix} m_A^2 & 0 \\ 0 & m_Z^2 \end{pmatrix}\begin{pmatrix} A^\mu \\ Z^\mu \end{pmatrix}$$

$$A_\mu = \frac{g' W^{(3)}_\mu + g_W B_\mu}{\sqrt{g_W^2 + g'^2}} \quad \text{with} \quad m_A = 0,$$

$$Z_\mu = \frac{g_W W^{(3)}_\mu - g' B_\mu}{\sqrt{g_W^2 + g'^2}} \quad \text{with} \quad m_Z = \tfrac{1}{2}v\sqrt{g_W^2 + g'^2}$$

$$m_A = 0 \quad \text{and} \quad m_Z = \tfrac{1}{2}v\sqrt{g_W^2 + g'^2}.$$

$$\frac{g'}{g_W} = \tan\theta_W$$

$$A_\mu = \cos\theta_W B_\mu + \sin\theta_W W^{(3)}_\mu,$$
$$Z_\mu = -\sin\theta_W B_\mu + \cos\theta_W W^{(3)}_\mu$$

$$m_Z = \frac{1}{2}\frac{g_W}{\cos\theta_W}v$$

$$m_W = \tfrac{1}{2}g_W v. \qquad v = 246\,\text{GeV} \qquad \frac{m_W}{m_Z} = \cos\theta_W$$

❑ Coupling to gauge bosons

$$(D_\mu\phi)^\dagger(D^\mu\phi) = \tfrac{1}{2}(\partial_\mu h)(\partial^\mu h) + \tfrac{1}{8}g_{\mathrm{W}}^2(W_\mu^{(1)} + iW_\mu^{(2)})(W^{(1)\mu} - iW^{(2)\mu})(v+h)^2$$

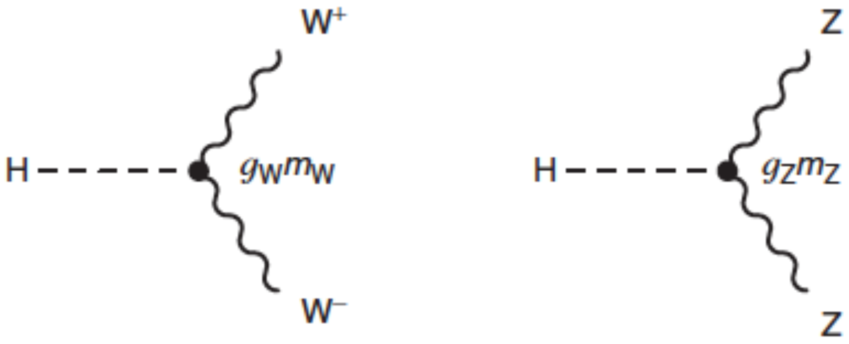
$$+ \tfrac{1}{8}(g_{\mathrm{W}} W_\mu^{(3)} - g' B_\mu)(g_{\mathrm{W}} W^{(3)\mu} - g' B^\mu)(v+h)^2$$

❑ coupling strength at the hW^+W^- vertex

$$\mathrm{W}^\pm = \frac{1}{\sqrt{2}}\left(\mathrm{W}^{(1)} \mp i\mathrm{W}^{(2)}\right)$$

$$\boxed{\tfrac{1}{4}g_{\mathrm{W}}^2 W_\mu^- W^{+\mu}(v+h)^2 = \tfrac{1}{4}g_{\mathrm{W}}^2 v^2 W_\mu^- W^{+\mu} + \tfrac{1}{2}g_{\mathrm{W}}^2 v W_\mu^- W^{+\mu} h + \tfrac{1}{4}g_{\mathrm{W}}^2 W_\mu^- W^{+\mu} hh.}$$

$$g_{\mathrm{HWW}} = \tfrac{1}{2}g_{\mathrm{W}}^2 v \equiv g_{\mathrm{W}} m_{\mathrm{W}}$$



- SSB occurs
  - Broken generators give rise to 3 massless Goldstone bosons

    $$\phi\,(x) = e^{i\,\frac{\sigma_i}{2}\,\theta^i(x)}\begin{pmatrix}0\\ v + H\,(x)\end{pmatrix}$$

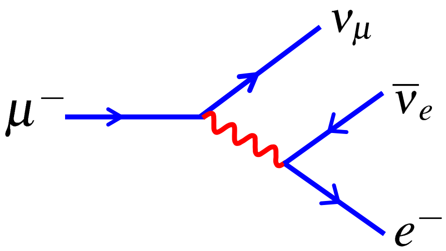
    - SU(2)$_L$ $invariance \implies \vec{\theta}(x)$ can be rotated away
    - Unitary Gauge: $\vec{\theta}(x) = 0$
    - W± & Z acquired mass – not photon
- Number of d.o.f. =?
  - Before SSB mechanism: 10 d.o.f
    - massless W± & Z bosons: 3 X 2 = 6 d.o.f
    - 4 real scalar fields: 4 X 1 = 4 d.o.f
  - After SSB: 10 d.o.f
    - 3 Goldstone modes ‘eaten’ by weak gauge bosons, becoming massive: 3 X 3 = 9 d.o.f
    - 1 remaining scalar particle H – Higgs boson

4.3 – Predictions

$$g\ sin\theta_W = g' cos\theta_W = e$$

- Ingredients to describe EW interaction within well-defined QFT
  - $M_Z = 91.1875 \pm 0.0021\ GeV;\ M_W = 80.399 \pm 0.023\ GeV$

$$M_Z \cos\theta_W = M_W = \frac{1}{2} v g \qquad \Longrightarrow \qquad sin^2\ \theta_W = 1 - \frac{M_W^2}{M_Z^2} = 0.223$$



- Independent estimate of $sin^2\theta_W$from muon-decay
  - Propagator $\dfrac{g^2}{M_W^2 - q^2} \approx \dfrac{g^2}{M_W^2} = \dfrac{4\pi\alpha}{sin^2\theta_W M_W^2} = 4\sqrt{2} G_F$
  - Lifetime $\tau_\mu = (2.1969803 \pm 0.0000022) \cdot 10^{-6} s$

$$\frac{1}{\tau_\mu} = \Gamma_\mu = \frac{G_F^2 M_\mu^5}{192\pi^2} f\left(\frac{m_e^2}{m_\mu^2}\right)(1 + \delta_{RC})\ \ ;\ \ f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 log x$$

  - Fermi coupling constant $G_F = (1.1663788 \pm 0.0000007) \cdot 10^{-5} GeV^{-2}$

Higher-order corrections

$$\alpha, M_W, G_F \qquad \Longrightarrow\ sin^2\theta_W = 0.215$$

- Fermi coupling ⟹ direct determination of EW scale – scalar vacuum expectation value

$$\mathrm{v} = \left(\sqrt{2}\ G_F\right)^{-1/2} = 246\ GeV$$

4.4 – The Higgs Boson

- $\mathcal{L}_S$ introduced new scalar particle into model – Higgs H

$$\mathcal{L}_S = D^\mu\phi^\dagger D_\mu\phi - \mu^2\phi^\dagger\phi - h\left(\phi^\dagger\phi\right)^2$$

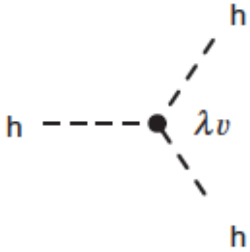
$$D^\mu\phi = \left(\partial^\mu + ig\widetilde{W}^\mu + \ ig'y_\phi B^\mu\ \right)\phi$$

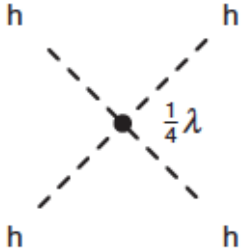
  - In terms of physical fields (unitary gauge), $\mathcal{L}_S$ takes form (*check*)

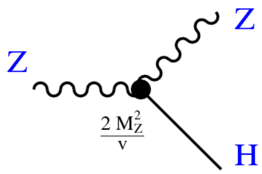
$$\mathcal{L}_S = \frac{1}{4}hv^4 + \mathcal{L}_H + \mathcal{L}_{HG^2}$$

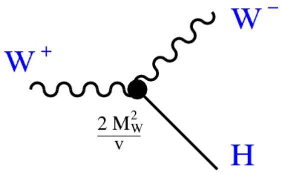
$$\mathcal{L}_H = \frac{1}{2}\ \partial_\mu H\partial^\mu H - \frac{1}{2}M_H^2H^2 - \frac{M_H^2}{2v}H^3 - \frac{M_H^2}{8v^2}\ H^2$$

$$\mathcal{L}_{HG^2} = M_W^2\ W_\mu^\dagger W^\mu\left\{1 + \frac{2}{v}H + \frac{H^2}{v^2}\right\} + \frac{1}{2}M_Z^2\ Z_\mu\ Z^\mu\ \left\{1 + \frac{2}{v}H + \frac{H^2}{v^2}\right\}$$



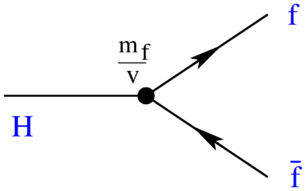


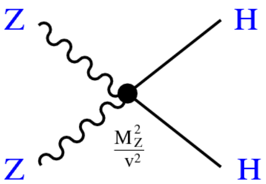


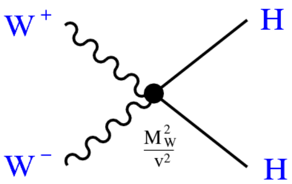


  - Higgs mass: $M_H = \sqrt{-2\mu^2} = \sqrt{2hv} = 125.09 \pm 0.24\ GeV$

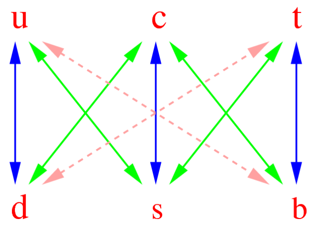
- Coupling of Higgs to gauge bosons $\propto m_V$
- Coupling of Higgs to fermions $\propto m_f$







4.5 – Fermion masses – see next 2 slides



- Fermion mass term not allowed – breaks gauge symmetry
  - would communicate LH & RH fields with ≠ properties
  - ⟶ would produce an explicit breaking gauge symmetry

$$\mathcal{L} = i\,\bar{\psi}\,\gamma^{\mu}\partial_{\mu}\psi - m\,\bar{\psi}\psi = \bar{\psi}_L i\,\gamma^{\mu}\partial_{\mu}\psi_L + \bar{\psi}_R i\,\gamma^{\mu}\partial_{\mu}\psi_R - m(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L)$$

$$\mathcal{L}_m = -m\,\bar{\psi}\psi = -m(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L)$$

- With additional scalar doublet, possible to write gauge-invariant fermion-scalar coupling

$$\mathcal{L}_Y = -c_1\left(\bar{u},\bar{d}\right)_L \begin{pmatrix}\phi^{(+)}\\ \phi^{(0)}\end{pmatrix} d_R - c_2\left(\bar{u},\bar{d}\right)_L \begin{pmatrix}\phi^{(0)*}\\ -\phi^{(-)}\end{pmatrix} u_R - c_3\left(\bar{\nu_e},\bar{e}\right)_L \begin{pmatrix}\phi^{(+)}\\ \phi^{(0)}\end{pmatrix} e_R + h.c.$$

  - Charge conjugate field $\phi^c \equiv i\sigma_2\,\phi^*$
  - Unitary gauge, Yukawa-type Lagrangian takes form (check)
  - SSB mechanism generates (does not predict) fermion masses
  - all Yukawa couplings are fixed in terms of masses

$$\mathcal{L}_Y = -\frac{1}{\sqrt{2}}(v+H)\{c_1\bar{d}d + c_2\bar{u}u + c_3\bar{e}e\}$$

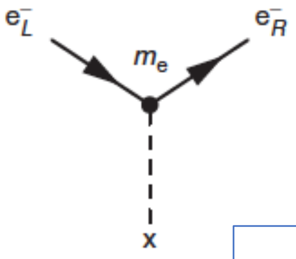
$$m_d = c_1\frac{v}{\sqrt{2}}\;;\quad m_u = c_2\frac{v}{\sqrt{2}}\;;\quad m_e = c_3\frac{v}{\sqrt{2}}$$

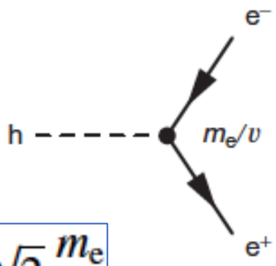
$$\mathcal{L}_Y = -\left(1+\frac{H}{v}\right)\{m_d\bar{d}d + m_u\bar{u}u + m_e\bar{e}e\}$$

Fermion masses

- Use BEH mechanism to generate masses of fermions
  - Mass term $-m\bar{\psi}\psi = -m\left(\bar{\psi}_R\psi_L + \bar{\psi}_L\psi_R\right)$ violates gauge symmetry
  - In SM, LH chiral fermions placed in SU(2) doublets *L* and RH fermions placed in SU(2) singlets *R*
  - two complex scalar fields of BEH mechanism in SU(2) doublet ϕ(x)
    - SU(2) local gauge transformation affects ϕ(x) as LH doublet *L*

$$\phi \longrightarrow \phi' = e^{i\frac{\sigma_i}{2}\theta^i(x)}\phi \qquad \bar{L} \equiv L^\dagger\gamma^0 \qquad \bar{L} \longrightarrow \bar{L}' = \bar{L}\, e^{-i\frac{\sigma_i}{2}\theta^i(x)}$$





    - $\bar{L}\phi$ invariant under $SU(2)_L$
    - $\bar{L}\phi R$ and $\bar{R}\phi^\dagger L$ invariant under $SU(2)_L X U(1)_Y$
    - $-g_f(\bar{L}\phi R + \bar{R}\phi^\dagger L)$ satisfies $SU(2)_L X U(1)_Y$ gauge symmetry

$$\mathcal{L}_e = -g_e\left[\left(\bar{\nu}_e\ \ \bar{e}\right)_L\begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix}e_R + \bar{e}_R\left(\phi^{+*}\ \ \phi^{0*}\right)\begin{pmatrix}\nu_e\\ e\end{pmatrix}_L\right]$$

$$\boxed{g_e = \sqrt{2}\frac{m_e}{v}}$$

    - After SSB

$$\phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v+h(x)\end{pmatrix} \quad \boxed{\mathcal{L}_e = -\frac{g_e}{\sqrt{2}}v\left(\bar{e}_Le_R + \bar{e}_Re_L\right) - \frac{g_e}{\sqrt{2}}h\left(\bar{e}_Le_R + \bar{e}_Re_L\right)} \quad \boxed{\mathcal{L}_e = -m_e\bar{e}e - \frac{m_e}{v}\bar{e}eh}$$

- VEV occurs in lower (neutral) component of Higgs doublet,
  - $-g_f(\overline{L}\phi R + \overline{R}\phi^\dagger L)$ only generate masses for fermion in lower component of SU(2)$_L$ doublet
- construct conjugate doublet ϕ_c $\phi_c = -i\sigma_2\phi^* = \begin{pmatrix} -\phi^{0*} \\ \phi^- \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} -\phi_3 + i\phi_4 \\ \phi_1 - i\phi_2 \end{pmatrix}$

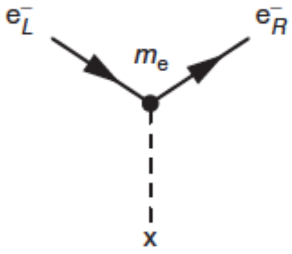
$$\mathcal{L}_u = g_u \begin{pmatrix} \bar{u} & \bar{d} \end{pmatrix}_L \begin{pmatrix} -\phi^{0*} \\ \phi^- \end{pmatrix} u_R \qquad \mathcal{L}_u = -\frac{g_u}{\sqrt{2}} v\,(\bar{u}_L u_R + \bar{u}_R u_L) - \frac{g_u}{\sqrt{2}} h\,(\bar{u}_L u_R + \bar{u}_R u_L)$$

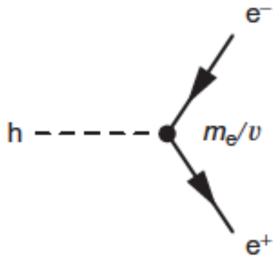
$$g_u = \sqrt{2} m_u / v \qquad \mathcal{L}_u = -m_u \bar{u}u - \frac{m_u}{v}\bar{u}uh$$

- Yukawa couplings of fermions to Higgs field

$$g_f = \sqrt{2}\frac{m_f}{v}$$

$$v = 246\,\mathrm{GeV}$$





- M_t~173.5±1GeV ➔ g_t~1
- If m(neutrinos) ➔ g_{nu} <~10^{-12} !
- ➔ mechanism generating neutrino masses might be different – Seesaw mechanism?

