Interaction theory –Electrons

Lesson FYSKJM4710

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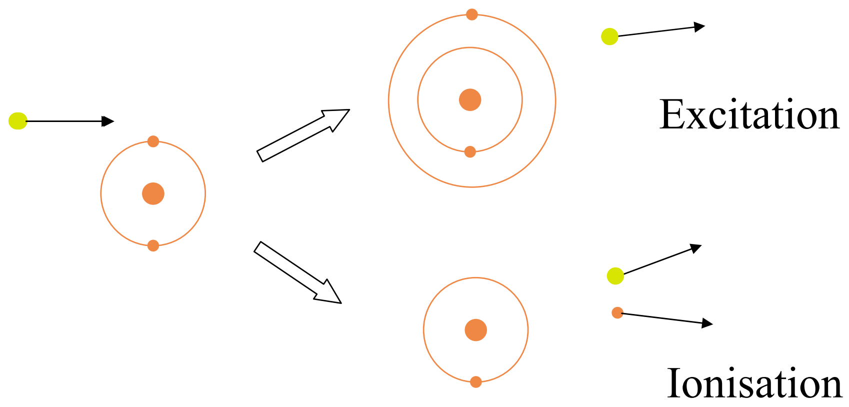
Source: F. H. Attix: Introduction to radiological physics and radiation dosimetry (ISBN 0-471-01146-0)

Excitation / ionization

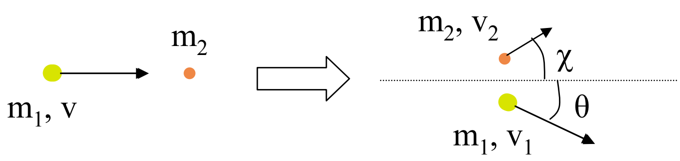
- Incoming charged particle interacts with atom / molecule:



- An ion pair is created

Elastic collision 1

- Interaction between two particles where kinetic energy is preserved:



- Classical mechanics:

$$T_0 = \frac{1}{2} m_1 v^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

$$m_1 v = m_1 v_1 \cos\theta + m_2 v_2 \cos\chi$$

$$0 = m_1 v_1 \sin\theta - m_2 v_2 \sin\chi$$

Elastic collision 2

$$\Rightarrow v_2 = \frac{2 m_1 v \cos\chi}{m_1 + m_2} \quad , \quad v_1 = v \sqrt{1 - \frac{4 m_1 m_2 \cos^2\chi}{(m_1 + m_2)^2}}$$

$$\tan\theta = \frac{\sin 2\chi}{\frac{m_1}{m_2} - \cos 2\chi}$$

- Equations give, among others, maximum energy transferred:

$$E_{max} = \frac{1}{2} m_2 v_{2,max}^2 = 4 \frac{m_1 m_2}{(m_1 + m_2)^2} T_0$$

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Elastic collision 3

a) $m_1 >> m_2$	b) $m_1 = m_2$	c) $m_1 << m_2$
$0 \le \chi \le \pi/2$ $0 \le \theta \le \tan^{-1}(\frac{m_2}{m_1}\sin 2\chi)$ $E_{max} = 4\frac{m_2}{m_1}T_0$	$0 \le \chi \le \pi/2$ $0 \le \theta \le \pi/2$ $E_{max} = T_0$	$0 \le \chi \le \pi/2$ $0 \le \theta \le \pi$ $E_{max} = 4\frac{m_1}{m_2}T_0$

- Proton-electron collision:

  $\theta_{max} = 0.03^o$, $E_{max} = 0.2$ %
- Electron-electron (or e.g. neutron-neutron) coll.:

  $\theta_{max} = 90^o$, $E_{max} = 100$ %

Elastic collision – cross section

- Rutherford showed that the cross section is:

$$\frac{d\sigma}{d\Omega} \propto \frac{1}{\sin^4(\theta/2)}$$

$\rightarrow$ small scattering angles are most probable

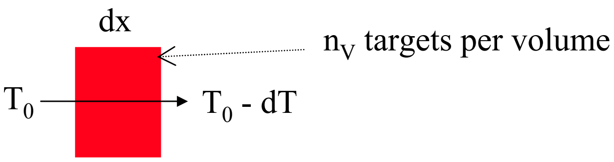
- With respect to energy:

