<table>
<tr><th>$n\ (= i)$</th><th>$p_n =$</th><th>Comments</th></tr>
<tr><td>1</td><td>$N$</td><td>All $N$ places are available</td></tr>
<tr><td>2</td><td>$\dfrac{N \cdot (N - 1)}{2}$</td><td>$N$ places for the first, only $N - 1$ places for the second vacancy. Exchanging both vacancies does not change the situation - we have to divide by 2</td></tr>
<tr><td>3</td><td>$\dfrac{N \cdot (N - 1) \cdot (N - 2)}{2 \cdot 3}$</td><td>Exchanging vacancies does not change the microstate, we have to divide by the number of all possible exchanges $= 6 = 2 \cdot 3$.</td></tr>
<tr><td colspan="3">Make sure you understand the exchange argument: Here is the detailed reasoning:<br>For vacancy No. 1 on place 1 , you have two possibilities: No. 2 on place 2, No. 3 on place 3 or No 2 on place 3 and No. 3 on place 2.<br>You can do the same thing for No. 2 on place 2 (exchange No. 1 and No. 3) and for No. 3 on place 3., so you have 2 options 3 times = 6 indistinguishable arrangements.</td></tr>
<tr><td>...</td><td>...</td><td>and so on</td></tr>
<tr><td>$n$</td><td>$\dfrac{N \cdot (N - 1) \cdot (N - 2) \cdot .. \cdot (N - (n + 1))}{2 \cdot 3 \cdot .... \cdot n}$</td><td>The obvious law for $n$ vacancies.<br><br>$\{1 \cdot 2 \cdot 3 \cdot .... \cdot n\}$ of course is simply n!</td></tr>
<tr><td>$n$</td><td>$\dfrac{\{N \cdot (N - 1) \cdot (N - 2) \cdot .. \cdot (N - (n + 1))\} \cdot \{(N - n)!\}}{n! \cdot \{(N - n)!\}}$</td><td>Extend the fraction by $(N - n)!$</td></tr>
<tr><td>$n$</td><td>$\dfrac{N!}{n! \cdot (N - n)!}$</td><td>This is a standard expression in combinatorics and called the binomial coefficient.</td></tr>
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Here are a few hints and problems in dealing with faculties and approximations.

- Having $\boldsymbol{n} \ll \boldsymbol{N}$, **i.e.** $\boldsymbol{n/(N - n) \approx n/N = c_V}$ = concentration of vacancies does *not* allow us to approximate $\mathbf{d/d}\boldsymbol{n}\{(\boldsymbol{N} - \boldsymbol{n}) \cdot \ln(\boldsymbol{N} - \boldsymbol{n})\}$ by simply doing $\mathbf{d/d}\boldsymbol{n}\{\boldsymbol{N} \cdot \ln \boldsymbol{N}\} = \mathbf{0}$.
  - This is so because **d/d*n*** gives the *change* of $\boldsymbol{N - n}$ with $\boldsymbol{n}$ and that not only *might* be large even if $\boldsymbol{n} \ll \boldsymbol{N}$**,** but *will* be large because $\boldsymbol{N}$ is essentially constant and the only change comes from $\boldsymbol{n}$.
- The derivative of $\boldsymbol{u(x)} \cdot \mathbf{v}(\boldsymbol{x})$ is: $\mathbf{d/d}\boldsymbol{x}(\boldsymbol{u} \cdot \mathbf{v}) = \mathbf{d}\boldsymbol{u}/\mathbf{d}\boldsymbol{x} \cdot \mathbf{v}(\boldsymbol{x}) + \mathbf{dv/d}\boldsymbol{x} \cdot \boldsymbol{u}(\boldsymbol{x})$.
  - The derivative of $\ln \boldsymbol{x}$ is: $\mathbf{d/d}\boldsymbol{x}(\ln \boldsymbol{x}) = \mathbf{1}/\boldsymbol{x}$
- Easy mistake: Don't forget the *inner derivative*, it produces an important *minus* sign: