FYS3410 - Vår 2016 (Kondenserte fasers fysikk)

http://www.uio.no/studier/emner/matnat/fys/FYS3410/v16/index.html

Pensum: Introduction to Solid State Physics
by Charles Kittel (Chapters 1-9 and 17, 18, 20)

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2016 FYS3410 Lectures (based on C.Kittel's Introduction to SSP, Chapters 1-9, 17,18,20)

Module I – Periodic Structures and Defects (Chapters 1-3, 20)

M18/1: 9-12 am	Introduction. Crystal bonding. Periodicity and lattices, Brag diffraction and Laue condition, reciprocal space	3h
W20/1 cancelled		
M25/1: 9-12 am	Ewald construction, interpretation of a diffraction experiment , Brag planes, and Brillouin zones	3h
W27/1 cancelled		
M01/2: 10-12 am	Elastic strain and structural defects in crystals	2h
W03/2: 9-10 am	Atomic diffusion in solids	1h
M08/2: 10-12 am	Summary of Module I	2h

Module II – Phonons (Chapters 4 and 5)

W10/2: 9-10 am	Vibrations in monoatomic and diatomic chains of atoms	1h
M15/2: 10-12am	Periodic boundary conditions, phonons and density of states (DOS)	2h
W17/2: 9-10 am	Planck distribution	1h
M22/2 : 10-12am	Lattice heat capacity: Dulong-Petit, Einstein, and Debye models	2h
W24/2 cancelled		
M29/2: 9-12am	Comparison of different models for lattice heat capacity, thermal conductivity with phonons	3h
W02/3: 9-10 am	Thermal expansion	1h
M07/3: 10-12am	Summary of Module II.	2h

Module III – Electrons (Chapters 6, 7, 18 - pp.528-530, and Appendix D)

W09/3: 9-10 am	Free electron gas (FEG) versus free electron Fermi gas (FEFG)	1h
M14/3: 10-12am	DOS of FEFG in 3D. Effect of temperature – Fermi-Dirac distribution	2h
W16/3: 9-10 am	Heat capacity of FEFG in 3D	1h
W30/3: 9-10 am	DOS in 2D - quantum wells	1h
M04/4: 10-12am	DOS in 1D and 0D, i.e. quantum wires and quantum dots; transport properties of electrons	2h
W06/4: 9-10 am	Origin of the energy band gap	
M11/4: 10-12am	Nearly free electron model. Kronig-Penney model. Empty lattice approximation.	2h
W13/4: 9-10 am	Number of orbitals in a band	1h
M18/4: 10-12am	Summary of Module III.	2h

Module IV – Semiconductors and interfaces (Chapters 8, 9-pp 223-231, 17)

W20/4: 9-10 am	Metals versus semiconductors. Surfaces and interfaces.	1h
M25/4: 9-12 am	Effective mass method.	3h
W27/4: 9-10 am	Intrinsic carrier generation – elctrons and holes.	1h
M02/5: 9-12 am	Localized levels for hydrogen-like impurities – donors and acceptors. Doping.	3h
W04/5: 9-10 am	Carrier statistics in semiconductors	1h
M09/5: 9-12 am	p-n junctions	3h
W11/5: 9-10 am	Optoelectronic semiconductor properties and devices	1h
M18/5: 9-12 am	Device demonstrations. Summary of Module IV	3h

Repetition

M23/5 9-12 am	The course in a nutshell	2h
W25/5, M30/5 and W1/6 cancelled		

Exam during week 22 (tentatively 30-31/5)

Condensed Matter Physics

Condensed Matter Physics

↓

Solid State Physics of Crystals

Condensed Matter Physics

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Solid State Physics of Crystals

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Properties of Waves in Periodic Lattices

Condensed Matter Physics

Solid State Physics of Crystals

Properties of Waves in Periodic Lattices

Elastic waves	Electron waves
Vibrations	Free electrons
Phonon DOS	Electron DOS
Planck distribution	Fermi-Dirac distribution