$$\frac{d\sigma}{dE} \propto \frac{1}{E^2}$$

- $\rightarrow$ small energy transfers most probable

Stopping power

- S=dT/dx; expected energy loss per unit lenght

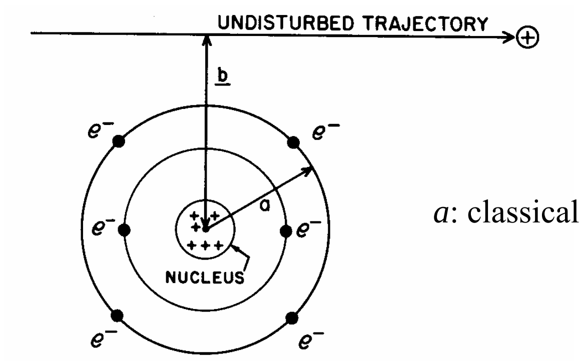


$$dT = \langle E n_V dx \sigma \rangle = n_V dx \int_{E_{min}}^{E_{max}} \frac{d\sigma}{dE} E dE = \rho \left(\frac{N_A Z}{A} \right) dx \int_{E_{min}}^{E_{max}} \frac{d\sigma}{dE} E dE$$

$$\left(\frac{dT}{\rho dx} \right) = \frac{S}{\rho} = \left(\frac{N_A Z}{A} \right) \int_{E_{min}}^{E_{max}} \frac{d\sigma}{dE} E dE$$

Impact parameter

- Charged particles: Coulomb interactions
- Most impprtant: interactions with electrons
- Impact parameter *b*:



a: classical atomic radius

Soft collisions 1

- *b* >> *a* : incoming particle passes atom at long distance
- Weak forces, small energy transfers to the atom
- Inelastic collisions: Predominantly excitations, some ionizations
- Energy transfer range from ”*Emin*” to ”*H*”
- Hans Bethe. Quantum mechanical considerations
- In the following, theory for heavy charged particles

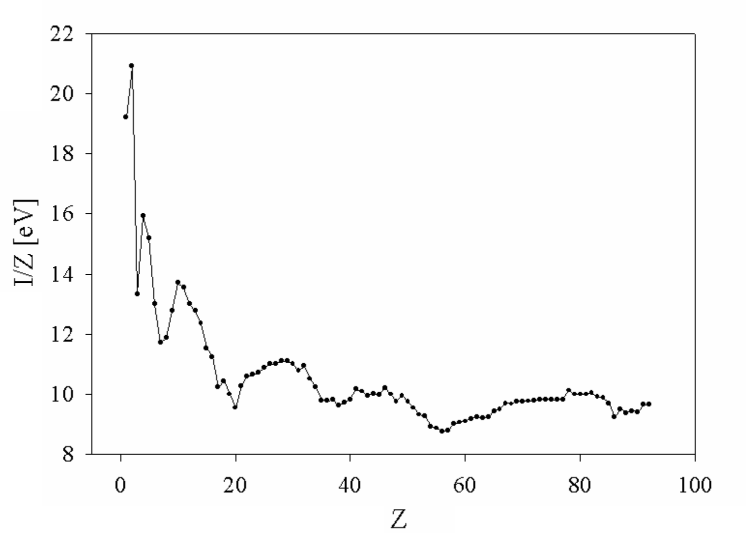
Soft collisions 2

$$\frac{S_{c,soft}}{\rho} = \left(\frac{dT_{soft}}{\rho dx}\right)_c = \frac{N_A Z}{A}\frac{2\pi r_0^2 m_e c^2 z^2}{\beta^2}\ln\left[\frac{2m_e c^2\beta^2 H}{I^2(1-\beta^2)} - \beta^2\right]$$

- r_0: classical electron radius = $e^2/4\pi\varepsilon_0 m_e c^2$
- I: mean excitation potential
- $\beta = v/c$
- z: charge of incoming particle
- ρ: density of medium
- $N_A Z/A$: numbers of electron per gram
- H: maksimum energy transferred by soft collisions

Soft collisions 2

- Quantum mechanics (atomic structure) is reflected in the mean excitation potential



Hard collisions 1

- $b << a$: charged pass particle pass 'through' atom
- Large (but few) energy transfers
- Energy transfers from H to E_{max}
- May be considered as an elastic collision between free particles (binding energy is negligible)

$$\frac{S_{c,hard}}{\rho} = \left(\frac{dT_{hard}}{\rho dx}\right)_c = \frac{N_A Z}{A}\frac{2\pi r_0^2 m_e c^2 z^2}{\beta^2}\ln\left[\frac{E_{max}}{H} - \beta^2\right]$$