5 – Electroweak phenomenology

Study Thomson, MPP, ch. 15, 16

- Gauge and scalar sectors of SM Lagrangian – 4 parameters:
  - *g, g′, μ^2 , h* or α, θ_W, M_W , M_H
  - Alternatively, choose as free parameters
    - $G_F = (1.166\,378\,8 \pm 0.000\,000\,7) \cdot 10^{-5} GeV^{-2}$
    - α^{-1} = 137.035 999 084 ± 0.000 000 051
    - $M_Z = 91.187\,5 \pm 0.002\,1\ GeV$
    - $M_H = 125.09 \pm 0.24\ GeV$

$$M_W = 80.399 \pm 0.023\ GeV$$

$$sin^2\theta_W = 0.223$$

$$sin^2\theta_W = 1 - \frac{M_W^2}{M_Z^2}$$

$$M_W^2\ sin^2\theta_W = \frac{\pi\alpha}{\sqrt{2}G_F}$$

$$\Rightarrow \quad sin^2\theta_W = 0.212 \qquad M_W = 80.94\ GeV$$

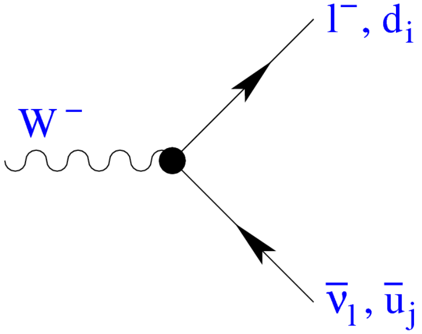
Radiative corrections

$$\alpha^{-1}(M_Z^2) = 128.93 \pm 0.05$$

$$\Rightarrow \quad sin^2\theta_W = 0.231 \qquad M_W = 79.96\ GeV$$

5 – Electroweak phenomenology

- Decay width of weak bosons (calculate)



$$\boxed{\Gamma(W^- \longrightarrow \bar{\upsilon}_l l^-) = \frac{G_F M_W^3}{6\pi\sqrt{2}}} \qquad \boxed{\Gamma(W^- \longrightarrow \bar{u}_i d_j) = N_C |V_{ij}|^2 \frac{G_F M_W^3}{6\pi\sqrt{2}}}$$

- $\bar{u}_i = \bar{u}, \bar{c}$; $\begin{pmatrix} d' \\ s' \end{pmatrix} \approx \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$ $\Rightarrow$ $Br(W^- \longrightarrow \bar{\upsilon}_l l^-) = \frac{\Gamma(W^- \longrightarrow \bar{\upsilon}_l l^-)}{\Gamma(W^- \longrightarrow all)} = \frac{1}{3 + 2N_C} = 11.1\%$

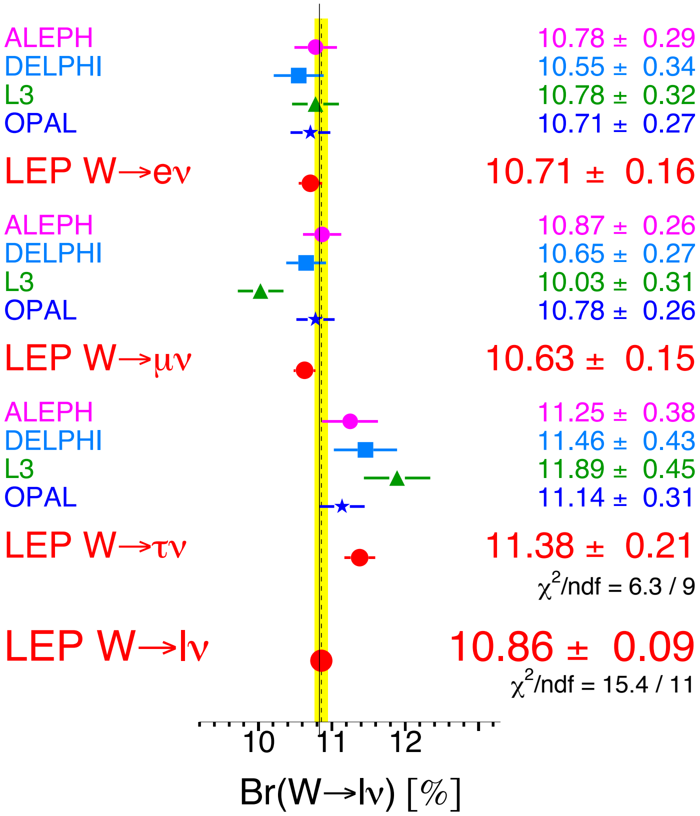
- QCD: $N_C \left\{1 + \frac{\alpha_s(M_Z)}{\pi}\right\} \approx 3.115$ $\Rightarrow$ $Br(W^- \longrightarrow \bar{\upsilon}_l l^-) = 10.8\%$

- Experiment:
- Universal couplings!

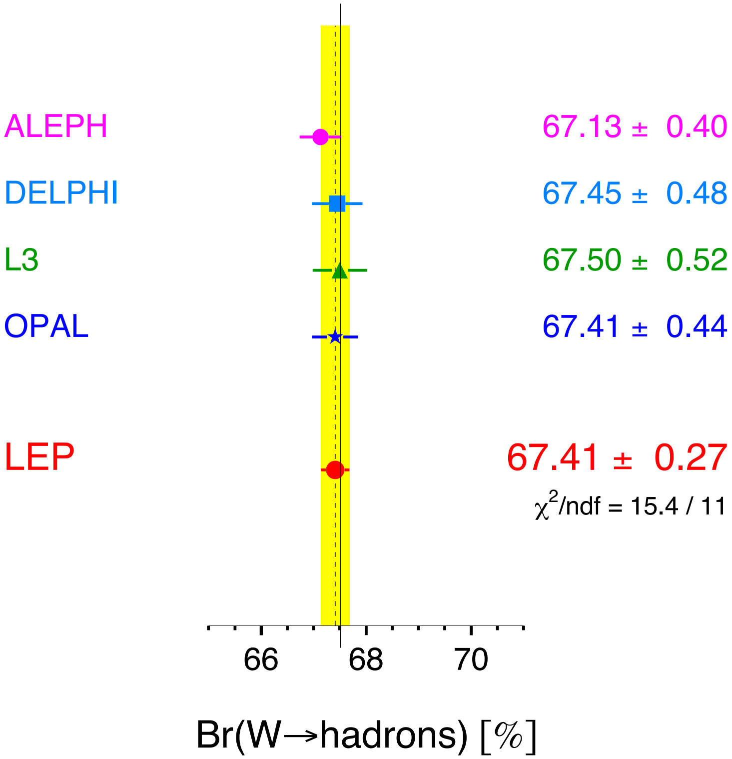
$$Br(W^- \longrightarrow \bar{\upsilon}_e e^-) = (10.75 \pm 0.13)\%$$
$$Br(W^- \longrightarrow \bar{\upsilon}_\mu \mu^-) = (10.57 \pm 0.15)\%$$
$$Br(W^- \longrightarrow \bar{\upsilon}_\tau \tau^-) = (11.25 \pm 0.20)\%$$
$$Br(W^- \longrightarrow \bar{\upsilon}_l l^-) = (10.80 \pm 0.09)\%$$

- W total width: $\Gamma_W = 2.09 GeV$ (2.085 ± 0.042) ; $\Gamma_Z = 2.48 GeV$ (2.4952 ± 0.0023)

W Leptonic Branching Ratios

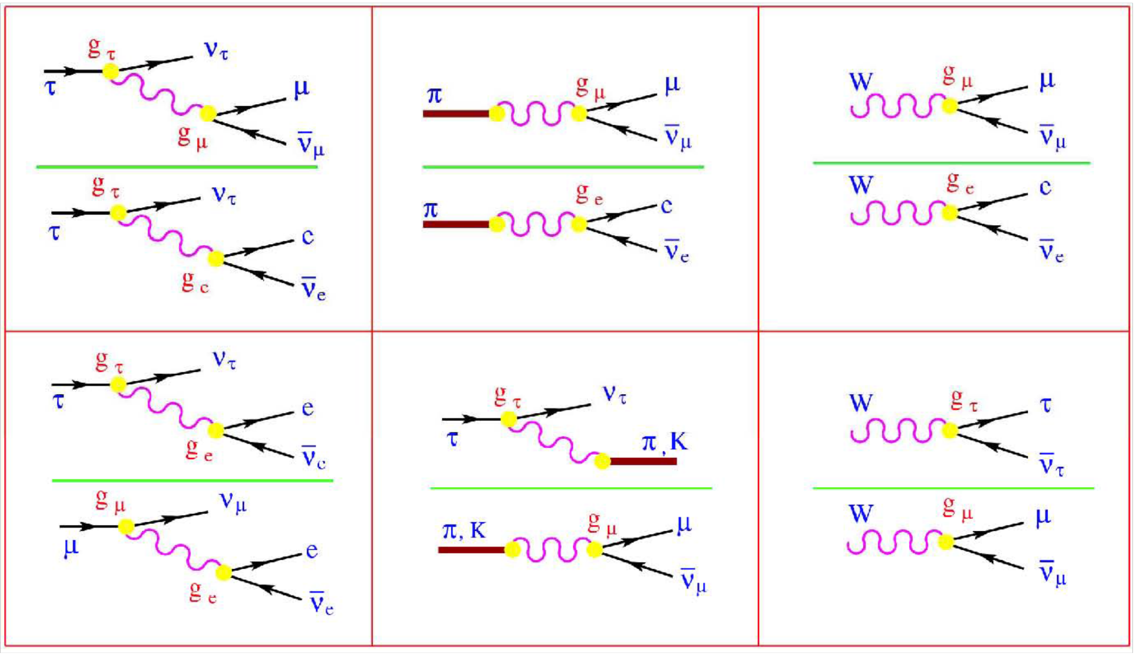


W Hadronic Branching Ratio



☐ Lepton universality

- ☐ g_μ/g_e ➔
- ☐ g_τ/g_μ ➔



g_μ/g_e

$B_{\tau\to\mu}/B_{\tau\to e}$	1.0000 ± 0.0020
$B_{\pi\to\mu}/B_{\pi\to e}$	1.0021 ± 0.0016
$B_{K\to\mu}/B_{K\to e}$	1.004 ± 0.007
$B_{K\to\pi\mu}/B_{K\to\pi e}$	1.002 ± 0.002
$B_{W\to\mu}/B_{W\to e}$	0.997 ± 0.010

g_τ/g_μ

$B_{\tau\to e}\ \tau_\mu/\tau_\tau$	1.0006 ± 0.0022
$\Gamma_{\tau\to\pi}/\Gamma_{\pi\to\mu}$	0.996 ± 0.005
$\Gamma_{\tau\to K}/\Gamma_{K\to\mu}$	0.979 ± 0.017
$B_{W\to\tau}/B_{W\to\mu}$	1.039 ± 0.013

g_τ/g_e

$B_{\tau\to\mu}\ \tau_\mu/\tau_\tau$	1.0005 ± 0.0023
$B_{W\to\tau}/B_{W\to e}$	1.036 ± 0.014

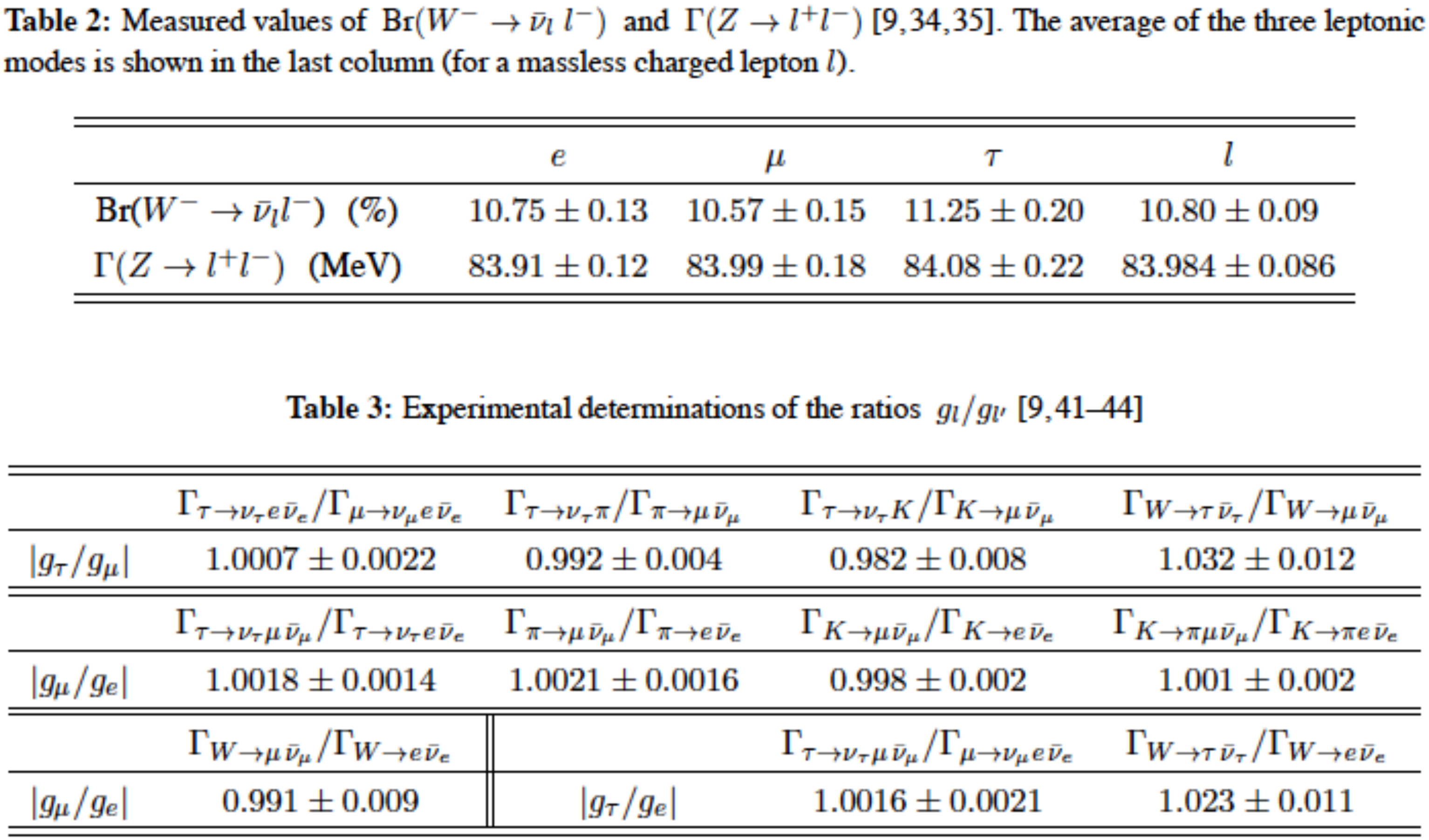


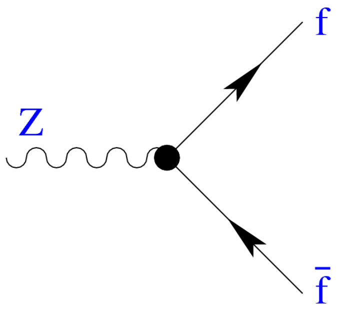
Table 2: Measured values of $\mathrm{Br}(W^- \to \bar{\nu}_l\, l^-)$ and $\Gamma(Z \to l^+l^-)$ [9,34,35]. The average of the three leptonic modes is shown in the last column (for a massless charged lepton l).

	e	μ	τ	l
$\mathrm{Br}(W^- \to \bar{\nu}_l l^-)$ (%)	10.75 ± 0.13	10.57 ± 0.15	11.25 ± 0.20	10.80 ± 0.09
$\Gamma(Z \to l^+l^-)$ (MeV)	83.91 ± 0.12	83.99 ± 0.18	84.08 ± 0.22	83.984 ± 0.086

Table 3: Experimental determinations of the ratios $g_l/g_{l'}$ [9,41–44]

	$\Gamma_{\tau\to\nu_\tau e\bar{\nu}_e}/\Gamma_{\mu\to\nu_\mu e\bar{\nu}_e}$	$\Gamma_{\tau\to\nu_\tau\pi}/\Gamma_{\pi\to\mu\bar{\nu}_\mu}$	$\Gamma_{\tau\to\nu_\tau K}/\Gamma_{K\to\mu\bar{\nu}_\mu}$	$\Gamma_{W\to\tau\bar{\nu}_\tau}/\Gamma_{W\to\mu\bar{\nu}_\mu}$
$\lvert g_\tau/g_\mu\rvert$	1.0007 ± 0.0022	0.992 ± 0.004	0.982 ± 0.008	1.032 ± 0.012
	$\Gamma_{\tau\to\nu_\tau\mu\bar{\nu}_\mu}/\Gamma_{\tau\to\nu_\tau e\bar{\nu}_e}$	$\Gamma_{\pi\to\mu\bar{\nu}_\mu}/\Gamma_{\pi\to e\bar{\nu}_e}$	$\Gamma_{K\to\mu\bar{\nu}_\mu}/\Gamma_{K\to e\bar{\nu}_e}$	$\Gamma_{K\to\pi\mu\bar{\nu}_\mu}/\Gamma_{K\to\pi e\bar{\nu}_e}$
$\lvert g_\mu/g_e\rvert$	1.0018 ± 0.0014	1.0021 ± 0.0016	0.998 ± 0.002	1.001 ± 0.002
	$\Gamma_{W\to\mu\bar{\nu}_\mu}/\Gamma_{W\to e\bar{\nu}_e}$		$\Gamma_{\tau\to\nu_\tau\mu\bar{\nu}_\mu}/\Gamma_{\mu\to\nu_\mu e\bar{\nu}_e}$	$\Gamma_{W\to\tau\bar{\nu}_\tau}/\Gamma_{W\to e\bar{\nu}_e}$
$\lvert g_\mu/g_e\rvert$	0.991 ± 0.009	$\lvert g_\tau/g_e\rvert$	1.0016 ± 0.0021	1.023 ± 0.011

Z-boson decay width (calculate)



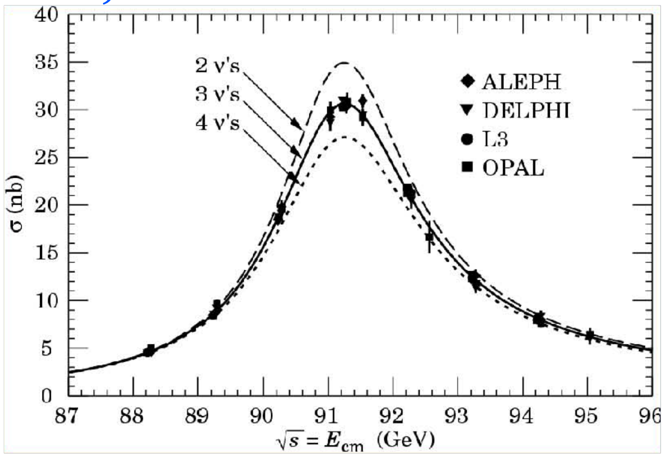
- $Z \longrightarrow l^+l^-, \nu_l\bar{\nu}_l$

$$\Gamma(Z \longrightarrow f\bar{f}) = N_C \frac{G_F M_Z^3}{6\pi\sqrt{2}}\left(|v_f|^2 + |a_f|^2\right) \qquad N_l = 1; N_q = 3$$

- Z invisible width – number of light neutrino species

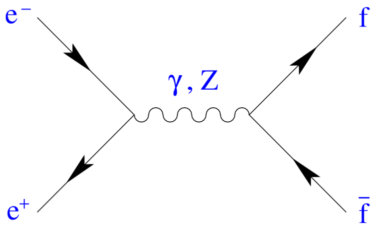
$$\frac{\Gamma_{inv}}{\Gamma_{ll}} = \frac{\Gamma(Z\longrightarrow invisible)}{\Gamma(Z\longrightarrow l^+l^-)} = N_\nu \frac{\Gamma(Z\longrightarrow \nu_l\bar{\nu}_l)}{\Gamma(Z\longrightarrow l^+l^-)} = N_\nu \frac{2}{(1-4sin^2\theta_W)^2+1} = 1.955N_\nu\ (1.989N_\nu)$$

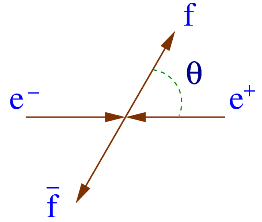
  - Experiment: $\frac{\Gamma_{inv}}{\Gamma_{ll}} = 5.943 \pm 0.016$ $\qquad N_\nu = 3.04\ (N_\nu = 2.99)$
  - Final LEP result: $N_\nu = 2.9840 \pm 0.0082$
    - from Z line-shape at LEP



- Z total width:
  - predicted $\Gamma_Z = 2.48GeV$
  - measured $\Gamma_Z = 2.4952 \pm 0.0023$ GeV