Free electron gas versus free electron Fermi gas

- Free electron gas (FEG) - Drude model
    - FEG explaing Ohm’s law
    - FEG expaling Hall effect
    - FEG explaing Wiedemann-Franz law
    - FEG heat capacity

- Free electron Fermi gas (FEFG) – a gas of electrons subject to Pauli principle

- One electron system – wave functions – orbits; FEFG in 1D in ground state

- FEFG in 3D in ground state

- Fermi-Dirac distribution and electron occupancy at T>0

Free electron gas and free electron Fermi gas

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Free electron gas (FEG) - Drude model

There could be different opinions what particular discovery was the main breakthrough for the acceleration of the condensed matter physics but a very prominent kick-off was by the discovery of the electron by J.J. Thompson in 1897. Soon afterwards (1900) P. Drude used the new concept to postulate a theory of electrical conductivity. Drude was considering why resistivity in different materials ranges from 10^{-8} Ω·m (Ag) to 10^{20} Ω·m (polystyrene)?

Drude was working prior to the development of quantum mechanics, so he began with a classical model, specifically:

(i) positive ion cores within an electron gas that follows Maxwell-Boltzmann statistics;

(ii) following the kinetic theory of gases - the electrons ae in form of free electron gas (FEG), so that individual electrons move in straight lines and make collisions only with the ion cores; electron-electron interactions are neglected;

(iii) Electrons lose energy gained from the electric field in colisions and the mean free path was approximately the inter-atomic spacing.

Drude (or FEG) model successfully exlpained Ohm amd Wiedemann-Franz laws, but failed to explain, e.g., electron heat capacity and the magnetic susceptibility of conduction electrons.

Free electron gas (FEG) - Drude model

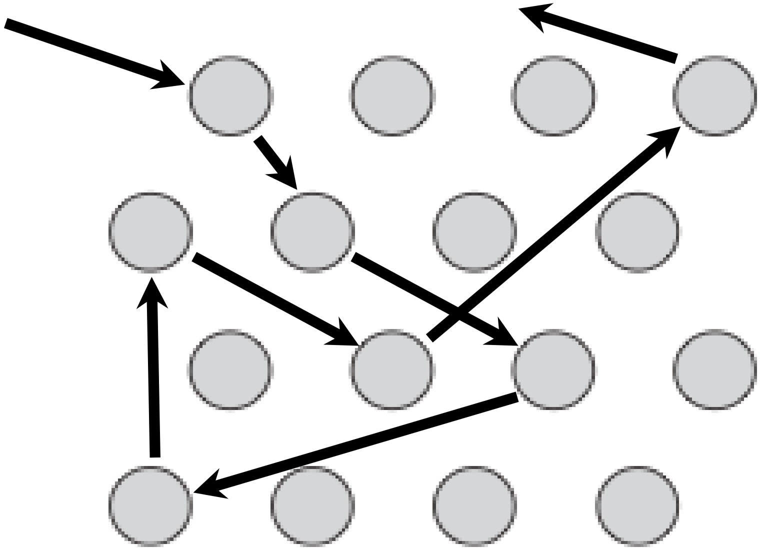
Velocity of electrons in FEG
e.g. at room temperature