Collision stopping power

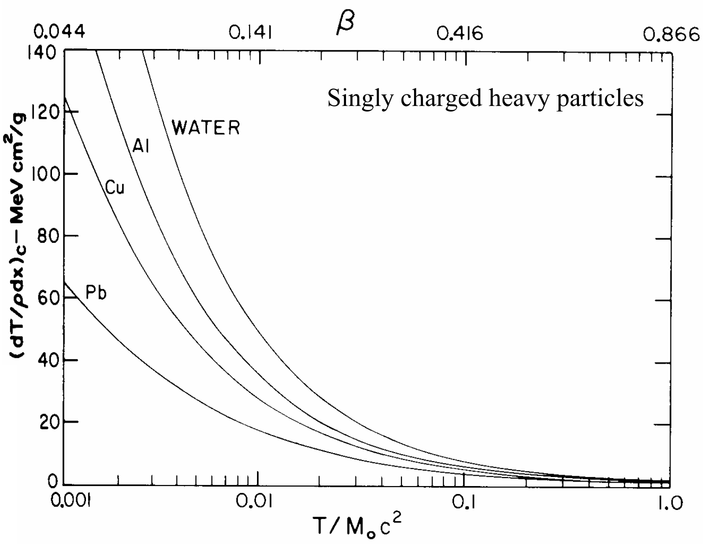
- For inelastic collisions, the total cross section is thus:

$$\frac{S_c}{\rho} = \frac{S_{c,\text{soft}}}{\rho} + \frac{S_{c,\text{hard}}}{\rho}$$

$$= 4\pi r_0^2 m_e c^2 \left(\frac{N_A Z}{A}\right)\left(\frac{z}{\beta}\right)^2 \left[\ln\left(\frac{2m_e c^2\beta^2}{(1-\beta^2)I}\right) - \beta^2\right]$$

- Important: Increases with z^2, decreases with v^2, not dependent on particle mass

S_c/ρ, different substances



- I and electron density (ZN_A/A) give differences

S_c/ρ, electrons and positrons

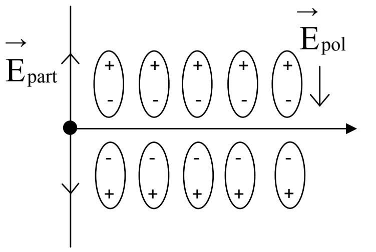
- Electron-electron scattering is more complicated; scattering between two identical particles
- $S_{c,\, hard}/\rho$ (el-el) is described by the Möller cross section
- $S_{c,\, hard}/\rho$ (pos-el) is described by the Bhabha c.s.
- $S_{c,\, soft}/\rho$ was given by Bethe, as for heavy particles
- Characteristics similar to that for heavy charged particles

Shell correction

- Derivation of S_c assumes $v \gg v_{atomic\ electrons}$
- When $v \sim v_{atomic\ electrons}$, no ionizations
- Most important for K-shell electrons
- Shell correction C/Z takes this into accout, and thus *reduces* S_c/ρ
- C/Z depends on particle energy and medium

Density correction

- Charged particles polarizes medium which is being traversed



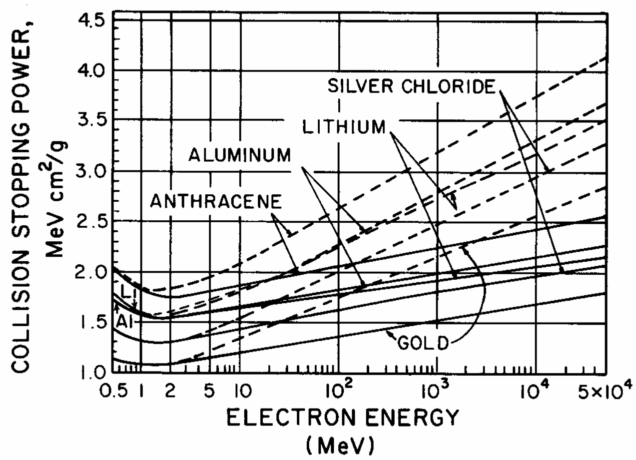
Charged (+z) particle

$$\vec{E}_{eff} = \vec{E}_{part} + \vec{E}_{pol}\ ,\quad \left|\vec{E}_{eff}\right| < \left|\vec{E}_{part}\right|$$

- Weaker interactions with remote atoms due to reduction in electromagnetic field strenght
- Polarization increases with energy and density
- Most important for electrons and positrons

Density correction

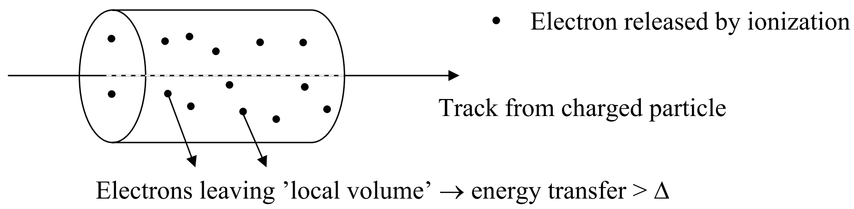
- Density correction δ *reduces* S_c/ρ for liquids and solids
- S_c/ρ (water vapor) > S_c/ρ (water)



Dashed line:
S_c/ρ without δ

Linear Energy Transfer 1

- LET_{Δ} is denoted the *restricted stopping power*
- dT/dx: mean energy loss per unit lenght – but how much is deposited 'locally'?