5.1 – Fermion-pair production at Z peak





☐ unpolarized e⁺ e⁻ beams

- ☐ differential cross-section at lowest order

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{8s} N_f \{ A(1 + cos^2\theta) + B\cos\theta - h_f\,[C(1 + cos^2\theta) + D\cos\theta] \}$$

$$A = 1 + 2v_e v_f \mathcal{R}e(\chi) + (v_e^2 + a_e^2)(v_f^2 + a_f^2)|\chi|^2$$
$$B = 4a_e a_f \mathcal{R}e(\chi) + 8v_e a_e v_f a_f |\chi|^2$$
$$C = 2v_e a_f \mathcal{R}e(\chi) + 2(v_e^2 + a_e^2) v_f a_f |\chi|^2$$
$$D = 4a_e v_f \mathcal{R}e(\chi) + 4v_e a_e (v_f^2 + a_f^2)|\chi|^2$$

  - ☐ h_f : sign of helicity of fermion f
  - ☐ $N_l = 1; N_q = N_C \left\{ 1 + \frac{\alpha_s(M_Z^2)}{\pi} + \cdots \right\}$
  - ☐ Z propagator

$$\chi = \frac{G_F M_Z^2}{2\sqrt{2}\,\pi\alpha} \frac{s}{s - M_Z^2 + i\,s\frac{\Gamma_Z}{M_Z}}$$

  - ☐ A,B,C,D from experiment
    - total cross section
    - forward–backward asymmetry,
    - polarization asymmetry,
    - forward–backward polarization asymmetry

$$\sigma(s) = \frac{4\pi\alpha^2}{3s} N_f A$$

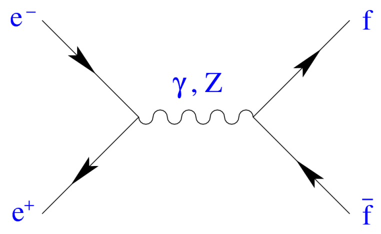
$$A_{FB}(s) \equiv \frac{N_f - N_B}{N_f + N_B} = \frac{3}{8}\frac{B}{A}$$

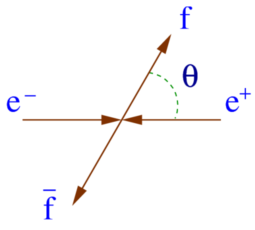
$$A_{Pol}(s) \equiv \frac{\sigma^{h_f=+1} - \sigma^{h_f=-1}}{\sigma^{h_f=+1} + \sigma^{h_f=-1}} = -\frac{C}{A}$$

$$A_{FB,Pol}(s) \equiv \frac{N_F^{h_f=+1} - N_F^{h_f=-1} - N_B^{h_f=+1} + N_B^{h_f=-1}}{N_F^{h_f=+1} + N_F^{h_f=-1} + N_B^{h_f=+1} + N_B^{h_f=-1}} = -\frac{3}{8}\frac{D}{A}$$

☐ Case $s = M_Z^2$

- ☐ $\mathcal{Re}(\chi)$ vanishes, γ exchange term negligible + $\frac{\Gamma_Z^2}{M_Z^2} \ll 1$





$$\boxed{\sigma^{0,f} \equiv \sigma(M_Z^2) = \frac{12\pi}{M_Z^2}\frac{\Gamma_e\Gamma_f}{\Gamma_Z^2}}$$

$$\boxed{A_{FB}^{0,f} \equiv A_{FB}(M_Z^2) = \frac{3}{4}\wp_e\wp_f}$$

$$\boxed{A_{Pol}^{0,f} \equiv A_{Pol}(M_Z^2) = \wp_f}$$

$$\boxed{A_{FB,Pol}^{0,f} \equiv A_{FB,Pol}(M_Z^2) = \frac{3}{4}\wp_e}$$

- ☐ $\wp_f$: average longitudinal polarization of fermion $f\,(=\tau)$:
  - Sensitive to New physics
  - $\wp_f$: Very sensitive function of $sin^2\theta_W$ due to $|v_l| = \frac{1}{2}|1 - 4sin^2\theta_W|$

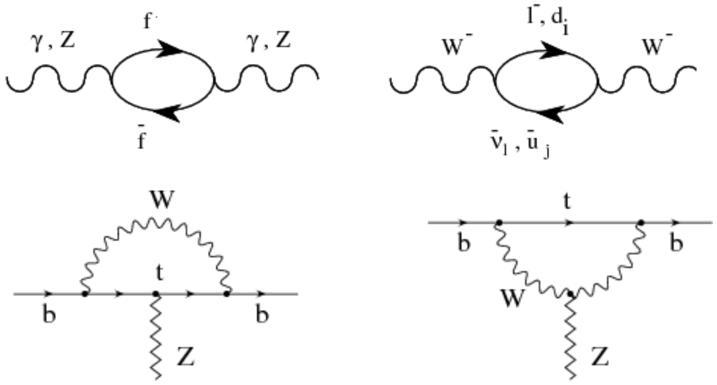
$$\wp_f \equiv -A_f = \frac{-2v_f a_f}{v_f^2 + a_f^2}$$

- ☐ Polarised e⁺ , e⁻ beams (SLC)
  - ☐ Left-right asymmetry between cross sections

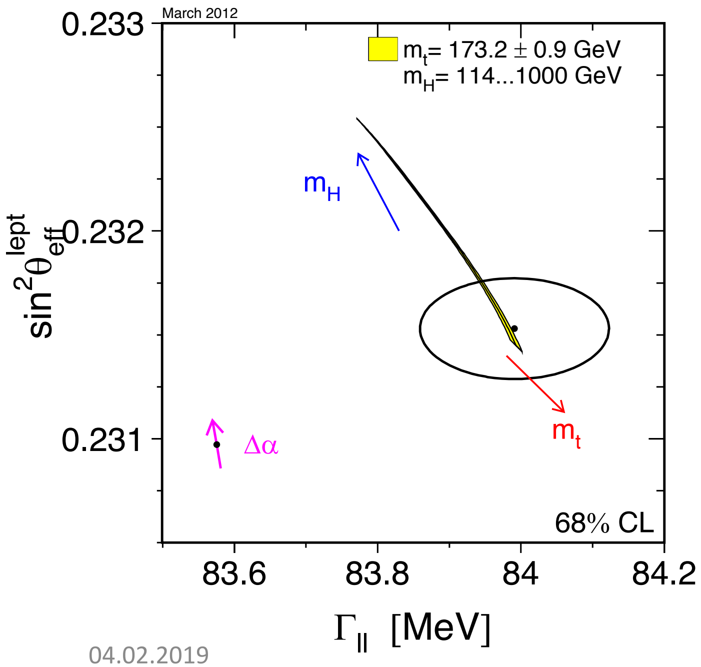
$$A_{LR}^0 \equiv A_{LR}(M_Z^2) = \frac{\sigma_L(M_Z^2) - \sigma_R(M_Z^2)}{\sigma_L(M_Z^2) + \sigma_R(M_Z^2)} = -\wp_e$$

$$A_{FB,LR}^{0,f} \equiv A_{FB,LR}(M_Z^2) = -\frac{3}{4}\wp_f$$

Higher order EW corrections

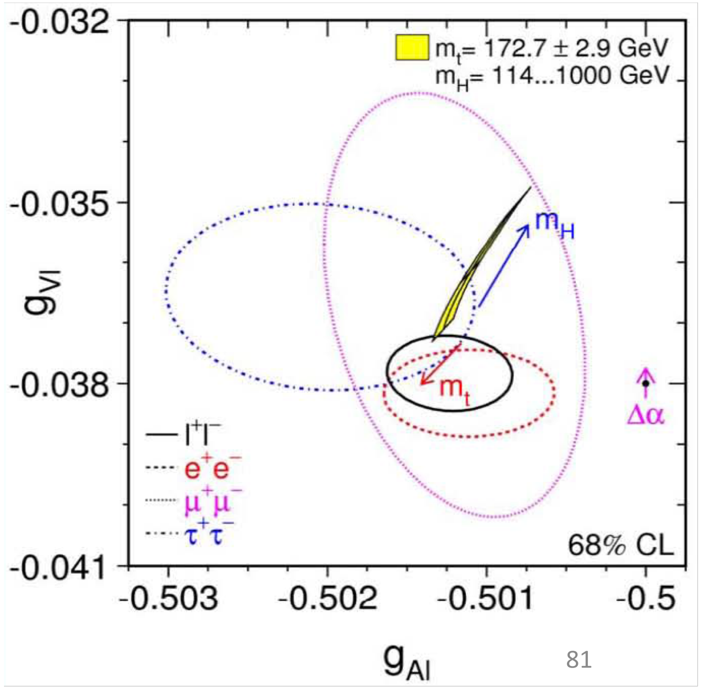


- Sensitive to heavier particles: Top, Higgs, ... New physics
  - Evidence of EW corrections
  - LEP & SLD measurements ➔ Low values of M_H preferred!
    - $sin^2\theta_{eff}^{lept}$ vs Γ_{ll}
    - corresponding effective vector and axial-vector couplings $v_l = 2g_{Vl}$vs $a_l = 2g_{Al}$



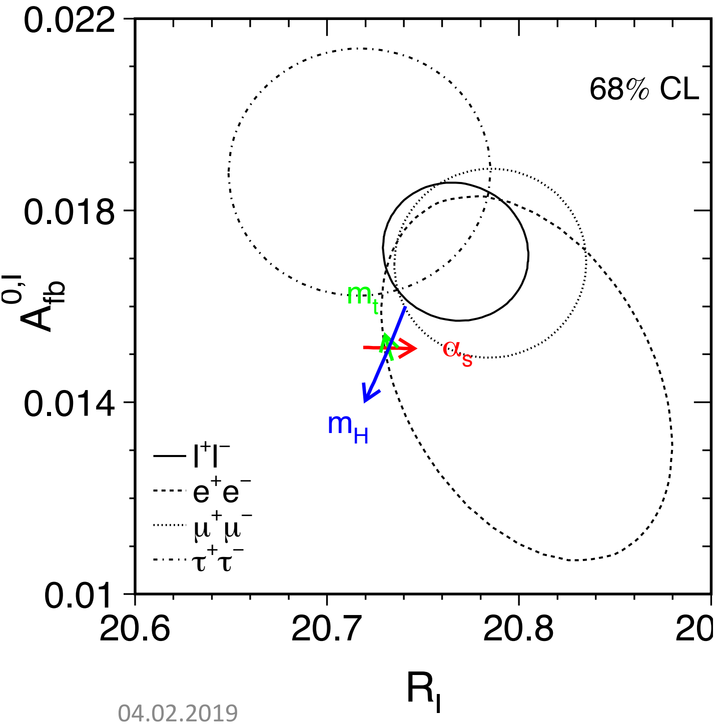
- *Shaded region: SM prediction*
- *Arrows point in direction of increasing values of m_t & M_H*
- *Point $\Delta\alpha$: prediction when only photon vacuum polarisation included in EW radiative corrections.*
- *Arrow $\Delta\alpha$ indicates variation induced by uncertainty in $\alpha(M_Z^2)$ – additional uncertainty to SM prediction-*

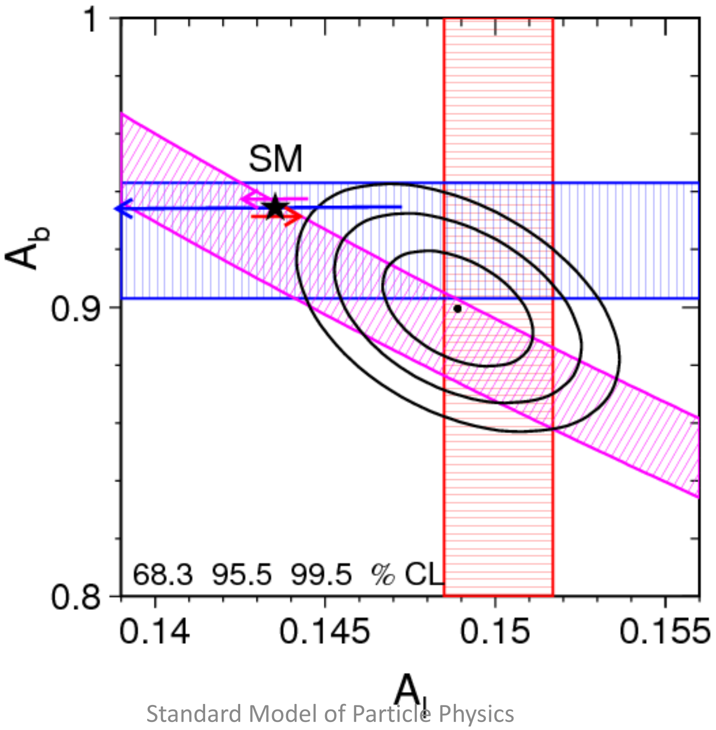
$\alpha(M_Z^2) = 128.93 \pm 0.05$

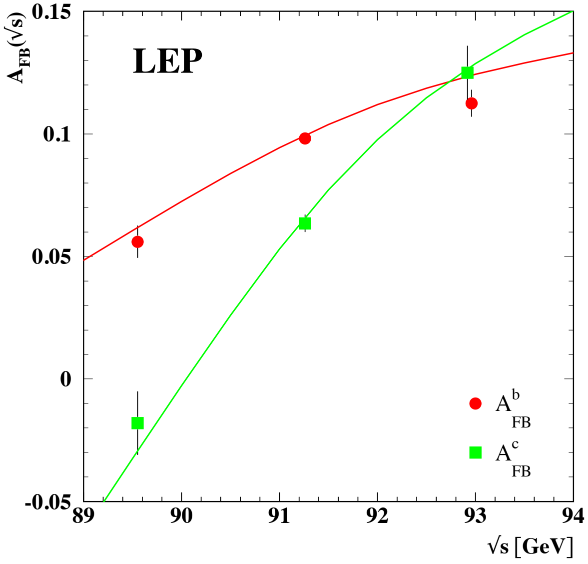


- ☐ $A_{FB}^{0,l}\ vs\ R_{ll}$
- ☐ $A_l\ vs\ A_b$

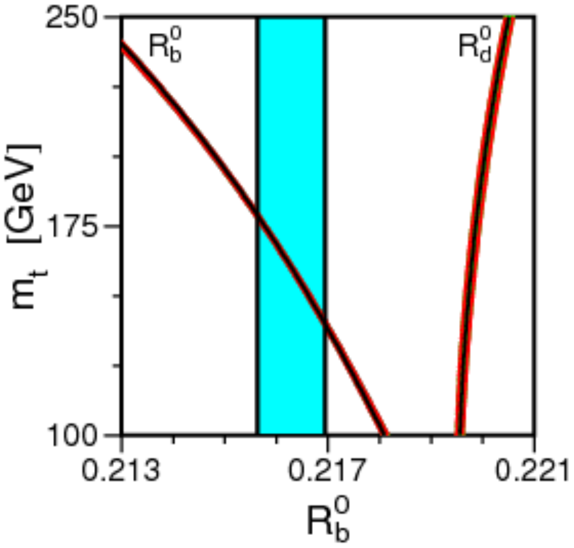
- *SM prediction contour*
- *Arrows point in direction of increasing values of* m_t *&* M_H $(M_H = 300^{+700}_{-186}GeV, M_t = 172.7 \pm 2.9GeV)$
- *Arrow* α_s *indicates variation induced by uncertainty in* $\alpha(M_Z^2)$ *– additional uncertainty to SM prediction-*

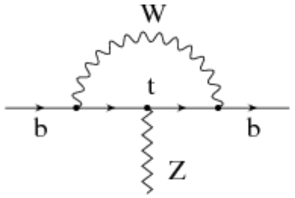


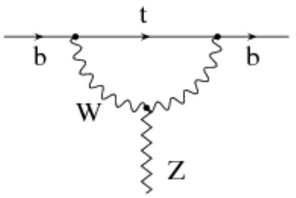


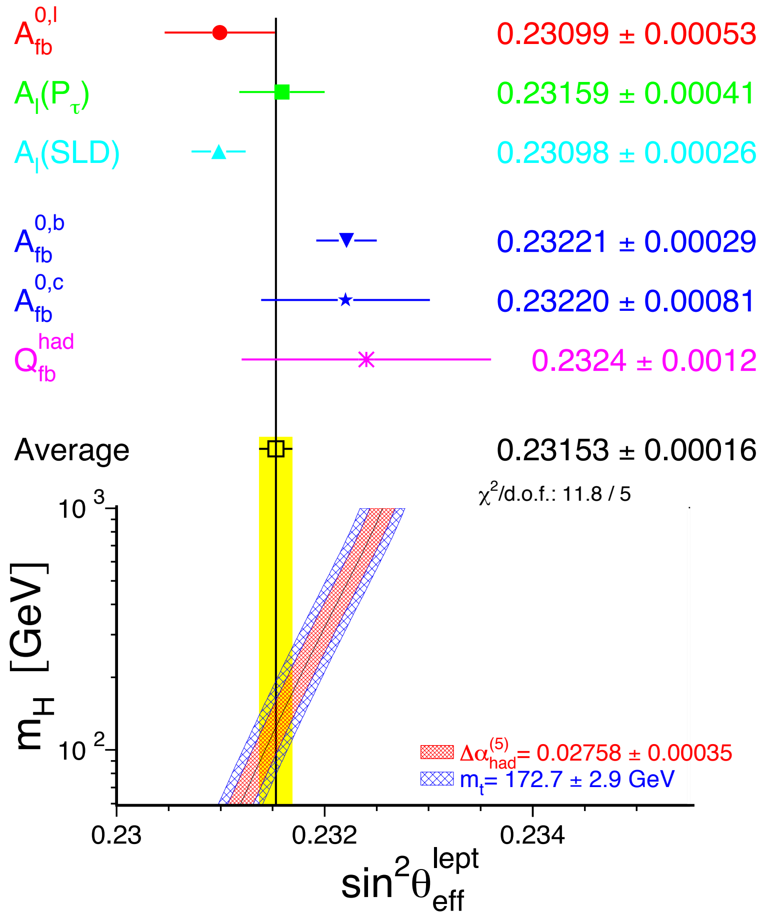


❑ SM prediction of ratios R_b and R_d $R_q = \frac{\Gamma(Z\longrightarrow q\bar{q})}{\Gamma(Z\longrightarrow hadrons)}$ as a function top mass. Measured value of R_b (vertical band) provides a determination of m_t





