FYS3410 - Vår 2011 (Kondenserte fasers fysikk)

http://www.uio.no/studier/emner/matnat/fys/FYS3410/index-eng.xml

Based on Introduction to Solid State Physics by Kittel

Course content

- Periodic structures, understanding of diffraction experiment and reciprocal lattice
- Imperfections in crystals: diffusion, point defects, dislocations
- Crystal vibrations: phonon heat capacity and thermal conductivity
- Free electron Fermi gas: density of states, Fermi level, and electrical conductivity
- Electrons in periodic potential: energy bands theory classification of metals, semiconductors and insulators
- Semiconductors: band gap, effective masses, charge carrier distributions, doping, pn-junctions
- Metals: Fermi surfaces, temperature dependence of electrical conductivity

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FYS3410 lecture schedule and exams: Spring 2011

W/19/1/2011:	Introduction and motivation. Periodicity and lattices	1h
M/24/1/2011:	Index system for crystal planes. Crystal structures	2h
W/26/1/2011:	Reciprocal space, Laue condition and Ewald construction	1h
M/31/1/2011:	Brillouin Zones. Interpretation of a diffraction experiment	2h
W/02/2/2011:	Crystal binding, elastic strain and waves	1h
M/07/2/2011:	Elastic waves in cubic crystals; defects in crystals	2h
W/09/2/2011:	Defects in crystals; case study – vacancies; diffusion	2h
M/14/2/2011:	Crystal vibrations and phonons	2h
W/16/2/2011:	Lattice heat capacity: Dulong-Petit and Einstein models	2h
M/21/2/2011:	Phonon density of states (DOS) and Debye model	2h
W/23/2/2011:	General result for DOS; role of anharmonic interactions	2h
M/28/2/2011:	Thermal conductivity and repetition of crystal vibrations	2h
W/02/3/2011:	no lectures	
M/07/3/2011:	no lectures	
W/09/3/2011:	no lectures	
M/14/3/2011:	Free electron Fermi gas in 1D and 3D – ground state	2h
W/17/3/2011:	Density of states, effect of temperature – FD distribution	1h
M/21/3/2011:	Heat capacity of FEFG	2h
W/23/3/2011:	Repetition	2h
M/28/3/2011:	Mid-term exam	

M/04/4/2011:	**Electrical and thermal conductivity in metals**	**2h**
W/06/4/2011:	**Bragg reflection of electron waves at the boundary of BZ**	**2h**
M/11/4/2011:	**Energy bands, Kronig - Penny model**	**2h**
W/13/4/2011:	**Empty lattice approximation; number of orbitals in a band**	**2h**
M/18/4/2011:	**Repetition of electrons in periodic potential**	**2h**
Påsk		
M/02/5/2011:	**no lectures**	
W/04/5/2011:	**no lectures**	
M/09/5/2010:	**Semiconductors, effective mass method, intrinsic carriers**	**2h**
W/11/4/2010:	**Impurity states in semiconductors and carrier statistics**	**2h**
M/16/5/2010:	**p-n junctions, Schottky contacts and heterojunctions**	**2h**
W/18/5/2010:	**Metals and Fermi surfaces**	**2h**
M/23/5/2010:	**Repetition**	**2h**
26-27/5/2010:	**Final Exam (Jonatan Slotte, Aalto University, Helsinki)**	

Lecture 16: Effect of temperature; Fermi-Dirac distribution

Fermi-Dirac distribution

- Describes the probability that an orbit at energy E will be occupied in an ideal electron gas under thermal equilibrium
- μ is chemical potential, f($\varepsilon = \mu$)=0.5; at 0K, $\varepsilon_F = \mu$