- S_c: energy transfers from E_{min} to E_{max}
- How much energy is deposited within the range of an electron given energy Δ?

Linear Energy Transfer 2

- Energy loss (soft + hard) per unit lenght for $E_{min} < E < \Delta$:

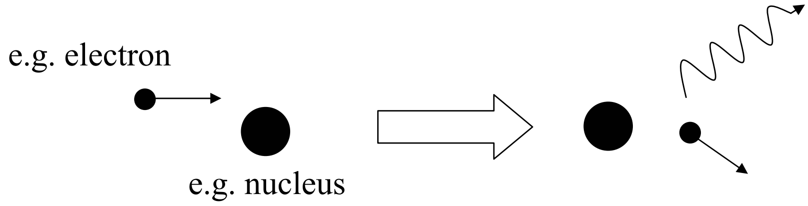
$$L_{\Delta} = \left(\frac{dT}{dx}\right)_{\Delta} = \rho\left(\frac{N_A Z}{A}\right)\int_{E_{min}}^{\Delta}\frac{d\sigma}{dE}EdE$$

$$= \rho 2\pi r_0^2 m_e c^2\left(\frac{N_A Z}{A}\right)\left(\frac{z}{\beta}\right)^2\left[\ln\left(\frac{2m_e c^2\beta^2\Delta}{(1-\beta^2)I}\right) - 2\beta^2\right]$$

- For $\Delta = E_{max}$, we have $L_{\infty} = S_c$; *unrestricted* LET
- LET_{Δ} is often given in [keV/μm]
- 30 MeV protons in water: $LET_{100\ eV} / L_{\infty} = 0.53$

Brehmsstrahlung 1

- Photon may be emitted from charged particle accelerated in the field from an electron or nucleus



- Larmor's formula (classical electromagnetism) for radiated effect from accelerated charged particle:

$$P = \frac{(ze)^2 a^2}{6\pi\varepsilon_0 c^3}$$

Brehmsstrahlung 2

- For particle accelerated in nuclear field:

$$F = ma = \frac{zZe^2}{4\pi\varepsilon_0 r^2} \quad \Rightarrow \quad a = \frac{zZe^2}{4\pi\varepsilon_0 m r^2}$$

$$\Rightarrow P \propto \left(\frac{Z}{m}\right)^2$$

- Comparison of protons and electrons:

$$\frac{P_p}{P_e} = \left(\frac{m_e}{m_p}\right)^2 \approx \frac{1}{1836^2}$$

- Brehmsstrahlung not important for heavy charged particles

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Brehmsstrahlung 3

- Energy loss by brehmsstrahlung is called *radiative loss*
- Maksimum energy loss is the totale kinetic energy *T*
- Radiative loss per unit lenght: *radiative stopping power*:

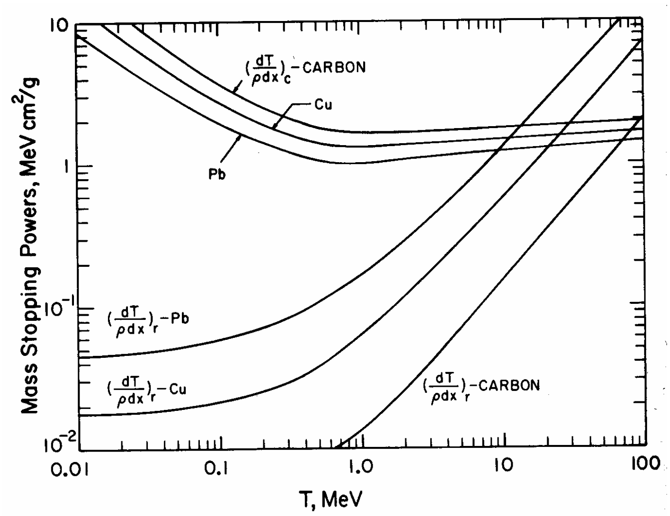
$$\left(\frac{S}{\rho}\right)_r = \left(\frac{dT}{\rho dx}\right)_r \approx \alpha r_0^2 \frac{N_A Z^2}{A}(T + m_e c^2)\overline{B}_r(T,Z)$$

- $\overline{B}_r(T,Z)$ weakly dependent on T and Z
- Brehmsstrahlung increases with energy and atomic number

Total stopping power, electrons

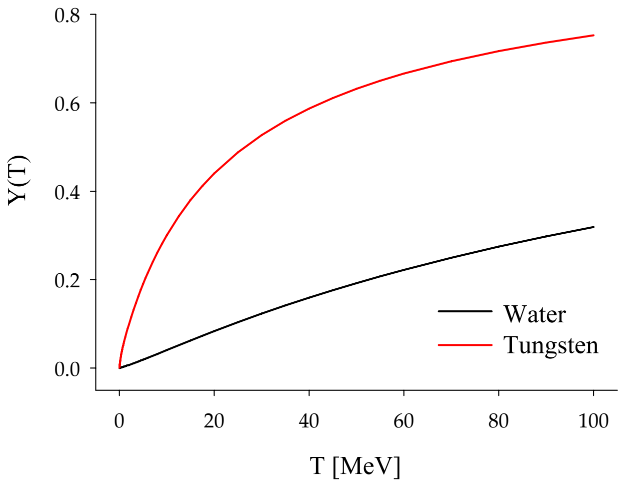
- Total mass stopping power:

$$\left(\frac{dT}{\rho dx}\right)_{tot} = \left(\frac{dT}{\rho dx}\right)_c + \left(\frac{dT}{\rho dx}\right)_r$$

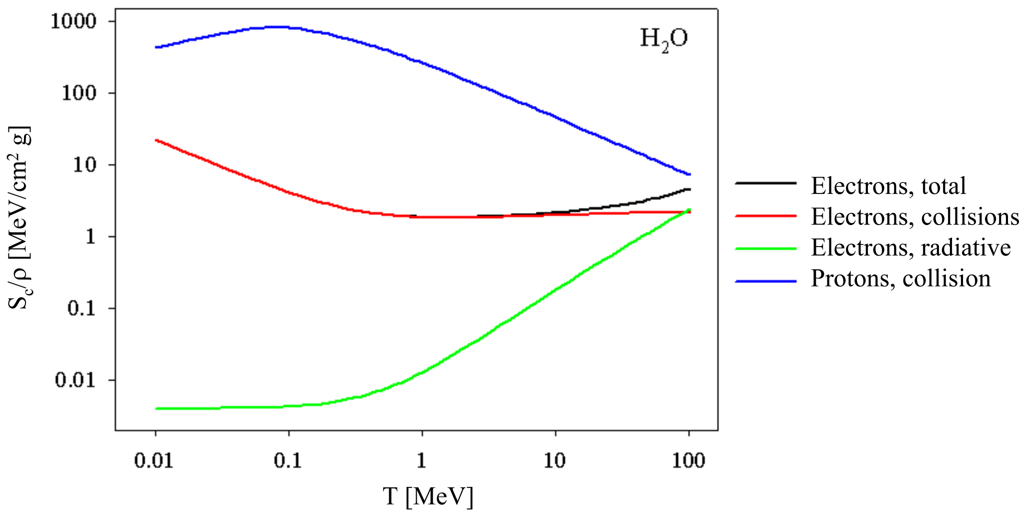


Radiation yield

$$Y(T) = \frac{(dT/\rho dx)_r}{(dT/\rho dx)_c + (dT/\rho dx)_r} = \frac{S_r}{S} \approx \frac{TZ}{n} \qquad n=750\ \text{MeV}$$

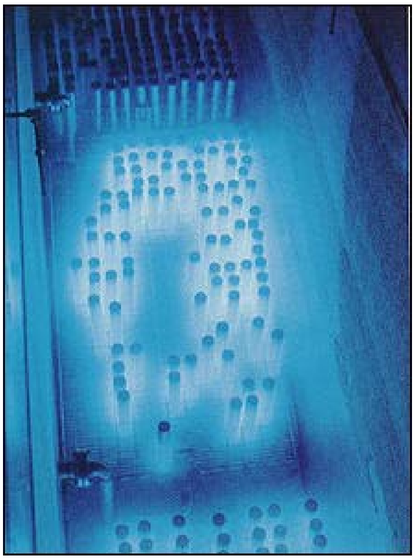


S_c/ρ, protons and electrons



Cerenkov effect

- High energy electrons (v > c/n) polarizes medium (e.g. water) and blueish light (+ UV) is emitted
- Low energy loss