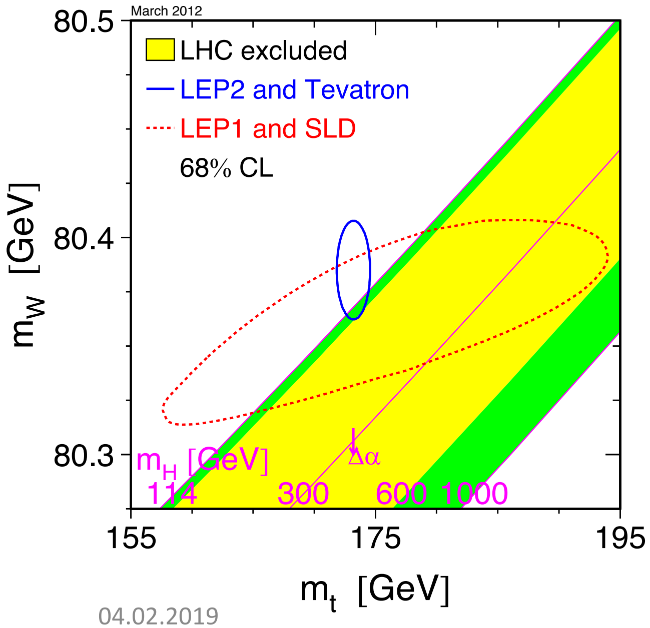


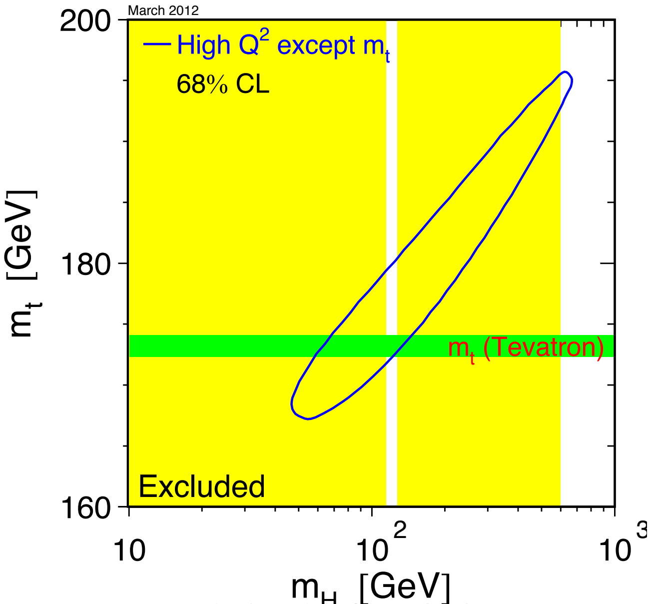


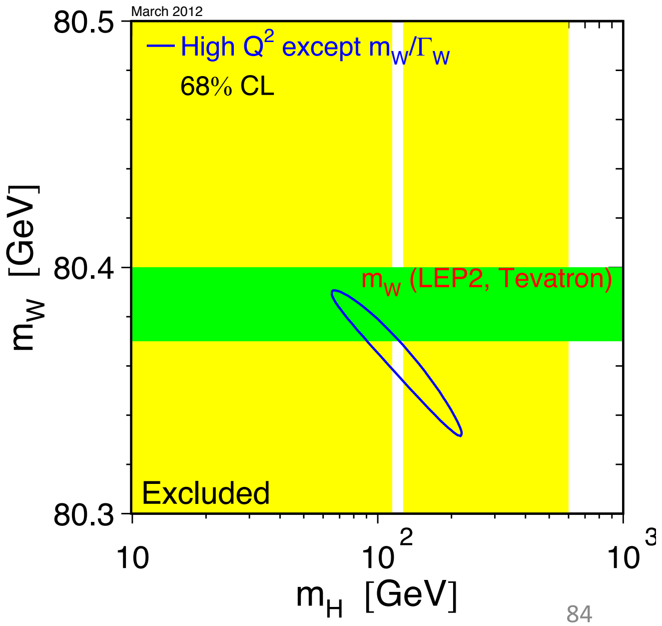
EW precision measurements and SM constraints

- *Comparison indirect constraints on m_W & m_t SM prediction as function of m_H in region favoured by theory (< 1000 GeV) not excluded by direct searches (114 GeV-158 GeV & > 175 GeV).*
- *Arrow $\Delta\alpha$ indicates variation induced by uncertainty in $\alpha(M_Z^2)$*
- $\alpha(M_Z^2) = 128.93 \pm 0.05$

- *68% CL contour in m_t (m_W) and m_H for fit to all high-Q^2 data except direct measurement of m_t (m_W) indicated by shaded horizontal band of ± σ*
- *vertical band shows 95% CL exclusion limit on m_H from direct searches at LEP-II (up to 114 GeV) and Tevatron (158 – 175 GeV).*

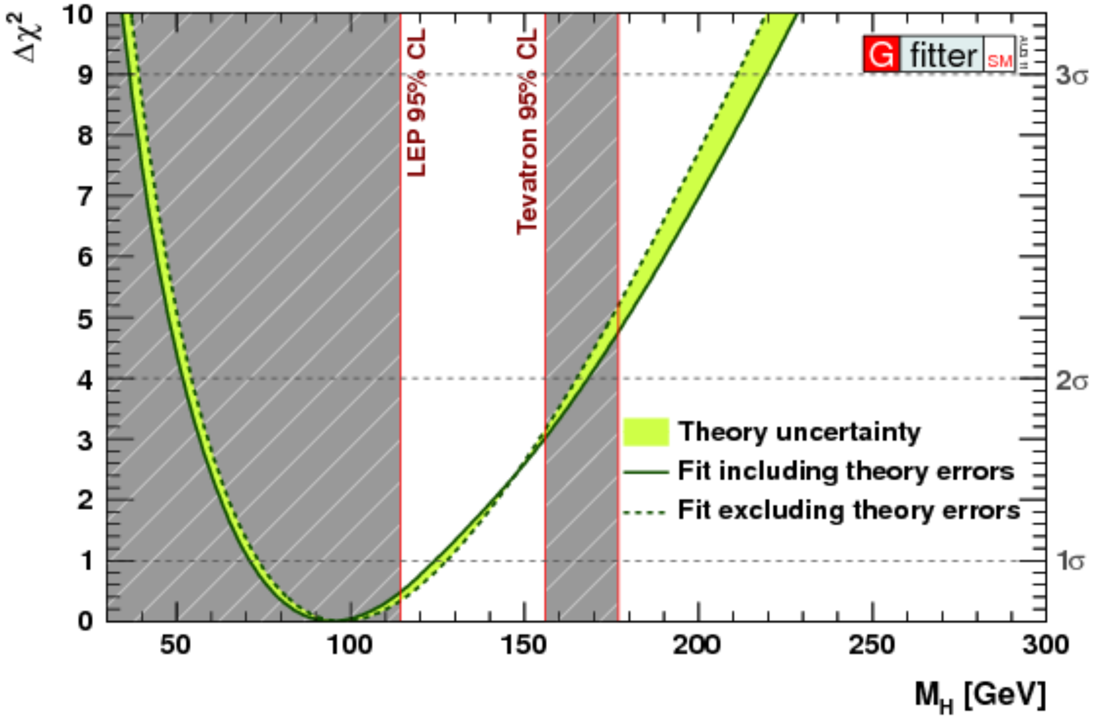


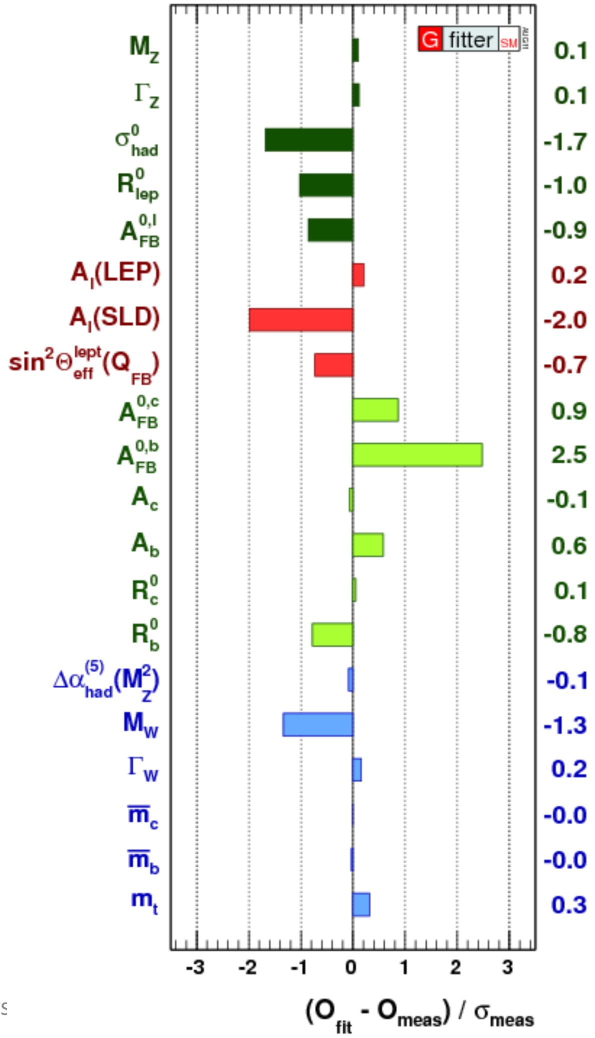




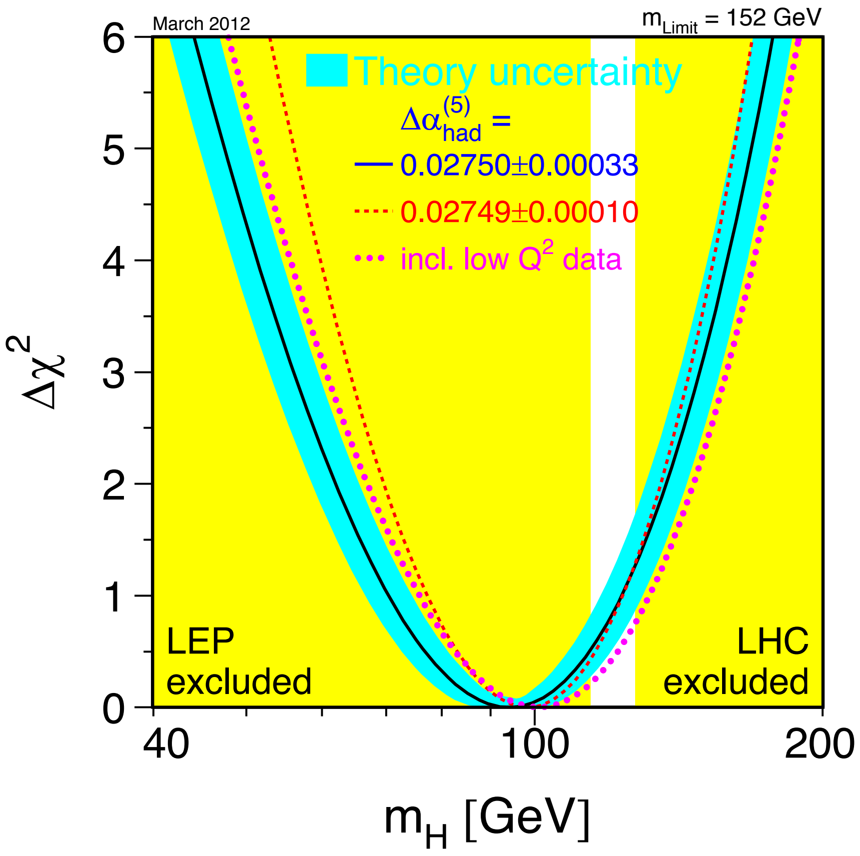
Constrained global EW SM-fit – $114.4\ GeV < M_H < 169\ GeV$

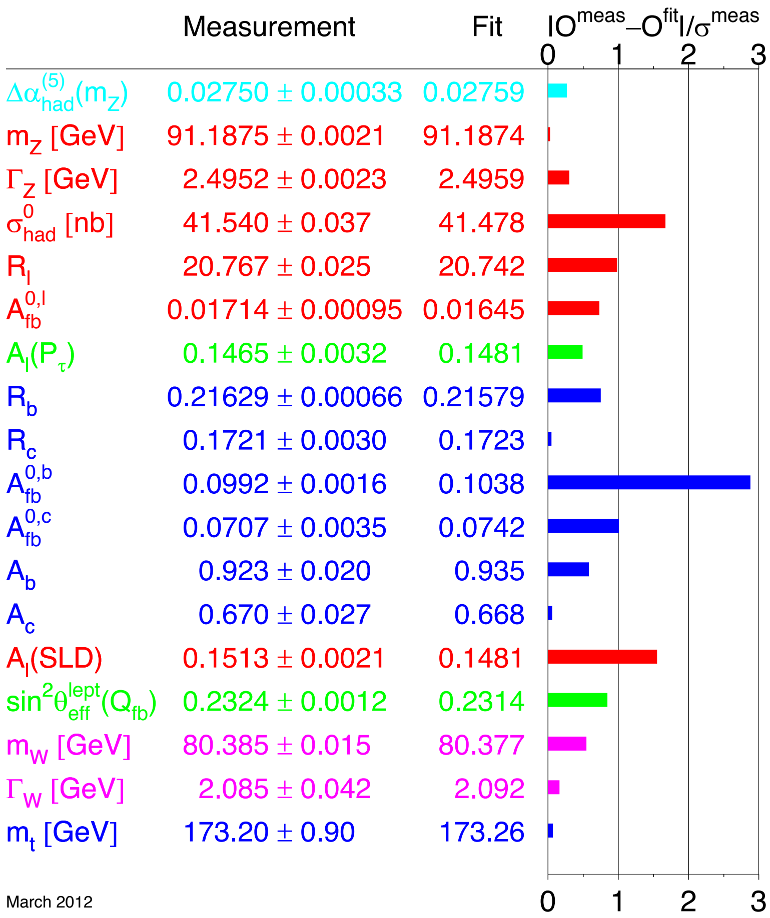
- 2011





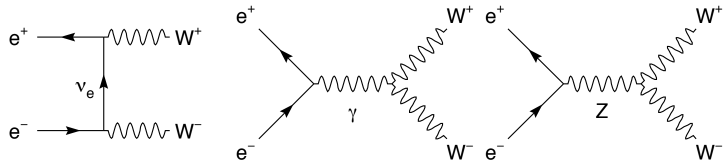
$114.4\ GeV < M_H < 160\ GeV$

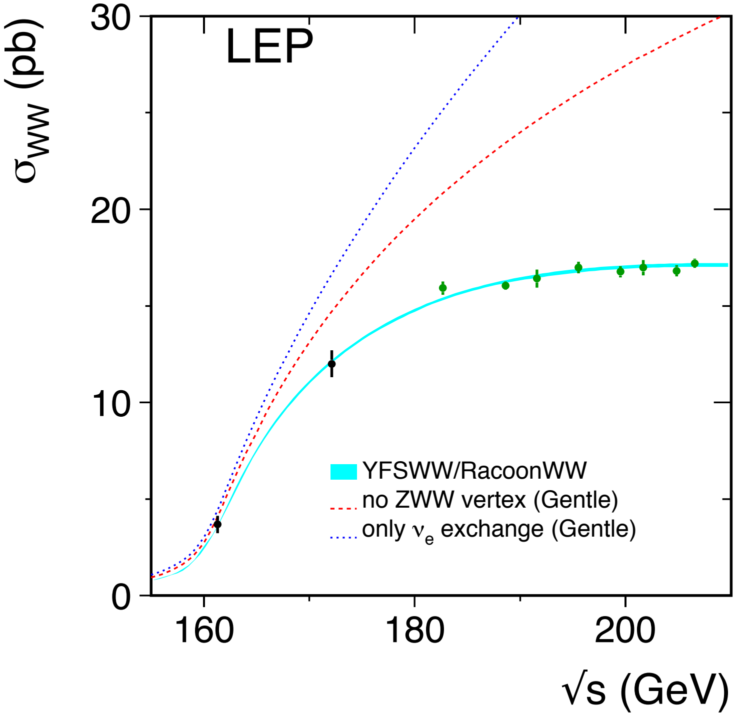


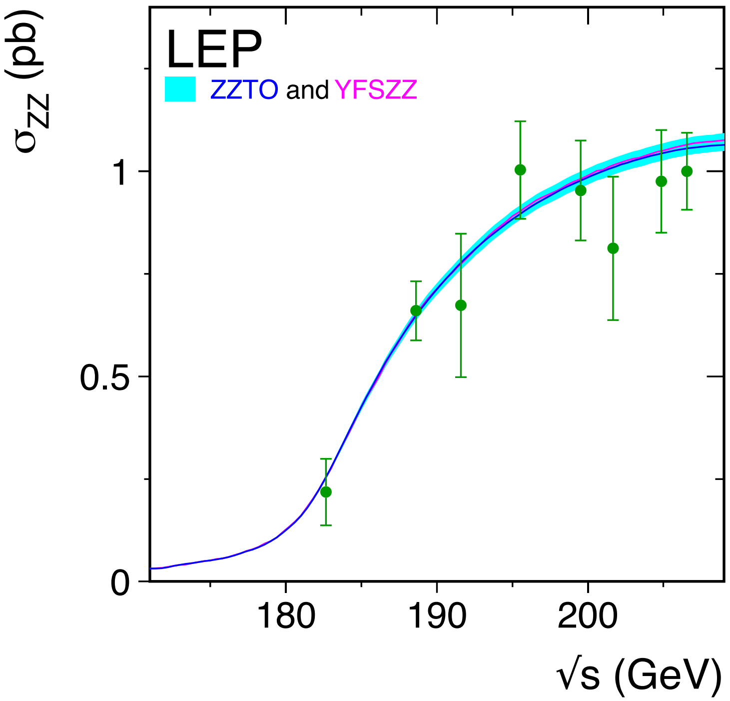


	Measurement	Fit
$\Delta\alpha^{(5)}_{had}(m_Z)$	0.02750 ± 0.00033	0.02759
m_Z [GeV]	91.1875 ± 0.0021	91.1874
Γ_Z [GeV]	2.4952 ± 0.0023	2.4959
σ^0_{had} [nb]	41.540 ± 0.037	41.478
R_l	20.767 ± 0.025	20.742
$A^{0,l}_{fb}$	0.01714 ± 0.00095	0.01645
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1481
R_b	0.21629 ± 0.00066	0.21579
R_c	0.1721 ± 0.0030	0.1723
$A^{0,b}_{fb}$	0.0992 ± 0.0016	0.1038
$A^{0,c}_{fb}$	0.0707 ± 0.0035	0.0742
A_b	0.923 ± 0.020	0.935
A_c	0.670 ± 0.027	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1481
$\sin^2\theta^{lept}_{eff}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.385 ± 0.015	80.377
Γ_W [GeV]	2.085 ± 0.042	2.092
m_t [GeV]	173.20 ± 0.90	173.26

Evidence of gauge boson self-interactions







Unitarity violation

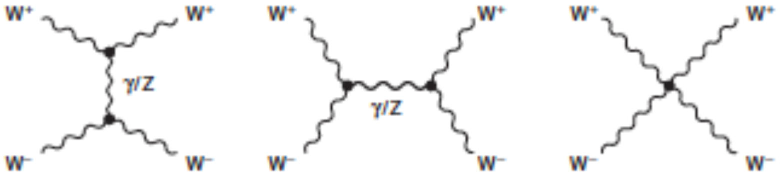


Fig. 17.1 The lowest-order Feynman diagrams for $W^+W^- \to W^+W^-$. The final diagram, corresponds to the quartic coupling of four W bosons.

