I. PROBLEM SESSION 5

A. Problem 5.1

a)Write down the equation of motion for an one dimensional chain of atoms with mass M and spring constant C
b)Find its traveling wave solution.
c)Find the dispersion law of the wave and explain its meaning.
d)Recall the concept of the Brillouin zone (BZ) and sketch the dispersion law in the first Brillouin zone.
e)Explain the concepts of the phase and group velocity. What happens to the phase and group velocity at the edge of the BZ.
f)How many vibrational modes has an one dimensional two-atomic lattice. What happens in the three dimensional case.

B. Problem 5.2

Continuous wave equation: Show that for long wavelengths the equation of motion

$$M\frac{d^2u_s}{dt^2} = C(u_{s+1} + u_{s-1} - 2u_s) \tag{1}$$

reduces to the continuum elastic wave equation

$$\frac{\partial^2 u}{\partial t^2} = v^2 \frac{\partial^2 u}{\partial x^2} \tag{2}$$

where v is the velocity of sound.

C. Problem 5.3

A linear chain of diatomic molecules is characterized by two spring constants. The springs that connects atoms in the same molecule have spring constant C_1, while the springs that connects atoms in different molecules have spring constant C_2. We assume $C_1 > C_2$. The translation vector has length $a = a_1 + a_2$ where a_1 is the distance between two atoms in the same molecule, and a_2 is the distance between the two closest atoms in adjacent molecules.
a)Show that the dispersion relation for this model, $\omega(k)$, is given by

$$m\omega^2 = C_1 + C_2 \pm \sqrt{C_1^2 + C_2^2 + 2C_1C_2cos(ka)} \tag{3}$$

where ω is the angular frequency and k is the wavenumber.
b)Find ω when $ka << 1$ (calculate to first order in ka). Also calculate ω when $k = \pi/a$.
c)Compute the group velocity v_g for $k = 0$.
d)Sketch the dispersion relation graphically, and give a physical interpretation of the two dispersion branches.
e)What happens at $C_1 = C_2$?