Other interactions

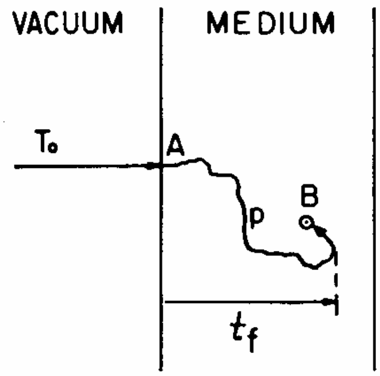
- *Nuclear interactions*: Inelastic process where charged particle (e.g. proton) excites nucleus →
  - Scattering of charged particle
  - Emission of neutron, photon, or α-particle ($^{4}_{2}He$)
- Not important below ~10 MeV (protons)

- *Positron annihilation*: Positron interacts with electron → a pair of photons with energy ≥ 2 x 0.511 MeV is created. Photons are emitted in opposite directions.
- Probability decreases as ~ 1/v

Range 1

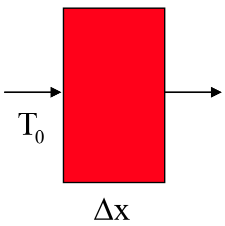
- *The range* $\mathscr{R}$ of a charged particle in matter is the (expectation value) of it's total pathlenght *p*
- *The projected range <t>* er is the (expectation value) of the largest depth t_f a charged particle can reach along it's incident direction

- Electrons:
  $\langle t\rangle < \mathscr{R}$
- Heavy charged particles:
  $\langle t\rangle \approx \mathscr{R}$



CSDA-range

- The range may be approximated by $\mathscr{R}_{CSDA}$ (continuous slowing down approximation)
- Energy loss per unit lenght *dT/dx* – gives implicitly a measure of the range:



$$T_0 - \Delta T = T_0 - \frac{dT}{dx}\Delta x$$

$$\Delta x = \frac{dx}{dT}\Delta T \quad , \quad \Rightarrow \Re = \sum_{i=1}^{n}\Delta x_i = \sum_{i=1}^{n}\left(\frac{dx}{dT}\right)_i \Delta T$$

$$\Rightarrow \Re_{CSDA} = \int_0^{T_0}\left(\frac{dT}{dx}\right)^{-1} dT$$

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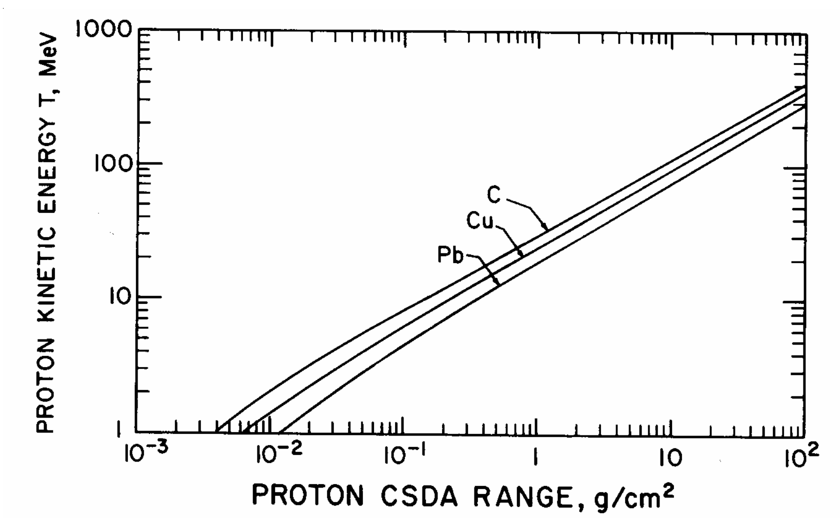
Range 3

- The range is often given multiplied by the density:

$$\Re_{CSDA} = \int_0^{T_0} \left(\frac{dT}{\rho dx} \right)^{-1} dT$$

- Unit thus becomes [cm] [g/cm^3] = [g/cm^2]
- Range of charged particle depends on:
  - Charge and kinetic energy
  - Density, electron density and mean excitation potential of absorber