$$\frac{1}{2}mv_t^2 = \frac{3}{2}k_BT$$

$$v_t = \sqrt{\frac{3k_BT}{m}}$$

$$v_t \approx 10^5\mathrm{ms}^{-1}$$

$$\lambda = \tau v_t$$

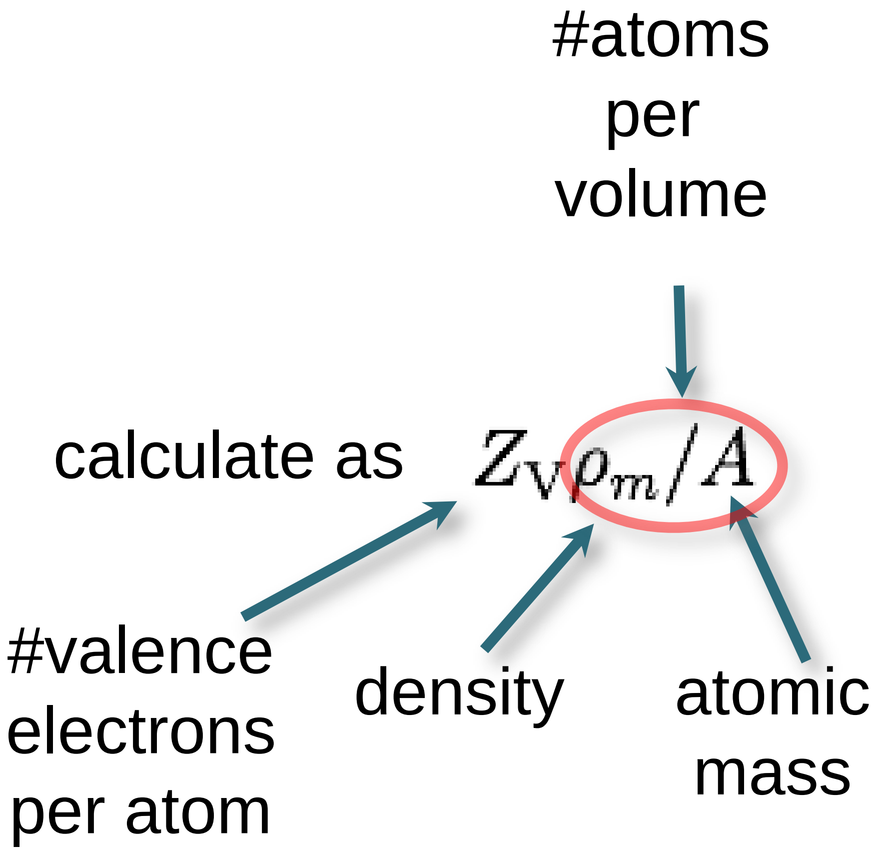


$$\lambda \approx 1\mathrm{nm}$$

$$v_t \approx 10^5\mathrm{ms}^{-1}$$

$$\tau \approx 1 \times 10^{-14}\ \mathrm{s}$$

Free electron gas (FEG) - Drude model



#atoms per volume

calculate as $Z_V\rho_m/A$

#valence electrons per atom

density

atomic mass

metal	Z_V	$n(10^{28}\ \mathrm{m}^{-3})$
Li	1	4.7
Na	1	2.65
K	1	1.4
Rb	1	1.15
Cs	1	0.91
Cu	1	8.47
Ag	1	5.86
Au	1	5.9
Be	2	24.7
Mg	2	8.61
Ba	2	3.15
Fe	2	17
Al	3	18.1
Pb	4	13.2
Sb	5	16.5
Bi	5	14.1

FEG explaining Ohm’s law

Apply an electric field. The equation of motion is

$$\frac{d\mathbf{v}}{dt}m_e = -e\mathbf{E}$$

integration gives

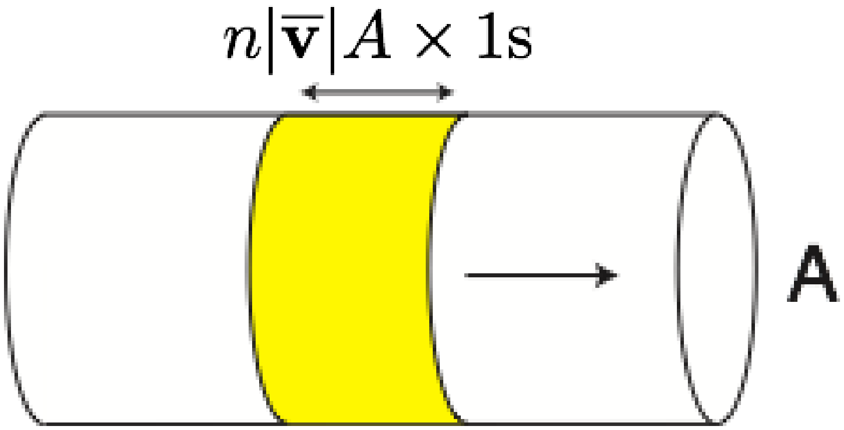
$$\mathbf{v}(t) = \frac{-e\mathbf{E}t}{m_e}$$

