I. PROBLEM SESSION 2

A. Problem 2.1

Consider the plane hkl in a crystal lattice. (a) Prove that the reciprocal lattice vector $\vec{G} = h\vec{b_1} + k\vec{b_2} + l\vec{b_3}$ is perpendicular to this plane. (b) Prove that the distance between two adjacent planes of the lattice is $d(hkl) = \frac{2\pi}{|\vec{G}|}$. (c) Show that for a simple cubic lattice that $d^2 = a^2/(h^2 + k^2 + l^2)$.

B. Problem 2.2

The primitive translation vectors of the hexagonal space lattice may be taken a:

$$\vec{a_1} = a\frac{\sqrt{3}}{2} * \hat{x} + \frac{a}{2}\hat{y}, \;\; \vec{a_2} = -a\frac{\sqrt{3}}{2}\hat{x} + \frac{a}{2}\hat{y}, \;\; \hat{a_3} = c\hat{z}. \tag{1}$$

-Show that the volume of this primitive cell is $\frac{\sqrt{3}}{2}a^2c$.
-Show that the primitive translation vectors of the reciprocal lattice is:

$$\vec{b_1} = \frac{2\pi}{\sqrt{3}a} * \hat{x} + \frac{2\pi}{a}\hat{y}, \;\; \vec{b_2} = -\frac{2\pi}{\sqrt{3}a}\hat{x} + \frac{2\pi}{a}\hat{y}, \;\; \hat{b_3} = \frac{2\pi}{c}\hat{z}. \tag{2}$$

Such that the lattice is its own reciprocal, but with a rotation of the axes.
-Describe and scetch the first Brilloin zone (reciprocal Wigner-Seitz) of the hexagonal space lattice.

C. Problem 2.3

Show that the maxima of diffraction signals obtained from a crystal correspond to reciprocal lattice points and map hkl-families of planes in the real space. (Tip - develop Laue conditions and use the Ewald construction).

D. Problem 2.4

Structure factor of diamond: If the cell of the diamond lattice is taken as the conventional cube (see chap1 Kittel), then the basis consist of eight atoms.
-Find the structure factor S in this basis.
-Find the zeros of S and show that the allowed reflections of the diamond structure satisfy $v_1 + v_2 + v_3 = 4n$, where the indices are even and n is any integer, or else all indices are odd (Fig. 18 Kittel).