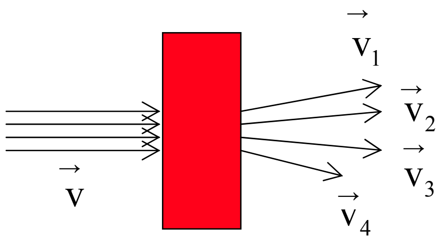
Range 4



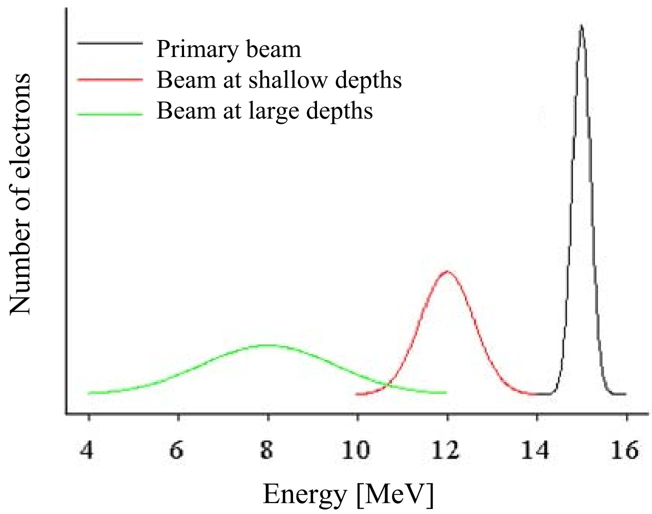
Multiple scattering and straggling

- In a beam of charged particles, one has:
  - Variations in energy deposition (straggling)
  - Variations in angular scattering

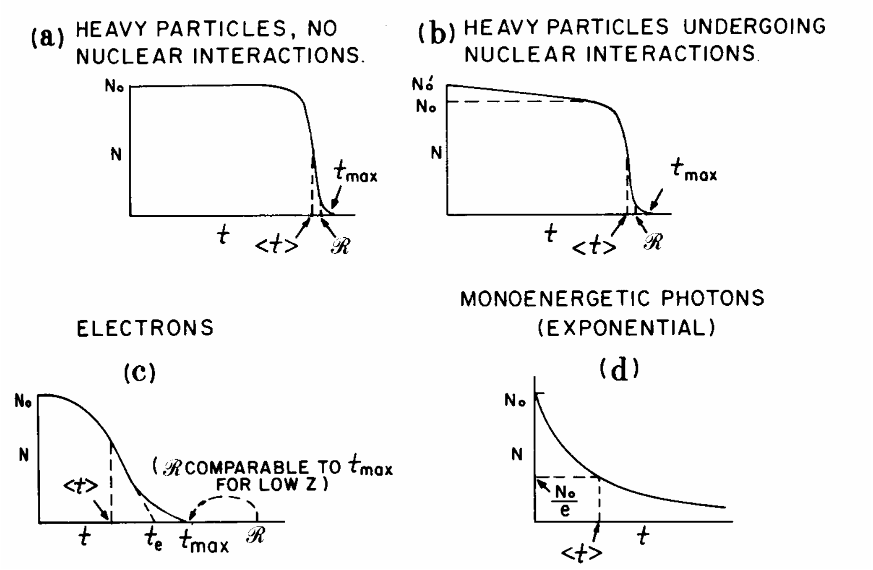
→ The beam, where all particles originally had the same velocity, will be smeared out as the particles traverses matter