$$f(\epsilon) = \frac{1}{\exp[(\epsilon - \mu)/k_B T] + 1} .$$

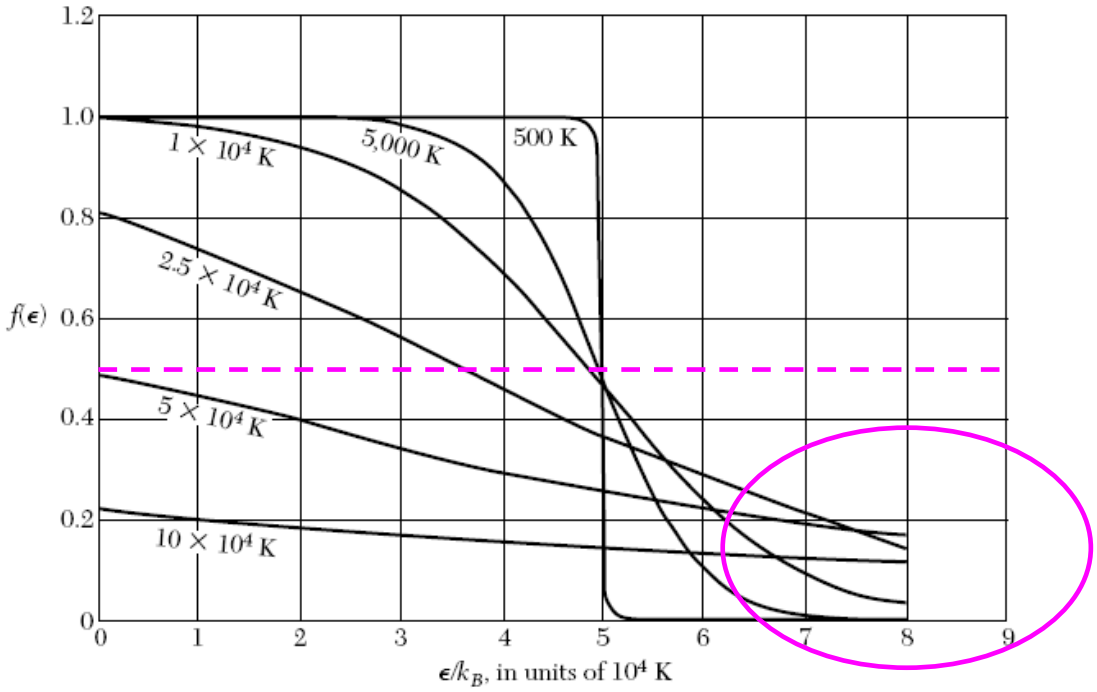


Figure 3 Fermi-Dirac distribution function (5) at the various labelled temperatures, for $T_F \equiv \epsilon_F/k_B = 50{,}000$ K. The results apply to a gas in three dimensions. The total number of particles is constant, independent of temperature. The chemical potential μ at each temperature may be read off the graph as the energy at which $f = 0.5$.

In comparison,

$$f_{Bose-Einstein}(E) = \frac{1}{\exp(\frac{\hbar\omega}{k_B T}) - 1}$$

High energy tail – approximation

$$E - E_F > 3k_B T$$

➔ **Boltzmann-Maxwell distribution**

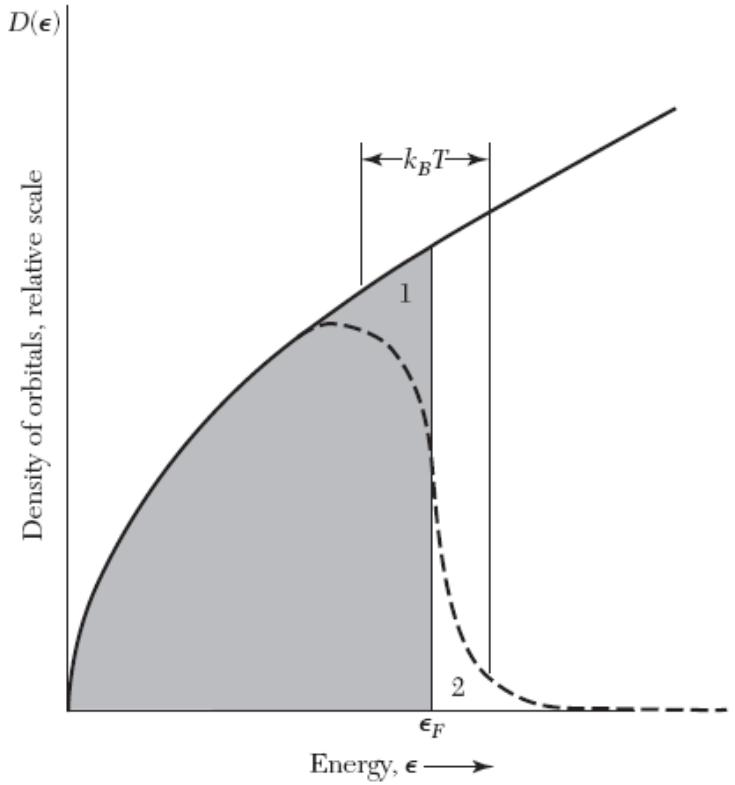
$$f_{Boltzmann-Maxwell}(E) = \exp(-\frac{E - E_F}{k_B T})$$

