I. PROBLEM SESSION 9

A. Problem 9.1

-Recall the nearly free electron model of electrons in a metal, what is the main assumptions behind the model compared to the free fermi gas model.
-In the nearly free electron model, what is the origin of the energy gap?
-How does the magnitude of the gap depend on the amplitude of the periodic potential?
-Recall the definition and general form of the Bloch functions. How do they depend on coordinates?

B. Problem 9.2

Square lattice, free electron energies
a) Show for a simple square lattice (two dimensions) that the kinetic energy of a free electron at the a corner of the first Brillouin zone is higher than that of an electron at the midpoint of a side face by a factor of 2.
b) What is the corresponding factor for a simple cubic lattice (three deimensions).
c) What implications might the result of b) have on the conductivity of divalent metals?

C. Problem 9.3

Free electron energies in the reduzed zone scheme:
Consider the free electron energy bands of an fcc crystal lattice in the approximation of an empty lattice, but in the reduced zone scheme (in which all $\vec{k}$'s are transformed to lie in the first brillouin zone). Plot roughly in the $[1,1,1]$ direction the energies of all bands up to six times the lowest band energy at the zone boundary at $\vec{k} = (2\pi/a)(1/2, 1/2, 1/2)$. Let this be the unit of energy. This problem shows why band edges need not necessarily be at the zone center. Several of the degeneracies (band crossings) will be removed when the crystal potential is taken into account.

D. Problem 9.4

(optional) Show that the chemical potential of a Fermi gas in two dimensions is given by

$$\mu(T) = k_B T log[e^{\frac{\pi n \hbar^2}{m k_B T}} - 1], \tag{1}$$

for n electrons per unit area. Note: the density of orbitals of a free electron gas in two dimensions is independent of energy: $D(\epsilon) = \frac{m}{\pi \hbar^2}$.