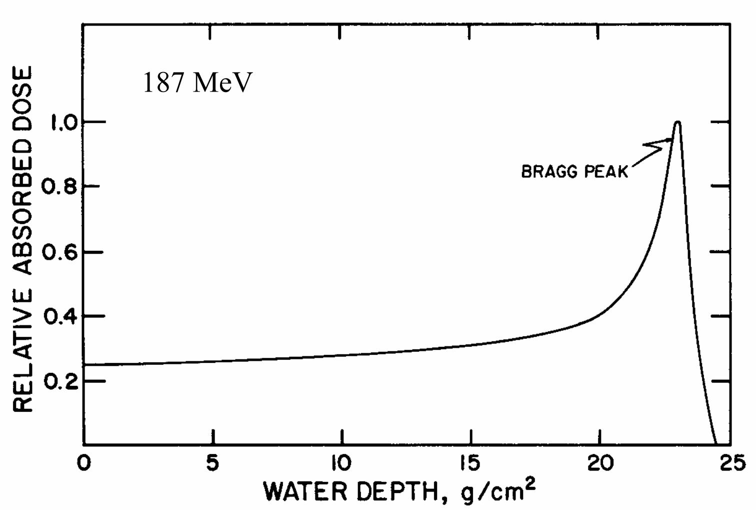
Straggling



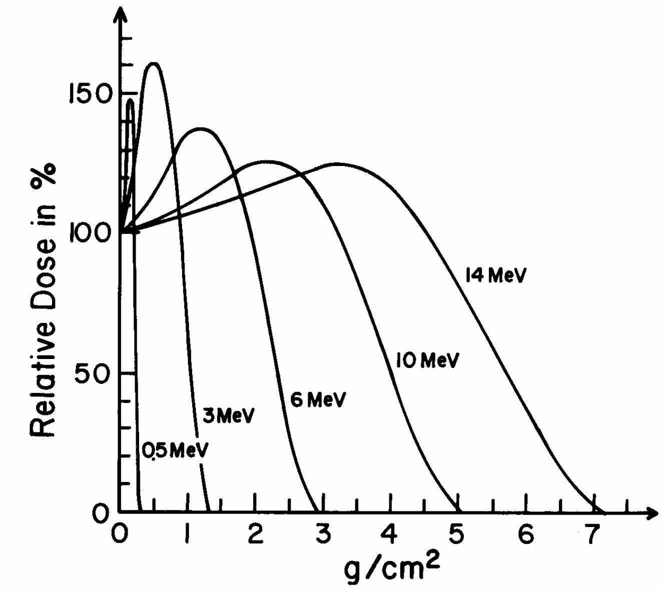
Projected range



Energy deposition, protons

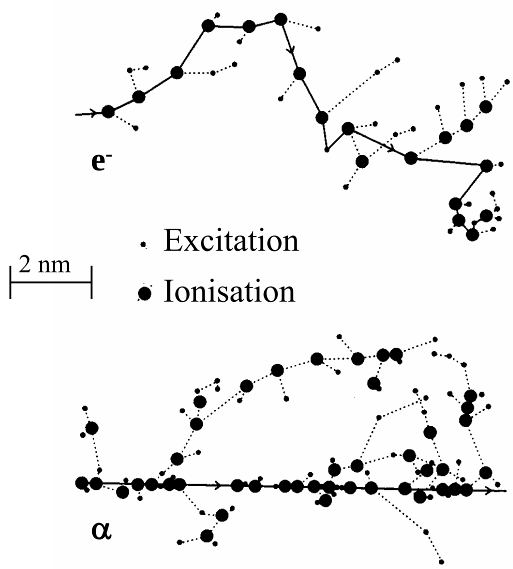


Energy deposition, electrons



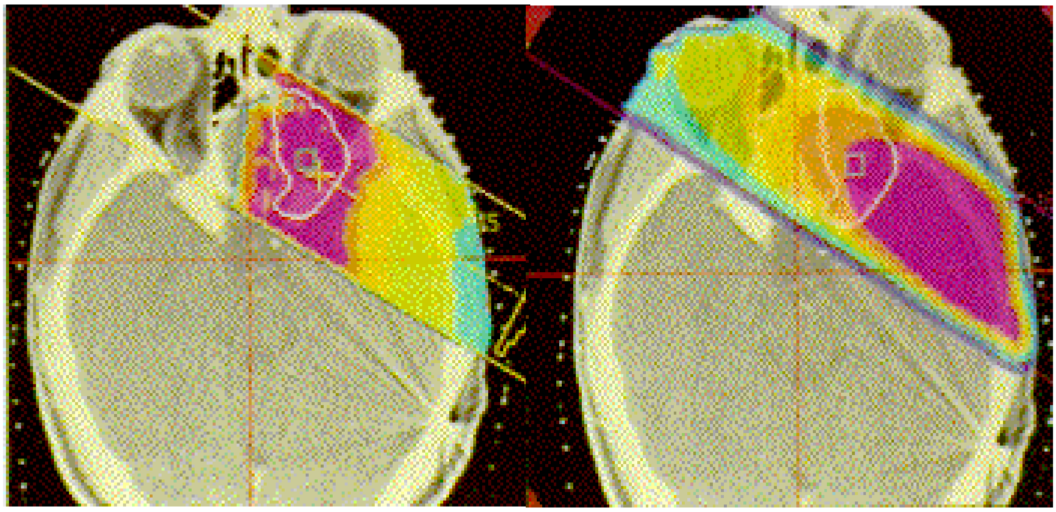
Monte Carlo simulations

- Monte Carlo simulations of the track of an electron (0.5 keV) and an α-particle (4 MeV) in water
- Note:
  e^- is most scattered
  α has the highest dT/dx



Hadron therapy

- Heavy charged particles may be used for radiation therapy – conforms better to the target than photons or electrons



Web pages

- For stopping powers:

http://physics.nist.gov/PhysRefData/Star/Text/contents.html

- For attenuation coefficients:

http://physics.nist.gov/PhysRefData/XrayMassCoef/cover.html