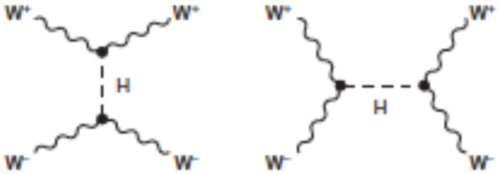
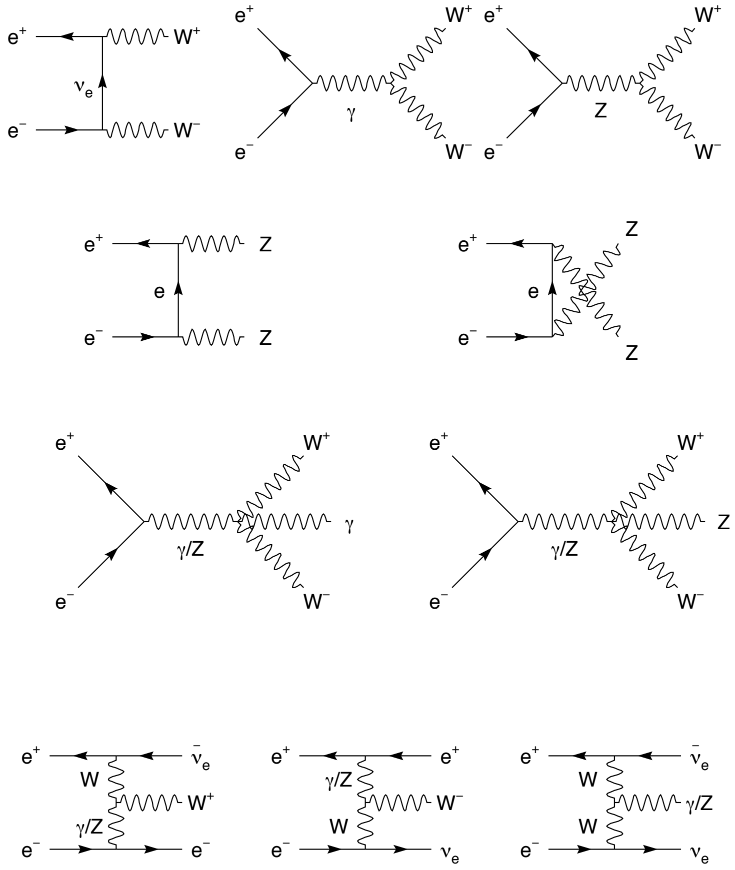
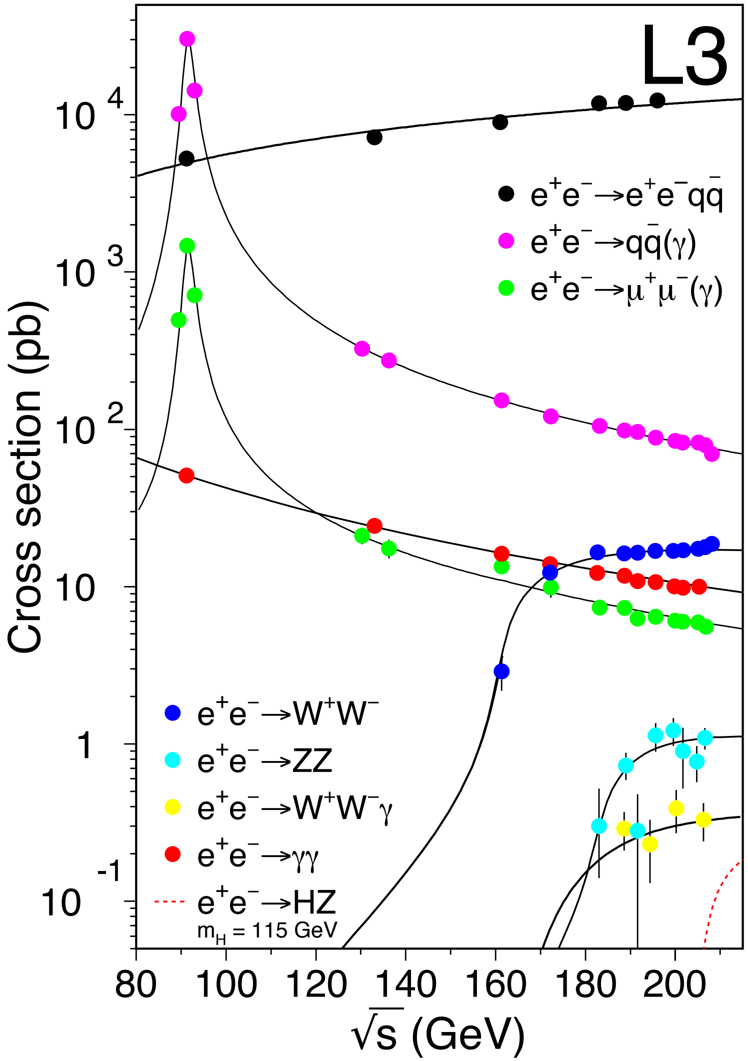


Fig. 17.2 Higgs boson exchange diagrams for $W^+W^- \to W^+W^-$.

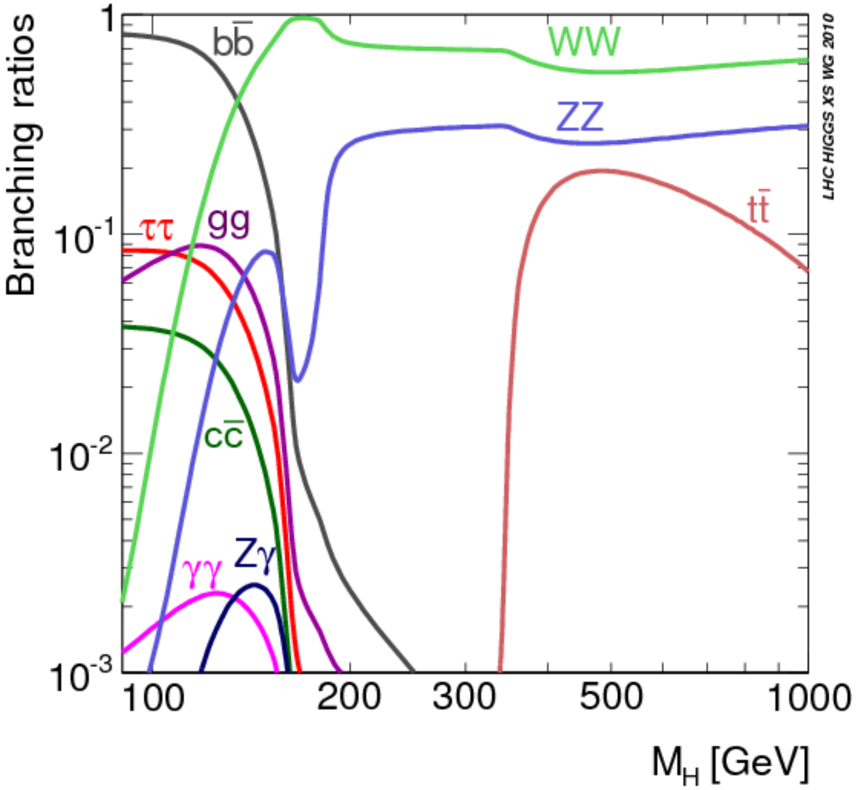
- Apparent violation of unitarity in $e^+e^- \to W^+W^-$ cross section
  - resolved by introduction of Z boson.
- Similar issue arises in $W^+W^- \to W^+W^-$ scattering process,
  - cross section calculated from Feynman diagrams (17.1) violates unitarity at ~1 TeV
    - Origin: WLWL → WLWL scattering with longitudinally polarized W.
  - Consequently, unitary violation inWW scattering can be associated with Wbosons being massive, since longitudinal polarisation states do not exist for massless particles.
  - unitarity violation of $W_LW_L \to W_LW_L$ cross section can be cancelled by diagrams 17.2 involving exchange of a scalar particle – Higgs boson in Standard Model
  - Cancellation can work only if couplings of scalar particle are related to EW couplings, which naturally occurs in the Higgs mechanism.

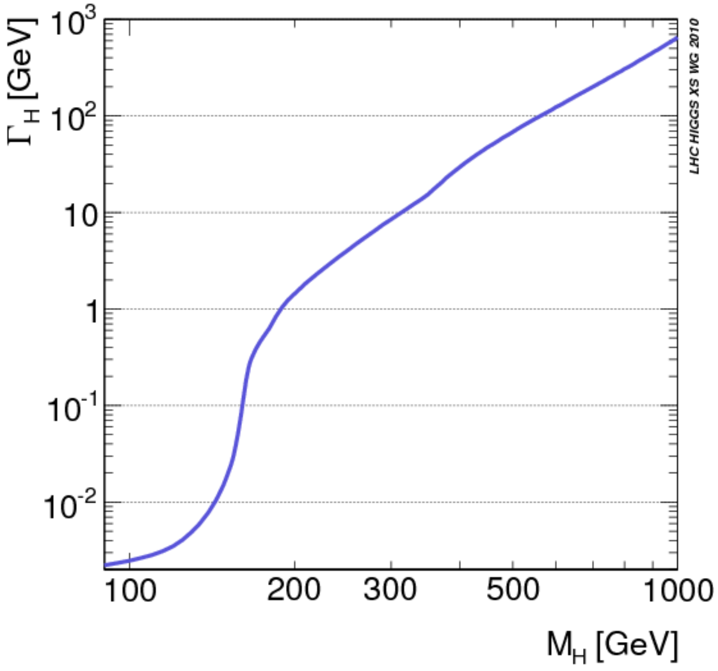
e^+e^- @ LEP – practice with CompHEP





Higgs Branching ratio and total Width according to SM





6 – Flavour Dynamics

$$\begin{bmatrix}\nu_e & u\\ e^- & d'\end{bmatrix}, \begin{bmatrix}\nu_\mu & c\\ \mu^- & s'\end{bmatrix}, \begin{bmatrix}\nu_\tau & t\\ \tau^- & b'\end{bmatrix}$$

- ☐ Fermions
  - ☐ 6 quark flavours (3 colours), 3 charged leptons, 3 neutrinos
  - ☐ 3 generations following $SU(2)_L \otimes U(1)_Y$ structure

$$\begin{bmatrix}\nu_l & q_u\\ l^- & q_d\end{bmatrix} = \begin{pmatrix}\nu_l\\ l^-\end{pmatrix}_L, \begin{pmatrix}q_u\\ q_d\end{pmatrix}_L, l_R^-, q_{uR}, q_{dR}$$

  - ☐ General Yukawa Lagrangian

$$\mathcal{L}_Y = -\sum_{jk}\left\{(\bar{u}'_j, \bar{d}'_j)_L \left[c^{(d)}_{jk}\begin{pmatrix}\phi^{(+)}\\ \phi^{(0)}\end{pmatrix} d'_{kR} + c^{(u)}_{jk}\begin{pmatrix}\phi^{(0)*}\\ -\phi^{(-)}\end{pmatrix} u'_{kR}\right] + (\bar{\nu}'_j, \bar{l}'_j)_L\, c^{(l)}_{jk}\begin{pmatrix}\phi^{(+)}\\ \phi^{(0)}\end{pmatrix} l'_{kR}\right\} + h.c.$$

    - ☐ Arbitrary coupling constants $c^{(d)}_{jk}$, $c^{(u)}_{jk}$, $c^{(l)}_{jk}$, arbitrary non-diagonal complex matrices M'_d, M'_u, M'_l
    - ☐ After SSB - unitary gauge

$$\mathcal{L}_Y = -\left(1 + \frac{H}{v}\right)\left\{\bar{d}'_L M'_d d'_R + \bar{u}'_L M'_u u'_R + \bar{l}'_L M'_l l'_R + h.c.\right\}$$

$$(M'_d)_{ij} \equiv c^{(d)}_{ij}\frac{v}{\sqrt{2}}\ ; \quad (M'_u)_{ij} \equiv c^{(u)}_{ij}\frac{v}{\sqrt{2}}\ ; \quad (M'_l)_{ij} \equiv c^{(l)}_{ij}\frac{v}{\sqrt{2}}$$

    - ☐ Matrix diagonalization → mass eigenstates d_j, u_j, l_j linear combinations of weak eigenstates d'_j, u'_j, l'_j

- Matrices M′$_{d,u,l}$ can be decomposed as

$$M'_d = H_d \cdot U_d = S_d^\dagger \cdot \mathcal{M}_d \cdot S_d \cdot U_d$$
$$M'_u = H_u \cdot U_u = S_u^\dagger \cdot \mathcal{M}_u \cdot S_u \cdot U_u$$
$$M'_l = H_l \cdot U_l = S_l^\dagger \cdot \mathcal{M}_l \cdot S_l \cdot U_l$$

$$H_f = H_f^\dagger$$
$$U_f \cdot U_f^\dagger = U_f^\dagger \cdot U_f = 1$$
$$S_f \cdot S_f^\dagger = S_f^\dagger \cdot S_f = 1$$

- $H_{d,u,l} = \sqrt{M'_{d,u,l}\ M'^\dagger_{d,u,l}}$: Hermitian positive-definite matrices, $U_{d,u,l}$: unitary matrices.
- $H_{d,u,l}$ can be diagonalized by unitary matrices $S_{d,u,l}$
- Resulting matrix $\mathcal{M}_d$: diagonal, Hermitian and positive definite

$$\mathcal{M}_d = diag(m_d, m_s, m_b, \dots);\ \mathcal{M}_u = diag(m_u, m_c, m_t, \dots);\ \mathcal{M}_l = diag\left(m_e, m_\mu, m_\tau, \dots\right)$$

- Yukawa Lagrangian → $\mathcal{L}_Y = -\left(1 + \frac{H}{v}\right)\left\{\bar{d}\mathcal{M}_d d + \bar{u}\mathcal{M}_u u + \bar{l}\mathcal{M}_l l\right\}$
- Mass eigen states defined as: $d_L \equiv S_d d'_L$ $\quad u_L \equiv S_u u'_L$ $\quad l_L \equiv S_l l'_L$

$$d_R \equiv S_d U_d d'_R \qquad u_R \equiv S_u U_u u'_R \qquad l_R \equiv S_l U_l l'_R$$

- Higgs couplings proportional to corresponding fermions masses

- ☐ Form of $\mathcal{L}_{NC}$ does not change when expressed in terms of mass eigenstates

$$\bar{f}'_L f'_L = \bar{f}_L f_L; \quad \bar{f}'_R f'_R = \bar{f}_R f_R \qquad (f = u, d, l)$$

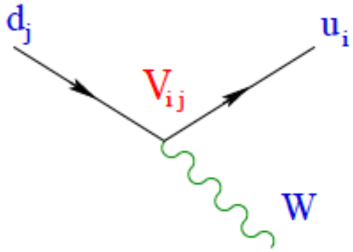
- ☐ No flavour-changing neutral currents (FCNC) in SM (GIM mechanism)
  - ☐ Consequence of treating all equal-charge fermions on same footing
- ☐ Form of $\mathcal{L}_{CC}$ altered – in general $S_u \neq S_d$

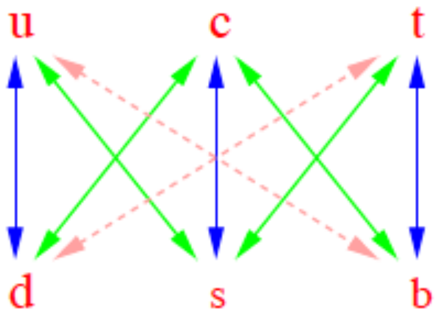
$$\bar{u}'_L d'_L = \bar{u}_L S_u S_d^\dagger d_L \equiv \bar{u}_L V d_L$$

- ☐ N_G X N_G unitary mixing matrix V – CKM matrix appears in quark charged-current sector

$$\mathcal{L}_{CC} = -\frac{g}{2\sqrt{2}}\left\{W_\mu^\dagger\left[\sum_{ij}\bar{u}_i\,\gamma^\mu(1-\gamma^5)V_{ij}d_j + \sum_l \bar{\nu}_l\,\gamma^\mu(1-\gamma^5)l\right] + h.c\right\}$$

  - ☐ Matrix V couples any ‘up-type’ quark with all ‘down-type’ quarks





- If neutrinos massless,
  - possible to redefine neutrino flavours, such to eliminate the analogous mixing in the lepton sector

$$\bar{\nu}'_L l'_L = \bar{\nu}'_L S_l^\dagger l'_L \equiv \bar{\nu}_L l_L$$

  - Lepton-flavour conservation in minimal SM without RH neutrinos
  - If sterile ν_R fields included in model → additional Yukawa term giving rise to neutrino mass matrix

$$(M'_\nu)_{ij} \equiv c_{ij}^{(\nu)} \frac{v}{\sqrt{2}}$$

  - Model can accommodate non-zero neutrino masses and lepton-flavour violation through a lepton mixing matrix V_L analogous to V_{CKM} in quark sector
  - However
    - Total lepton number $L \equiv L_e + L_\mu + L_\tau$ still conserved
    - Neutrino mass is tiny and strong bounds on Lepton-flavor violating decays

$$BR(\mu^\pm \longrightarrow e^\mp e^+ e^-) < 10^{-12}; \quad BR(BR(\mu^\pm \longrightarrow e^\pm \gamma) < 2.4 \cdot 10^{-12}$$

