

I. PROBLEM SESSION 10

A. Problem 10.1

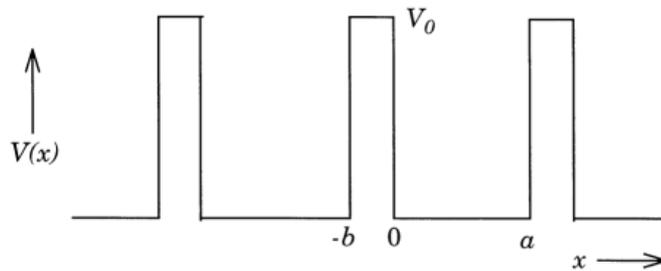
- Recall the nearly free electron model of electrons in a metal, what are the main assumptions behind the model compared to the free Fermi gas model.
- In the nearly free electron model, what is the origin of the energy gap?
- How does the magnitude of the gap depend on the amplitude of the periodic potential?
- Recall the definition and general form of the Bloch functions. How do they depend on coordinates?
- Describe the Kronig-Penney model. Write down the central equation for this model. How does it look in the different areas of the Brillouin zone (center compared edge)
- Attempt to explain the concept of crystal momentum.

B. Problem 10.2

Square lattice, free electron energies

- a) Show for a simple square lattice (two dimensions) that the kinetic energy of a free electron at the corner of the first Brillouin zone is higher than that of an electron at the midpoint of a side face by a factor of 2.
- b) What is the corresponding factor for a simple cubic lattice (three dimensions).
- c) What implications might the result of b) have on the conductivity of a metal.

C. Problem 10.3



Kronig-Penney model: The KP model is a strongly simplified one-dimensional quantum mechanical model of a crystal. A complex many body problem is reduced to that of a single electron in a periodic potential consisting of an infinite periodic array of rectangular potential barriers. Despite the simplifications, the electronic band structure obtained from this model shares many features with band structures that result from more sophisticated models.

- a) For the delta-function potential and with $P \ll 1$, find at $k = 0$ the energy of the lowest energy band. Here P is the parameter from Eq.21 in the book, $P = \frac{mU_0ba}{\hbar^2}$. b) Find the band gap for $k = \frac{\pi}{a}$.