I. PROBLEM SESSION 7

A. Problem 7.1

a) Attempt to explain what are the limits (consequences) of the harmonic approximation for vibrations (phonons) in a lattice.
b)In a perfect harmonic crystal the phonon states are stationary states. What does this imply about the termal conductivity of such a crystal?
c)Explain the concept of Umklapp processes, doe the rate of Umklapp processes depend on temperature, what happens at low temperatures?

B. Problem 7.2

Thermal expansion: Consider an infinite one-dimensional chain with nearest-neighbor interactions. Assume a potential $U(x) = cx^2 - gx^3 - x^4$, where $x = r - a$ is the displacement from the equilibrium spacing a. Calculate the average displacement $< x >$ using the Boltzmann distribution function $e^{-U(x)/k_BT}$. Show that $< x >$ is proportional to T and g when the anharmonic terms are small compared to k_BT.

C. Problem 7.3

Thermal conductivity: Derive the relation for the thermal current $j = -K\nabla T$, assuming a one dimensional small temperature gradient (in the x-direction). Assume that phonons emerging from collisions at a point x contribute to the energy density depending on local temperature $u = u(T(x))$. Each phonon will contribute to the current density, an amount equal to the product of its velocity component in the x-direction times its contribution to the energy density. However the average contribution of a phonon to the energy density depend on the local density of its last collision. Use that the distance between collisions on average is $l = c\tau$ where c comes from the dispersion relation and τ is the mean time between collisions.