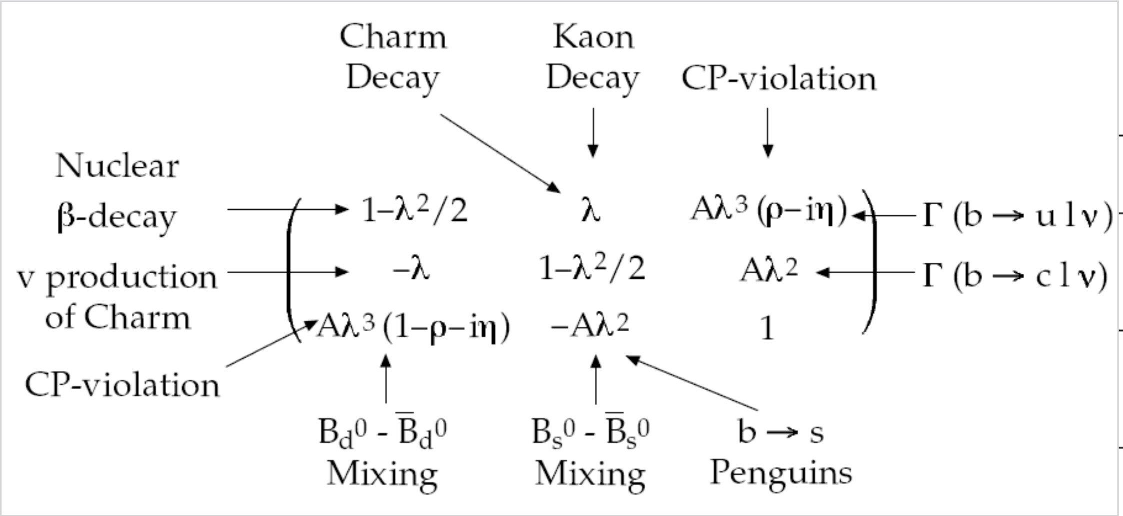
$$\mathcal{L}_Y = -\sum_{jk}\left\{\left(\bar{u}'_j, \bar{d}'_j\right)_L \left[c^{(d)}_{jk}\begin{pmatrix}\phi^{(+)}\\ \phi^{(0)}\end{pmatrix} d'_{kR} + c^{(u)}_{jk}\begin{pmatrix}\phi^{(0)*}\\ -\phi^{(-)}\end{pmatrix} u'_{kR}\right] + \left(\bar{\nu}'_j, \bar{l}'_j\right)_L c^{(l)}_{jk}\begin{pmatrix}\phi^{(+)}\\ \phi^{(0)}\end{pmatrix} l'_{kR}\right\} + h.c.$$

- Fermion masses and quark mixing matrix V determined by Yukawa couplings
  - however, coefficients $c^{(f)}_{ij}$ not known
  - bunch of arbitrary parameters
- A general $N_G X N_G$ unitary matrix characterized by N_G^2 parameters
  - $N_G(N_G-1)/2$ moduli and $N_G(N_G + 1)/2$ phases.
  - In case of V_{CKM}, many irrelevant parameters
  - Quark phases can be chosen arbitrarily
    - Under phase redefinitions $u_i \longrightarrow e^{i\phi_i}u_i$ and $d_j \longrightarrow e^{i\theta_j}d_j \Rightarrow V_{ij} \longrightarrow V_{ij}e^{i(\theta_j-\phi_i)}$
      - $2N_G - 1$ phases are unobservable.
      - Number of physical free parameters in V_{ij} reduced to $(N_G - 1)^2$
      - $N_G(N_G - 1)/2$ moduli and $(N_G - 1)(N_G - 2)/2$ phases
  - $N_G=3$ ➔ CKM
    - 3 angles and 1 phase

$$\mathbf{V} = \begin{bmatrix} c_{12}\,c_{13} & s_{12}\,c_{13} & s_{13}\,e^{-i\delta_{13}} \\ -s_{12}\,c_{23} - c_{12}\,s_{23}\,s_{13}\,e^{i\delta_{13}} & c_{12}\,c_{23} - s_{12}\,s_{23}\,s_{13}\,e^{i\delta_{13}} & s_{23}\,c_{13} \\ s_{12}\,s_{23} - c_{12}\,c_{23}\,s_{13}\,e^{i\delta_{13}} & -c_{12}\,s_{23} - s_{12}\,c_{23}\,s_{13}\,e^{i\delta_{13}} & c_{23}\,c_{13} \end{bmatrix}$$

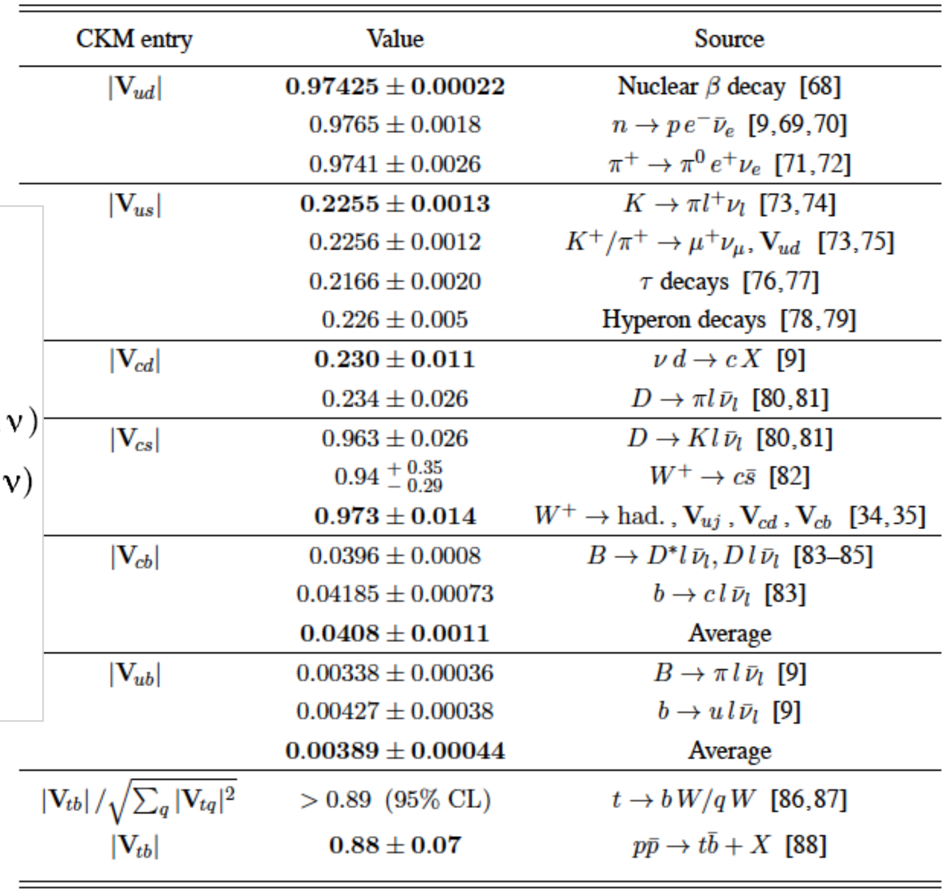
CKM matrix elements measurements

- Various measurements
  - to determine CKM parameters



- Wolfenstein parameterisation
  - $\lambda = s_{12}$
  - $A\lambda^2 = s_{23}$
  - $A\lambda^3(\rho - i\eta) = s_{13}e^{-i\delta}$

Table 4: Direct determinations of the CKM matrix elements $\mathbf{V}_{ij}$ [41]. The 'best' values are indicated in bold face.



CKM entry	Value	Source
$\lvert\mathbf{V}_{ud}\rvert$	$\mathbf{0.97425 \pm 0.00022}$	Nuclear β decay [68]
	0.9765 ± 0.0018	$n \to p e^- \bar{\nu}_e$ [9,69,70]
	0.9741 ± 0.0026	$\pi^+ \to \pi^0 e^+ \nu_e$ [71,72]
$\lvert\mathbf{V}_{us}\rvert$	$\mathbf{0.2255 \pm 0.0013}$	$K \to \pi l^+ \nu_l$ [73,74]
	0.2256 ± 0.0012	$K^+/\pi^+ \to \mu^+\nu_\mu, \mathbf{V}_{ud}$ [73,75]
	0.2166 ± 0.0020	τ decays [76,77]
	0.226 ± 0.005	Hyperon decays [78,79]
$\lvert\mathbf{V}_{cd}\rvert$	$\mathbf{0.230 \pm 0.011}$	$\nu\, d \to c\, X$ [9]
	0.234 ± 0.026	$D \to \pi l \bar{\nu}_l$ [80,81]
$\lvert\mathbf{V}_{cs}\rvert$	0.963 ± 0.026	$D \to K l \bar{\nu}_l$ [80,81]
	$0.94\,^{+0.35}_{-0.29}$	$W^+ \to c\bar{s}$ [82]
	$\mathbf{0.973 \pm 0.014}$	$W^+ \to$ had., $\mathbf{V}_{uj}$, $\mathbf{V}_{cd}$, $\mathbf{V}_{cb}$ [34,35]
$\lvert\mathbf{V}_{cb}\rvert$	0.0396 ± 0.0008	$B \to D^* l \bar{\nu}_l, D\, l\, \bar{\nu}_l$ [83–85]
	0.04185 ± 0.00073	$b \to c\, l\, \bar{\nu}_l$ [83]
	$\mathbf{0.0408 \pm 0.0011}$	Average
$\lvert\mathbf{V}_{ub}\rvert$	0.00338 ± 0.00036	$B \to \pi\, l\, \bar{\nu}_l$ [9]
	0.00427 ± 0.00038	$b \to u\, l\, \bar{\nu}_l$ [9]
	$\mathbf{0.00389 \pm 0.00044}$	Average
$\lvert\mathbf{V}_{tb}\rvert/\sqrt{\sum_q \lvert\mathbf{V}_{tq}\rvert^2}$	> 0.89 (95% CL)	$t \to b\, W/q\, W$ [86,87]
$\lvert\mathbf{V}_{tb}\rvert$	$\mathbf{0.88 \pm 0.07}$	$p\bar{p} \to t\bar{b} + X$ [88]

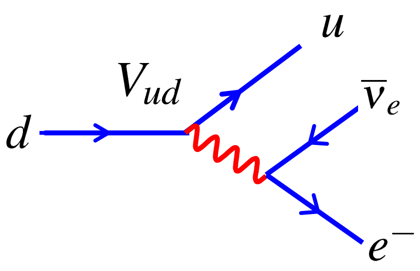
$$|\mathbf{V}_{ud}|^2 + |\mathbf{V}_{us}|^2 + |\mathbf{V}_{ub}|^2 \;=\; 1.0000 \pm 0.0007$$

Determination of the CKM Matrix

- ☐ Experimental determination of the CKM matrix elements
  - ☐ mainly from measurements of leptonic decays (well understood)
  - ☐ Easy to produce/observe meson decays,
    - ☐ however theoretical uncertainties associated with decays of bound states often limit precision
- ☐ Contrast this with the measurements of the PMNS matrix
  - ☐ Few theoretical uncertainties and experimental difficulties in dealing with neutrinos limits precision.

➊ **$|V_{ud}|$** **from nuclear beta decay**

$$\begin{pmatrix} \times & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$



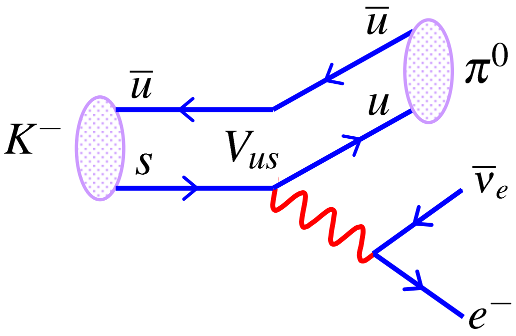
Super-allowed $0^+ \to 0^+$ beta decays are relatively free from theoretical uncertainties

$$\Gamma \propto |V_{ud}|^2$$

$$\boxed{|V_{ud}| = 0.97377 \pm 0.00027} \qquad (\approx \cos\theta_c)$$

❷ **$|V_{us}|$** from semi-leptonic kaon decays

$$\begin{pmatrix} \cdot & \times & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$



$$\Gamma \propto |V_{us}|^2$$

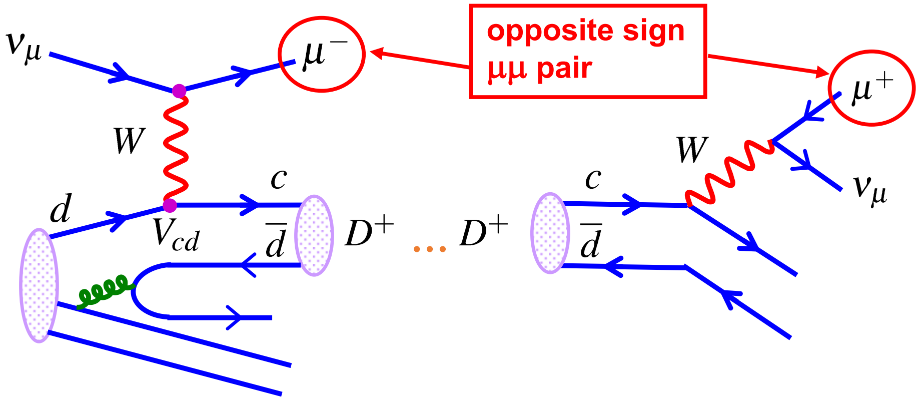
$$\boxed{|V_{us}| = 0.2257 \pm 0.0021} \qquad (\approx \sin\theta_c)$$

❸ **$|V_{cd}|$** from neutrino scattering

$$\nu_\mu + N \rightarrow \mu^+ \mu^- X$$

$$\begin{pmatrix} \cdot & \cdot & \cdot \\ \times & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$

Look for opposite charge di-muon events in ν_μ scattering from production and decay of a $D^+(c\overline{d})$ meson



opposite sign μμ pair

$$\text{Rate} \propto \underbrace{|V_{cd}|^2 \text{Br}(D^+ \rightarrow X\mu^+\nu_\mu)}$$

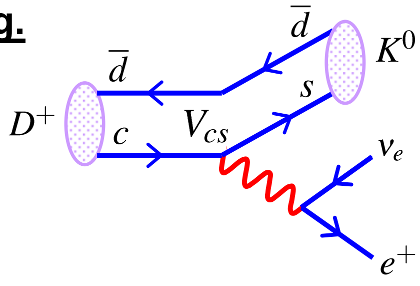
Measured in various collider experiments

⇨ $$\boxed{|V_{cd}| = 0.230 \pm 0.011}$$

❹ **$|V_{cs}|$** **from semi-leptonic charmed meson decays**

$$\begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \times & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$

e.g.



$$\Gamma \propto |V_{cs}|^2$$

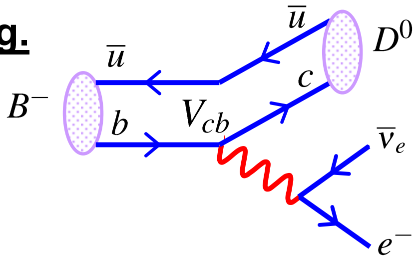
- **Precision limited by theoretical uncertainties**

$$\boxed{|V_{cs}| = 0.957 \underbrace{\pm 0.017}_{\text{experimental error}} \underbrace{\pm 0.093}_{\text{theory uncertainty}}}$$

❺ **$|V_{cb}|$** **from semi-leptonic B hadron decays**

$$\begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \times \\ \cdot & \cdot & \cdot \end{pmatrix}$$

e.g.



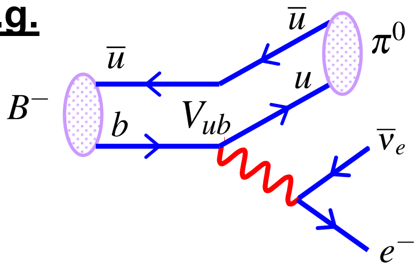
$$\Gamma \propto |V_{cb}|^2$$

$$\boxed{|V_{cb}| = 0.0416 \pm 0.0006}$$

❻ **$|V_{ub}|$** **from semi-leptonic B hadron decays**

$$\begin{pmatrix} \cdot & \cdot & \times \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$

e.g.



$$\Gamma \propto |V_{ub}|^2$$

$$\boxed{|V_{ub}| = 0.0043 \pm 0.0003}$$

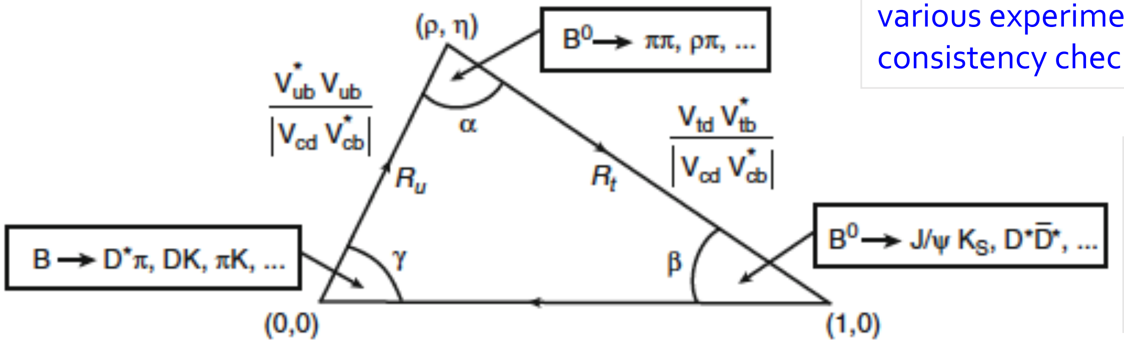
Unitarity triangle and CP violation

- ☐ Wolfenstein parameterisation
  - ☐ $\lambda = s_{12}$
  - ☐ $A\lambda^2 = s_{23}$
  - ☐ $A\lambda^3(\rho - i\eta) = s_{13}e^{-i\delta}$

$$\begin{bmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{bmatrix}.$$

- Unitarity conditions
  - Equation of triangle

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$



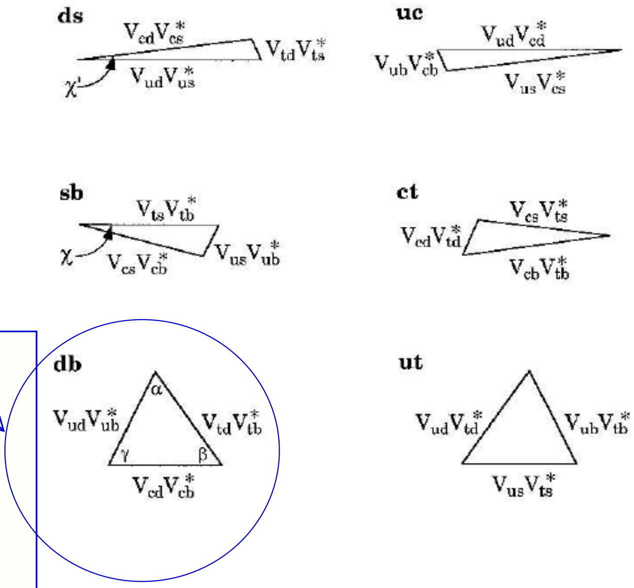
The position of the apex is fixed by various experiments and provides a consistency check of the SM

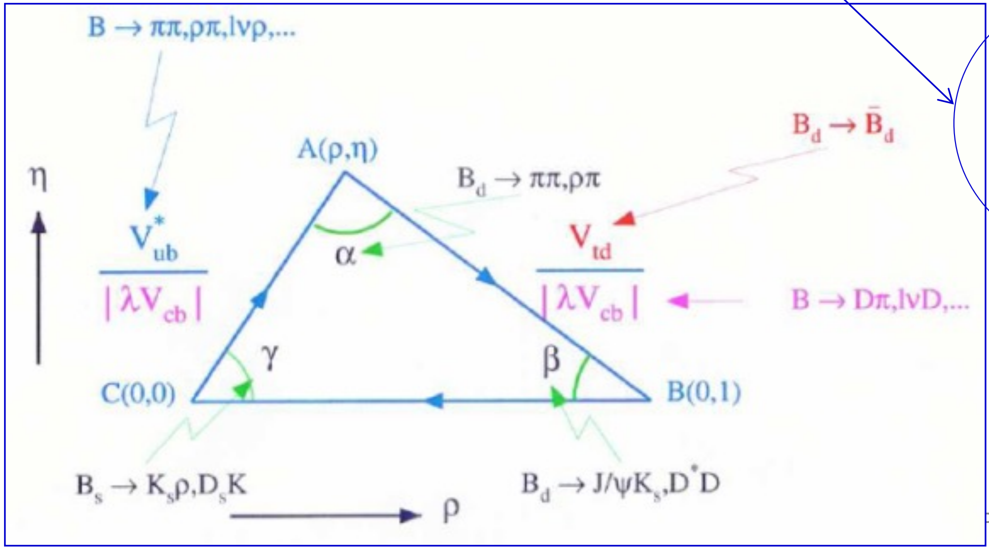
- If, for ex., β >o → CP violation!
- If triangle closed →3 generations!