DOS example: free electrons (parabolic dispersion)

$$3D-DOS(cm^{-3}eV^{-1})$$

$$\frac{d^3k}{(2\pi)^3}=\frac{4\pi k^2dk}{(2\pi)^3}=\frac{4\pi k(E)^2}{(2\pi)^3}\frac{dk}{dE}dE=g_{3D}(E)dE \Rightarrow g_{3D}(E)=2\cdot\frac{4\pi\left(\frac{2mE}{\hbar^2}\right)}{(2\pi)^3}\cdot\sqrt{\frac{2m}{\hbar^2}}\frac{1}{2\sqrt{E}}=\frac{1}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}E^{1/2}$$

- **# of states between E and E+dE**



$$\epsilon_F=\frac{\hbar^2}{2m}\left(\frac{3\pi^2N}{V}\right)^{2/3}.$$

$$N=\frac{V}{3\pi^2}\left(\frac{2m\epsilon}{\hbar^2}\right)^{3/2}$$

Note here: $N=N(\varepsilon)$
D(E) has a unit of ev^{-1}.

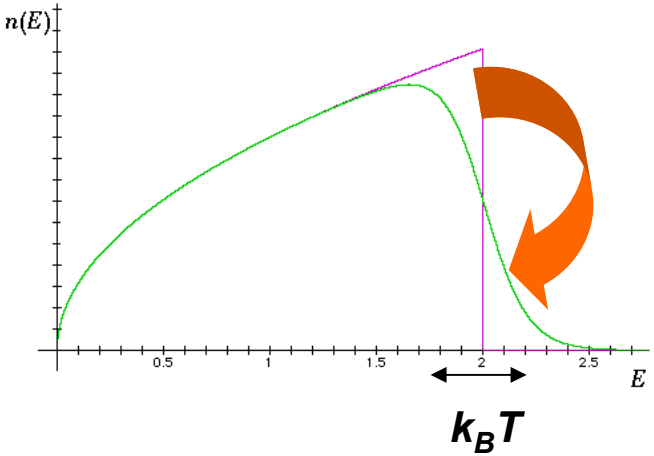
$$D(\epsilon)\equiv\frac{dN}{d\epsilon}=\frac{V}{2\pi^2}\cdot\left(\frac{2m}{\hbar^2}\right)^{3/2}\cdot\epsilon^{1/2}.$$

Figure 5 Density of single-particle states as a function of energy, for a free electron gas in three dimensions. The dashed curve represents the density $f(\epsilon, T)D(\epsilon)$ of filled orbitals at a finite temperature, but such that k_BT is small in comparison with ϵ_F. The shaded area represents the filled orbitals at absolute zero. The average energy is increased when the temperature is increased from 0 to T, for electrons are thermally excited from region 1 to region 2.

Heat capacity of a cold FEFG

One of the greatest successes of the free electron model and FD statistics is the explanation of the ***T*** dependence of the heat capacity of a metal.

$$C_V \equiv \frac{dQ(T)}{dT}\Big|_{V=const} = \frac{dE_t(T)}{dT}$$



To calculate the heat capacity, we need to know how the internal energy of the Fermi gas, $E_t(T)$, depends on temperature. By heating a Fermi gas, we populate some states above the Fermi energy E_F and deplete some states below E_F. This modification is significant within a narrow energy range ~ k_BT around E_F (we assume that the system is cold - strong degeneracy).

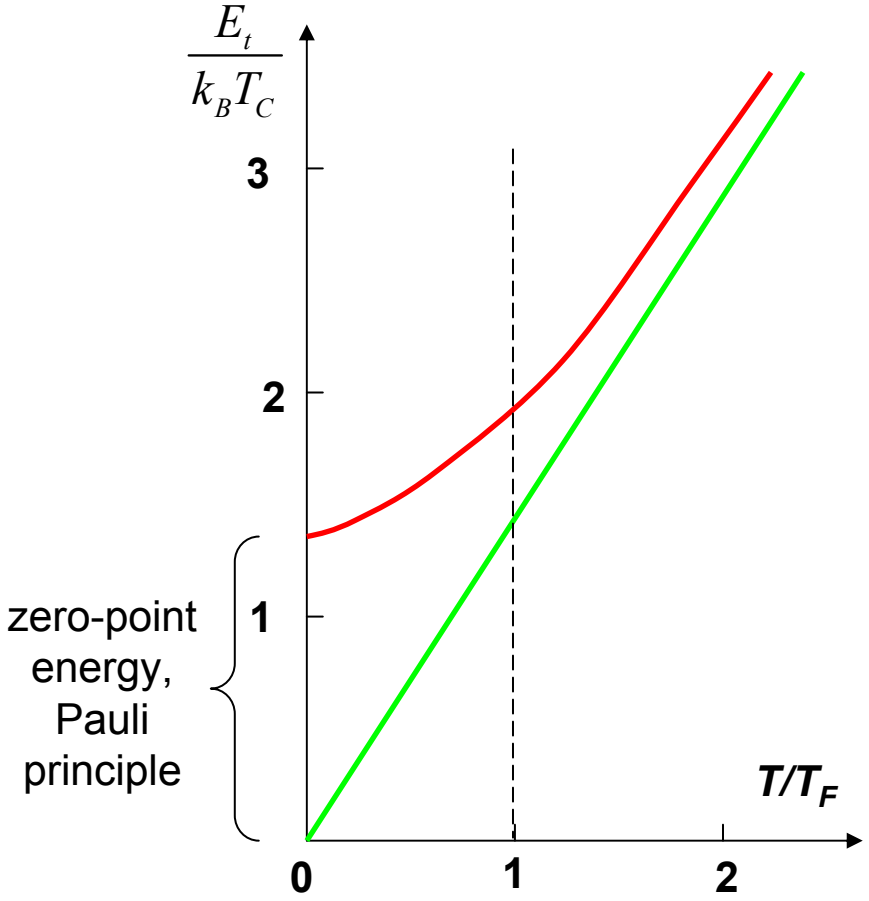
The fraction of electrons that we "transfer" to higher energies ~ k_BT/E_F, the energy increase for these electrons ~ k_BT. Thus, the increase of the internal energy with temperature is proportional to $n\times(k_BT/E_F)\times(k_BT) \sim n\,(k_BT)^2/E_F$.

