I. PROBLEM SESSION 8

A. Problem 8.1

a)Write down the expression for the electron energy levels in one dimension.
b)What is the meaning of the Fermi energy, as a explicit model use the one dimensional electron gas.
c)What is the relationship between the chemical potential and the Fermi energy.
d)Recall the expression for the electron density of states in the 3d electron gas. How does the Fermi energy depend on the electron concentration in this case.
e)How does the heat capacity of a metal depend on temperature.

B. Problem 8.2

Kinetic energy of the electron gas: Show that the kinetic energy of a three-dimensional electron gas of N free electrons at zero temperature is

$$U_0 = \frac{3}{5} N \epsilon_F. \tag{1}$$

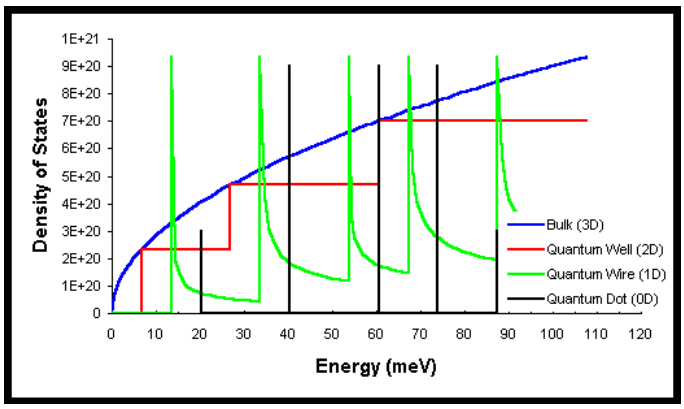
C. Problem 8.3

Pressure and bulk modulus of an electron gas: a) Derive a relation connecting the pressure and the volume of an electron gas at 0K. Hint: Use the result of the previous problem and the relation between the Fermi energy and the electron density. The result may be written as $pV = \frac{2}{3}U_0$
b) Show that the bulk modulus $B = -V\frac{\delta p}{\delta V}$ of an electron gas at $0K$ is

$$B = \frac{5p}{3} = \frac{10U_0}{9V}. \tag{2}$$

c) Estimate for potassium the value of the electron gas contribution to B. Use the table on page 139 in Kittel.

D. Problem 8.4



The figure abowe shows typical density of states for free electrons confined to 3, 2, 1 and 0 dimensions. Derive the energy dependence of the density of states for all four cases(you do not need to worry about constant factors) and compare with the figure. (Tip: in zero dimensions (often called quantum dots) we have localized states, as in the case of an atom) What is the underlying physical reason that the density of states is qualitatively different in the four cases?

Optional: In the figure there is multiple peaks, can you explain this? Can you find a criteria for when a material can be teoretically treated as an object of lower dimension (use your knowledge from quantum mechanics).