Six Unitarity Triangles (with same area) in a complex plane: $(0,0)$, $(1,0)$, (ρ,η) $\left[\left(\overline{\rho} \equiv \left(1-\frac{\lambda^2}{2}\right)\rho, \overline{\eta} \equiv \left(1-\frac{\lambda^2}{2}\right)\eta\right)\right]$.

$$\alpha \equiv arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right) \; ; \; \beta \equiv arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right) \; ; \; \gamma \equiv arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$$

$$\begin{array}{ll}
ds & V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* = 0 \\
sb & V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0 \\
\boxed{db} & \boxed{V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0} \\
uc & V_{ud}V_{cd}^* + V_{us}V_{cs}^* + V_{ub}V_{cb}^* = 0 \\
ct & V_{cd}V_{td}^* + V_{cs}V_{ts}^* + V_{cb}V_{tb}^* = 0 \\
tu & V_{ud}V_{td}^* + V_{us}V_{ts}^* + V_{ub}V_{tb}^* = 0
\end{array}$$





The position of the apex is fixed by various experiments and provides a consistency check of the SM.

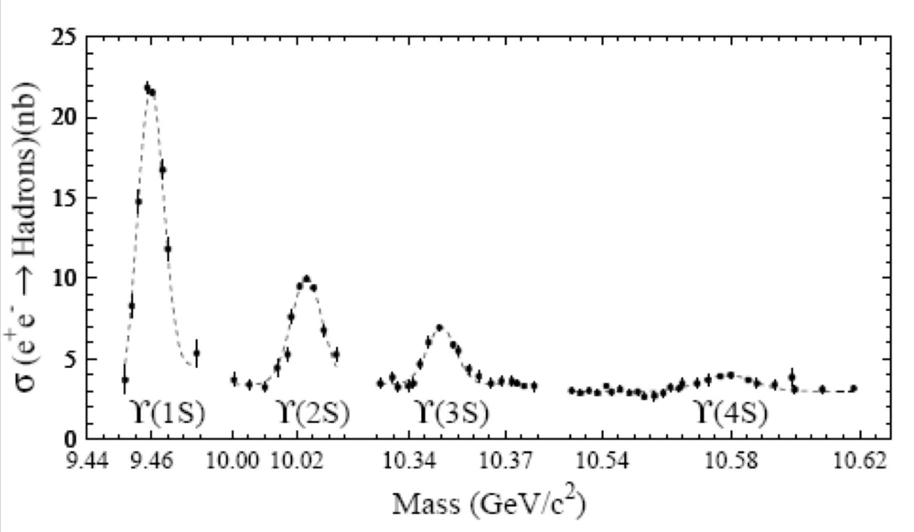
- If, for ex., β >0 → CP violation!
- If triangle closed →3 generations!

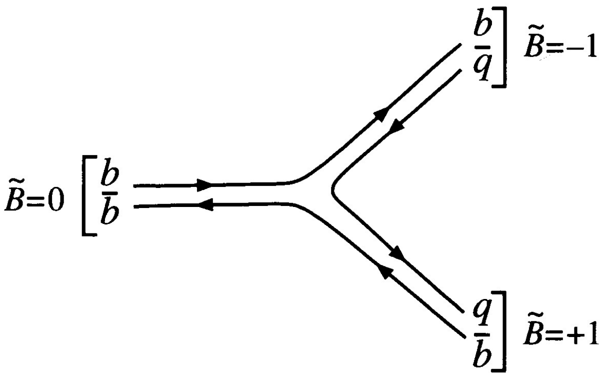
CP violation in B Decays

- What is a neutral B-meson?
  - Short lifetime → no beams of B mesons
  - B-factories $\Upsilon(4s)$: M=10.58GeV, Γ=20MeV

$$e^+e^- \rightarrow \Upsilon(4s) \rightarrow B_d^0\overline{B}_d^0 \;;\; B^+B^-$$
$$J^{PC} = 1^{--}$$

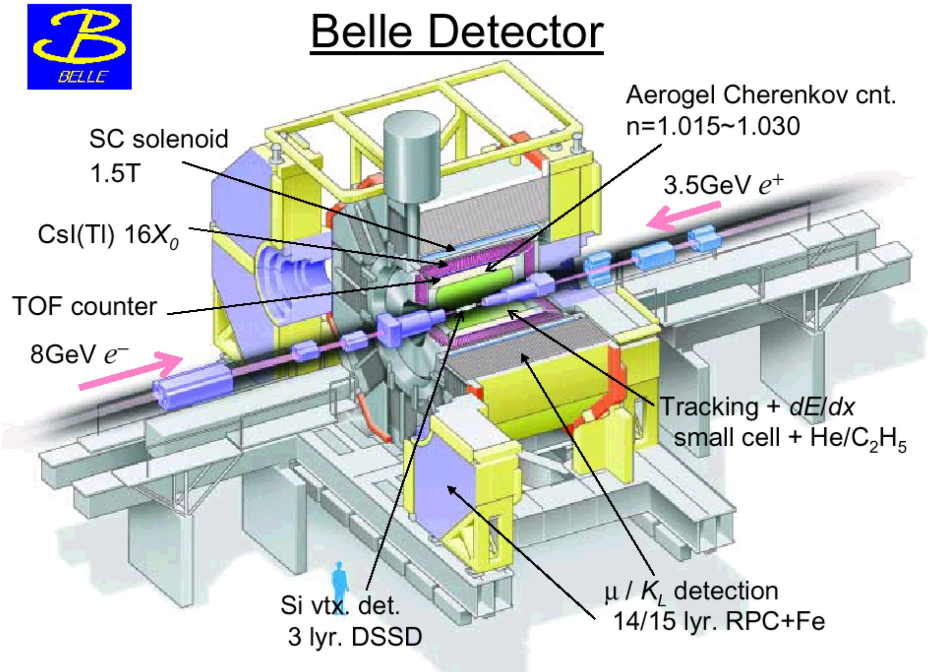
$$B^0(5279.58) \equiv d\overline{b} \;;\; \overline{B}^0 \equiv b\overline{d}$$
$$B^+(5279.26) \equiv u\overline{b} \;;\; B^- \equiv b\overline{u}$$
$$\tilde{B} = +1 \quad \tilde{B} = -1$$
$$\tau_B \sim 1.5ps$$
$$\text{Analog of } K_S^0, K_L^0 \rightarrow B_L^0, B_H^0$$





Experiments at asymmetric B factories

- BaBar at PEP-II, SLAC, US
- BELLE at KEK-B, Japan



$$A_{K\pi} \equiv \frac{\Gamma(\overline{B}^0 \to K^-\pi^+) - \Gamma(B^0 \to K^+\pi^-)}{\Gamma(\overline{B}^0 \to K^-\pi^+) + \Gamma(B^0 \to K^+\pi^-)}$$

$$A_{K\pi} = -0.095 \pm 0.013$$

Other decays studied where CP violation measured

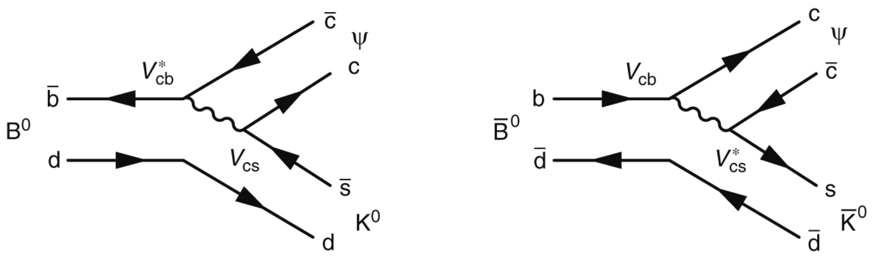
$$B^0 / \overline{B}^0 \to J/\Psi K_S$$

Question: how do we know that a Bo (or anti-Bo) is produced?

CP violation in B decays

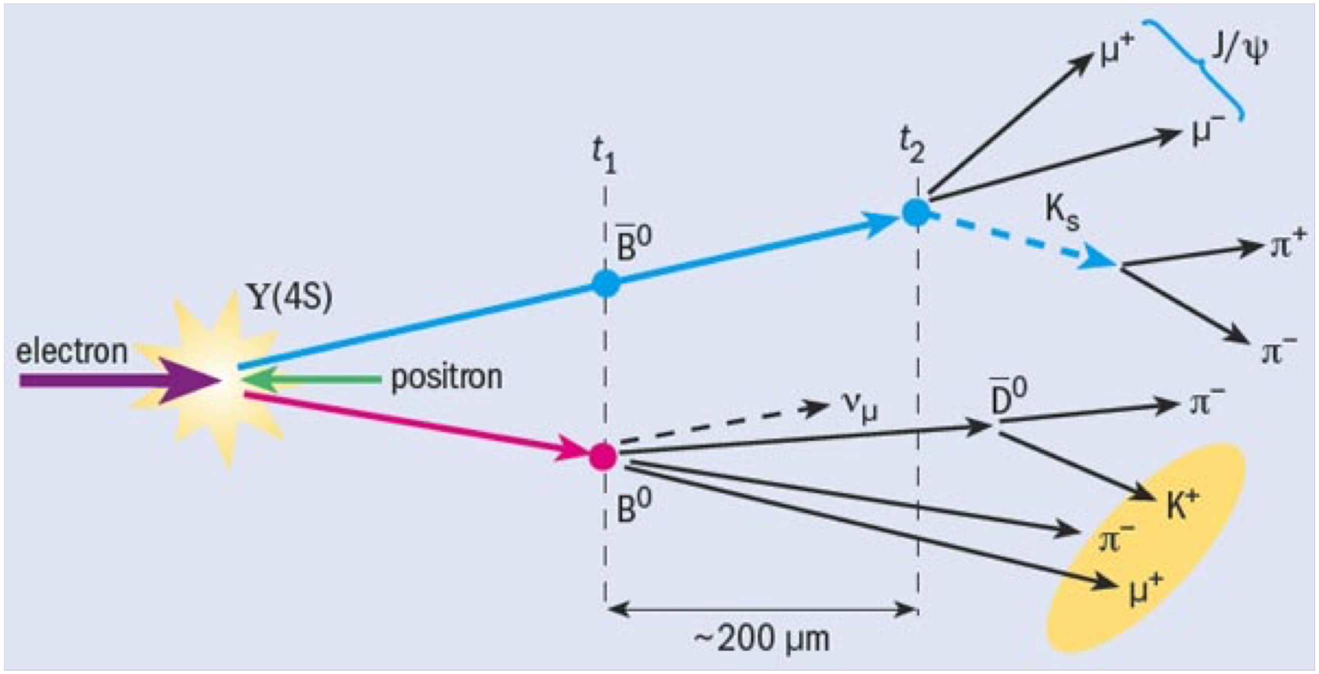
- ☐ Tag one B meson and study the other:
  - ☐ Sign of K, μ
  - ☐ Asymmetric collider

$$B^0 / \bar{B}^0 \to J/\Psi K_S$$

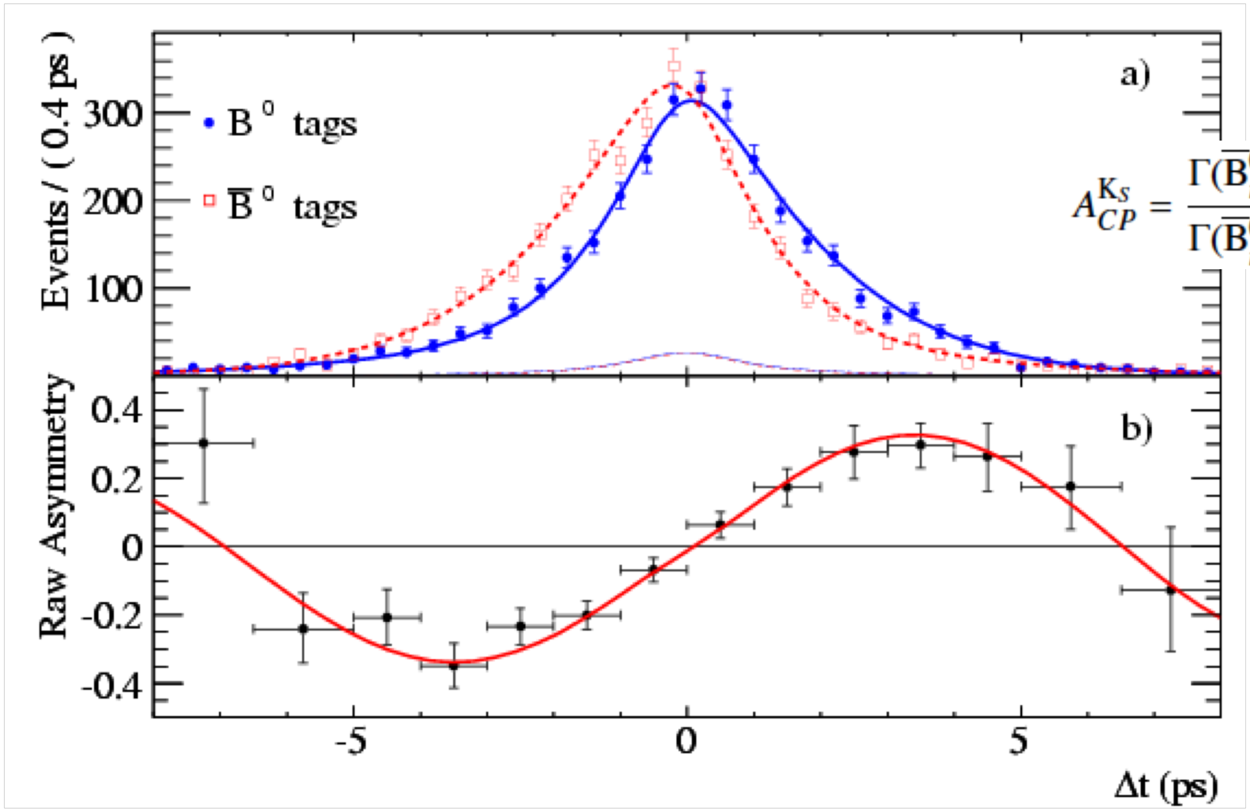


$$\bar{B}^0 \to J/\Psi K_S \to \mu^+\mu^-\pi^+\pi^-$$
$$B^0 \to \bar{D}^0\pi^-\mu^+\nu_\mu ; \bar{D}^0 \to \pi^- K^+$$

$$\beta\gamma >> 1 \Rightarrow \Delta t = t_2 - t_1 = \frac{z_2 - z_1}{\beta\gamma c}$$



CP violation in B decays



$$A_{CP}^{K_S} = \frac{\Gamma(\overline{B}^0_{t=0} \to \psi K_S) - \Gamma(B^0_{t=0} \to \psi K_S)}{\Gamma(\overline{B}^0_{t=0} \to \psi K_S) + \Gamma(B^0_{t=0} \to \psi K_S)} = \sin(\Delta m_d t)\,\sin(2\beta)$$

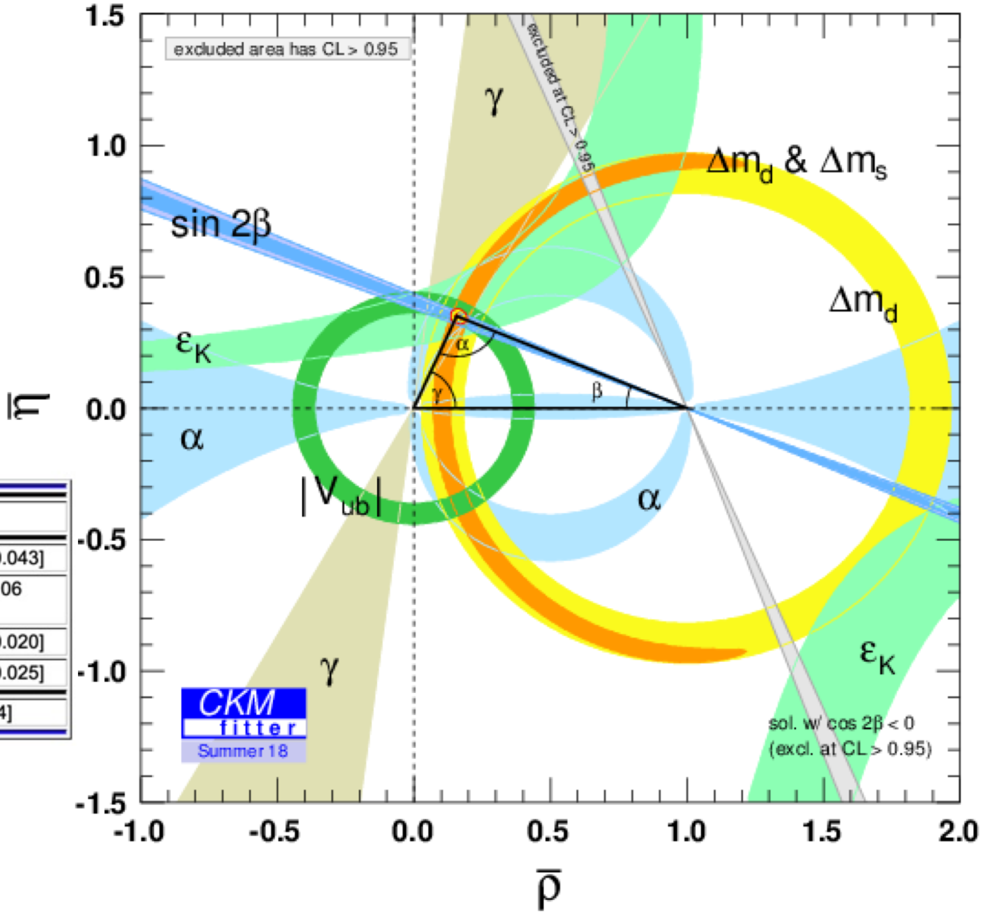
$$\sin 2\beta = 0.681 \pm 0.025$$

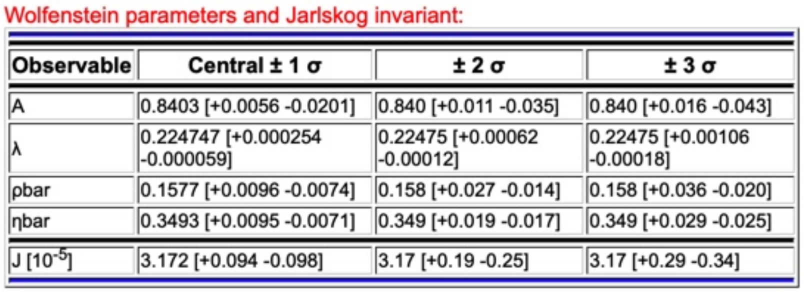
From other decay final states

$$\alpha = 92^\circ \pm 7^\circ \text{ and } \gamma = 82^\circ \pm 20^\circ$$

CKM overall fit

- ☐ *http://ckmfitter.in2p3.fr*





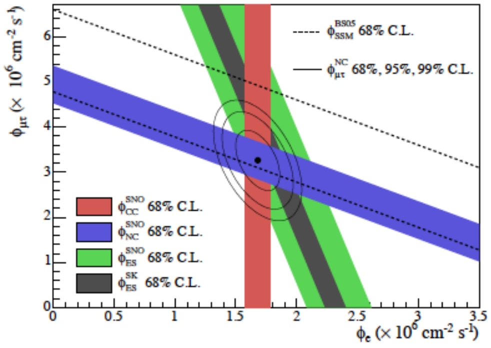
Wolfenstein parameters and Jarlskog invariant:

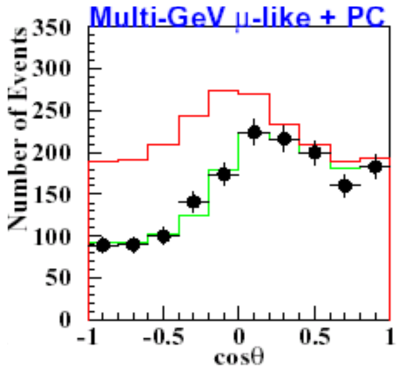
Observable	Central ± 1 σ	± 2 σ	± 3 σ
A	0.8403 [+0.0056 -0.0201]	0.840 [+0.011 -0.035]	0.840 [+0.016 -0.043]
λ	0.224747 [+0.000254 -0.000059]	0.22475 [+0.00062 -0.00012]	0.22475 [+0.00106 -0.00018]
ρbar	0.1577 [+0.0096 -0.0074]	0.158 [+0.027 -0.014]	0.158 [+0.036 -0.020]
ηbar	0.3493 [+0.0095 -0.0071]	0.349 [+0.019 -0.017]	0.349 [+0.029 -0.025]
J [10^{-5}]	3.172 [+0.094 -0.098]	3.17 [+0.19 -0.25]	3.17 [+0.29 -0.34]

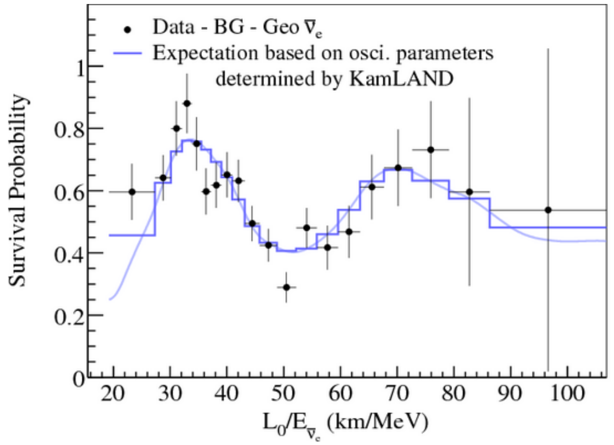
$$\lambda = 0.2253 \pm 0.0007,\ A = 0.811^{+0.022}_{-0.012},\ \rho = 0.13 \pm 0.02,\ \eta = 0.345 \pm 0.014.$$

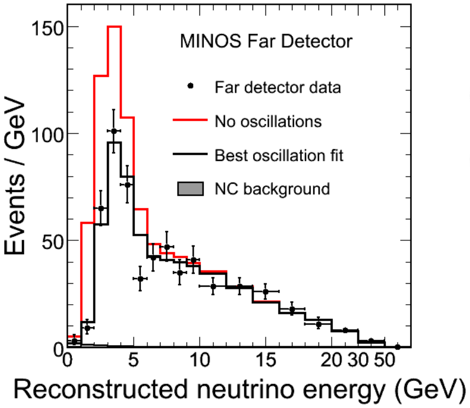
Interpretation of Solar and atmospheric Neutrino Data

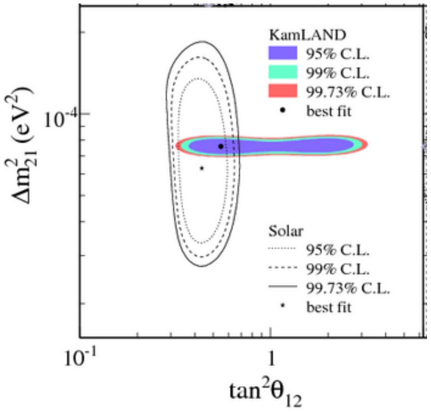
- Neutrinos have mass, mix and oscillate!
  - A combined analysis of all solar neutrino data gives
    $\Delta m^2_{solar} \approx 8\times10^{-5}eV^2, sin^2\theta_{solar} \approx 0.85$
  - Atmospheric neutrino Data consistent with
    $\Delta m^2_{atmos} \approx 2.5\times10^{-3}eV^2, sin^2\theta_{solar} \approx 1$
  - Supported by long-baseline accelerator and reactor experiments











$$\tan^2\theta_{12} = 0.47^{+0.06}_{-0.05}$$

$$|\Delta m^2_{21}| = (7.59 \pm 0.21) \times 10^{-5}\,\mathrm{eV}^2$$

2-neutrino flavour oscillations

- ❑ two-flavour oscillation probability

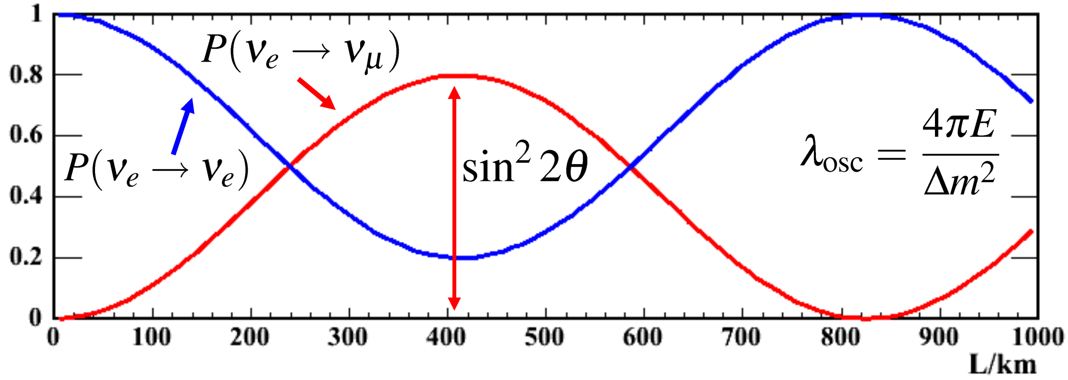
$$P(\nu_e \to \nu_\mu) = \sin^2 2\theta \sin^2 \left(\frac{\Delta m^2_{21} L}{4E} \right)$$

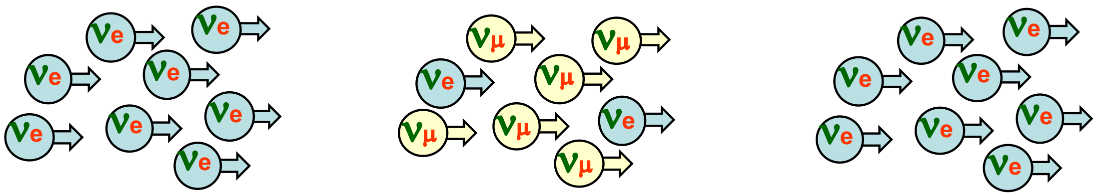
$$\Delta m^2_{21} = m^2_2 - m^2_1$$

- ❑ Corresponding two-flavour survival probability

$$P(\nu_e \to \nu_e) = 1 - \sin^2 2\theta \sin^2 \left(\frac{\Delta m^2_{21} L}{4E} \right)$$

$$\Delta m^2 = 0.003\,\mathrm{eV}^2, \qquad \sin^2 2\theta = 0.8, \qquad E_\nu = 1\,\mathrm{GeV}$$





3-neutrino flavour oscillations

- PMNS mass matrix

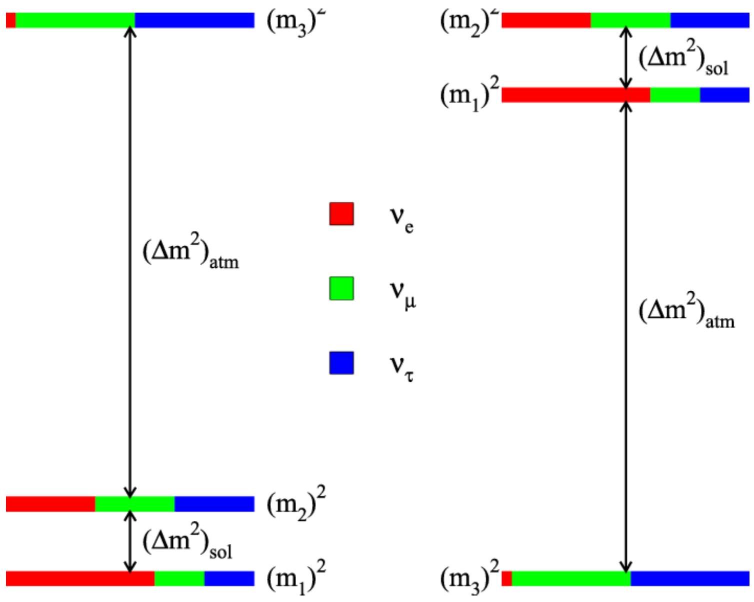
$$\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = \begin{pmatrix}U_{e1} & U_{e2} & U_{e3}\\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3}\\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3}\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix} \quad |U| = \begin{bmatrix}|U|_{e1} & |U|_{e2} & |U|_{e3}\\ |U|_{\mu 1} & |U|_{\mu 2} & |U|_{\mu 3}\\ |U|_{\tau 1} & |U|_{\tau 2} & |U|_{\tau 3}\end{bmatrix} = \begin{bmatrix}0.799\ldots0.844 & 0.516\ldots0.582 & 0.141\ldots0.156\\ 0.242\ldots0.494 & 0.467\ldots0.678 & 0.639\ldots0.774\\ 0.284\ldots0.521 & 0.490\ldots0.695 & 0.615\ldots0.754\end{bmatrix}$$

$$\phi_i \approx \frac{m_i^2}{2E}L$$

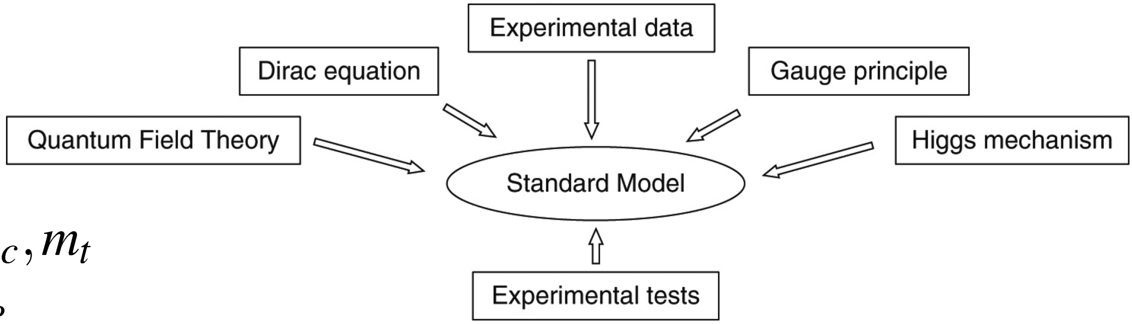
- 3-flavour oscillations

$$\begin{aligned} P(\nu_e \rightarrow \nu_\mu) &= 2\Re\{U_{e1}U_{\mu 1}^{*}U_{e2}^{*}U_{\mu 2}[e^{-i(\phi_1-\phi_2)}-1]\}\\ &+ 2\Re\{U_{e1}U_{\mu 1}^{*}U_{e3}^{*}U_{\mu 3}[e^{-i(\phi_1-\phi_3)}-1]\}\\ &+ 2\Re\{U_{e2}U_{\mu 2}^{*}U_{e3}^{*}U_{\mu 3}[e^{-i(\phi_2-\phi_3)}-1]\}\end{aligned}$$

- Normal or inverted mass hierarchy?



Standard Model and beyond



Experimental data

Dirac equation

Gauge principle

Quantum Field Theory

Standard Model

Higgs mechanism

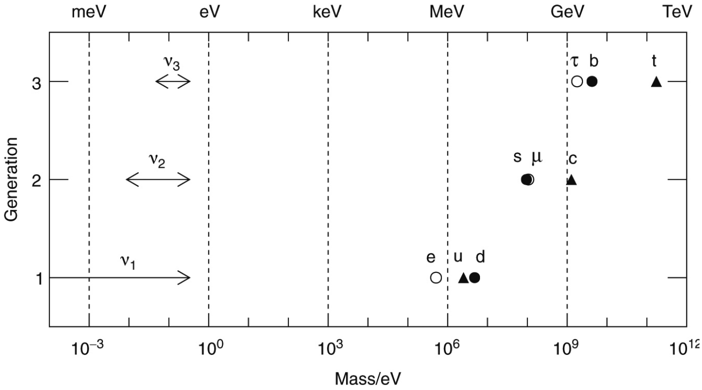
Experimental tests

- Too many parameters

$$m_{\nu_1}, m_{\nu_2}, m_{\nu_3}, m_e, m_\mu, m_\tau, m_d, m_s, m_b, m_u, m_c, m_t$$

$$\theta_{12}, \theta_{13}, \theta_{23}, \delta \quad \lambda, A, \rho, \eta \quad e, G_F, \theta_W, \alpha_S \quad m_H, \theta_{CP}$$

  - 14 associated to the Higgs field, 8 with flavour sector, 3 with the gauge sector
  - Forgetting neutrinos masses within each generation are similar
  - Coupling constants of similar order of magnitude, GUT?



- Open questions
  - What is Dark Matter (DM)?
    - Can it be directly detected? Produced at colliders?
    - Does Supersymmetry (SUSY) exist?
    - Hierarchy problem, DM candidates, gauge unification

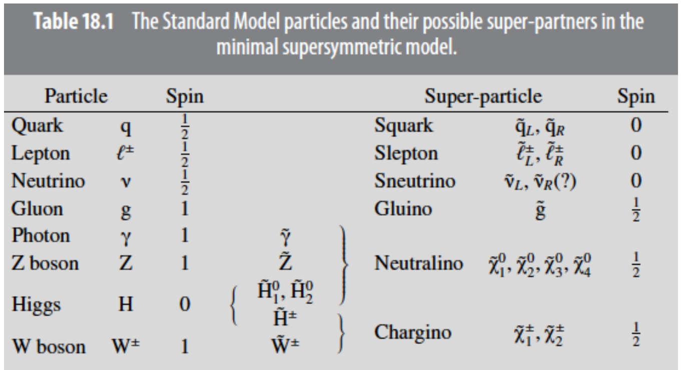
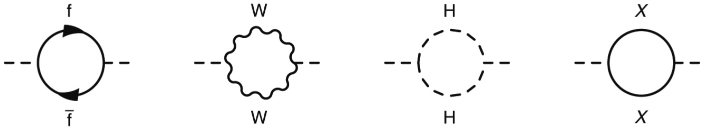


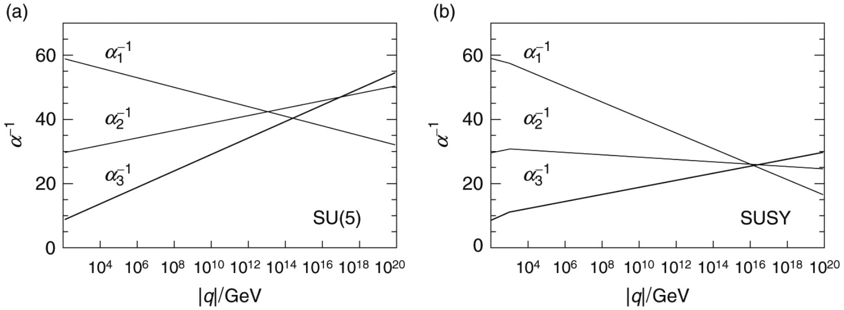
Table 18.1 The Standard Model particles and their possible super-partners in the minimal supersymmetric model.

Particle		Spin		Super-particle		Spin
Quark	q	$\frac{1}{2}$		Squark	$\tilde{q}_L, \tilde{q}_R$	0
Lepton	$\ell^\pm$	$\frac{1}{2}$		Slepton	$\tilde{\ell}^\pm_L, \tilde{\ell}^\pm_R$	0
Neutrino	ν	$\frac{1}{2}$		Sneutrino	$\tilde{\nu}_L, \tilde{\nu}_R(?)$	0
Gluon	g	1		Gluino	$\tilde{g}$	$\frac{1}{2}$
Photon	γ	1	$\tilde{\gamma}$			
Z boson	Z	1	$\tilde{Z}$	Neutralino	$\tilde{\chi}^0_1, \tilde{\chi}^0_2, \tilde{\chi}^0_3, \tilde{\chi}^0_4$	$\frac{1}{2}$
Higgs	H	0	$\tilde{H}^0_1, \tilde{H}^0_2$ $\tilde{H}^\pm$			
W boson	$W^\pm$	1	$\tilde{W}^\pm$	Chargino	$\tilde{\chi}^\pm_1, \tilde{\chi}^\pm_2$	$\frac{1}{2}$



☐ Can the forces be unified?

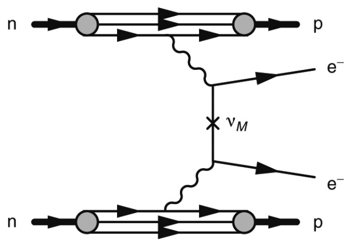
$$\alpha^{-1} : \alpha_W^{-1} : \alpha_S^{-1} \approx 128 : 30 : 9$$



☐ What is the nature of the Higgs boson?

☐ Flavour and the origin of CP violation

☐ Are neutrinos Majorana particles?



☐ ...