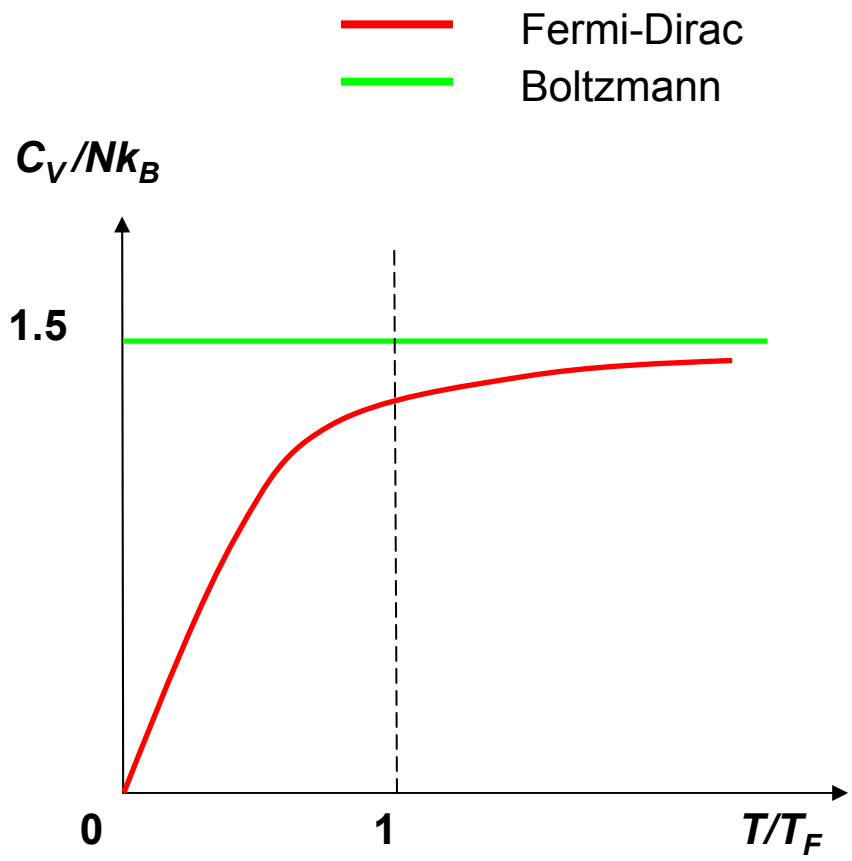
$$C_V = \frac{dE_t(T)}{dT} \propto N\frac{k_B^2 T}{E_F}$$

$$C_e = \frac{\pi^2}{2} n k_B \frac{k_B T}{E_F} \quad \longleftrightarrow \text{compare} \quad C_V = \frac{3}{2} n k_B \quad \text{for an ideal gas}$$

The Fermi gas heat capacity is much smaller (by $k_BT/E_F<<1$) than that of a classical ideal gas with the same energy and pressure. The small heat capacity is a direct consequence of the Pauli principle: most of the electrons cannot change their energy, only a small fraction k_BT/E_F of the electrons are excited out of the ground state.

Comparison between Boltzmann and FD Distributions





$$T_F \approx T_C = \frac{h^2}{2\pi m k_B}\left(\frac{n}{2.6}\right)^{2/3}$$