and if τ is the average time between collisions then the average drift speed is

$$\overline{\mathbf{v}} = \frac{-e\mathbf{E}\tau}{m_e}$$ for $E \approx 10\mathrm{Vm}^{-1}$ we get $\overline{v} = 10^{-2}\mathrm{ms}^{-1}$

remember: $v_t = 10^5\mathrm{ms}^{-1}$

FEG explaining Ohm’s law



number of electrons passing in unit time

$$n|\overline{\mathbf{v}}|A$$

current of negatively charged electrons

$$-en|\overline{\mathbf{v}}|A$$

current density

$$\mathbf{j} = n\overline{\mathbf{v}}(-e)$$

and with $\overline{\mathbf{v}} = \frac{-e\mathbf{E}\tau}{m_e}$ we get

Ohm’s law

$$\mathbf{j} = \frac{ne^2\tau}{m_e}\mathbf{E}$$

FEG explaining Ohm's law

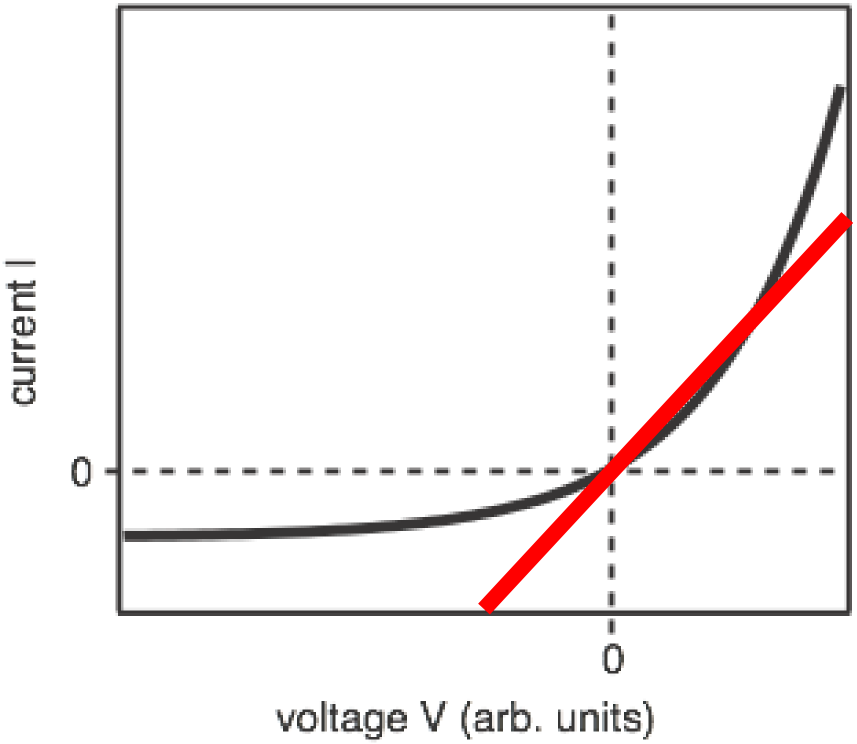
$$j = \sigma E = \frac{E}{\rho}$$

$$\mathbf{j} = \frac{ne^2\tau}{m_e}\mathbf{E}$$

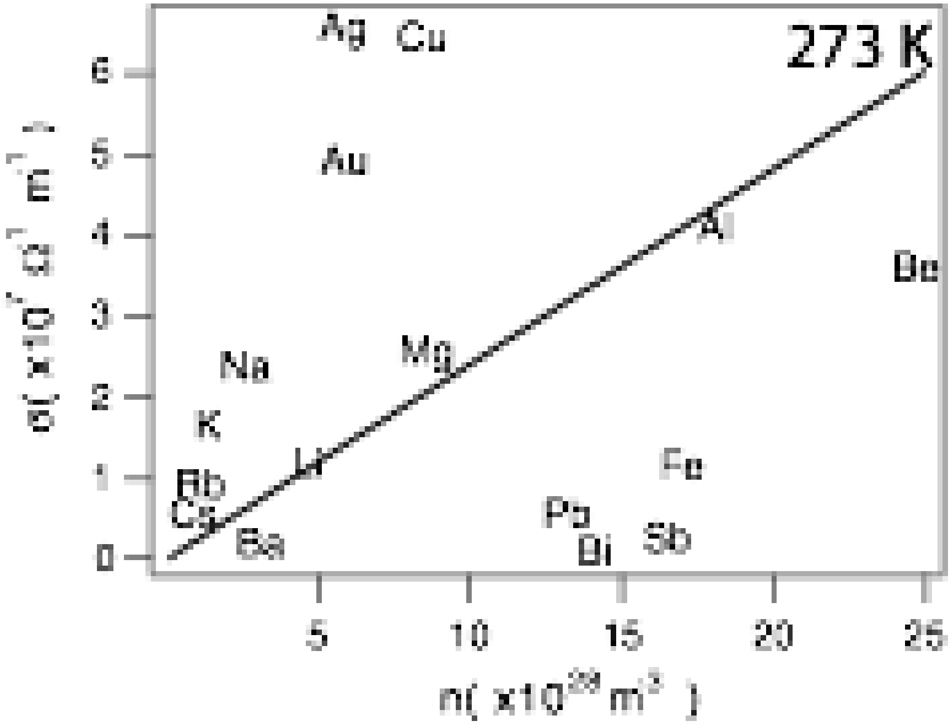
$$\sigma = \frac{ne^2\tau}{m_e} = n\mu e$$

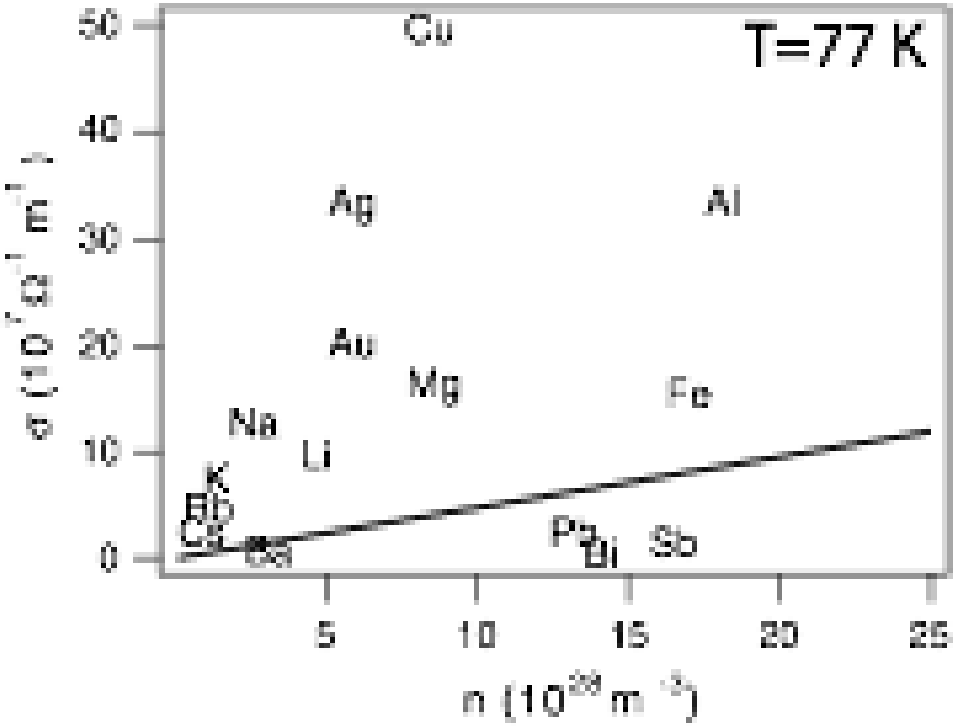
$$\rho = \frac{m_e}{ne^2\tau} = \frac{1}{n\mu e}$$

$$\mu = \frac{e\tau}{m_e}$$



FEG explaining Ohm’s law





$$\sigma = \frac{ne^2\tau}{m_e}$$

line

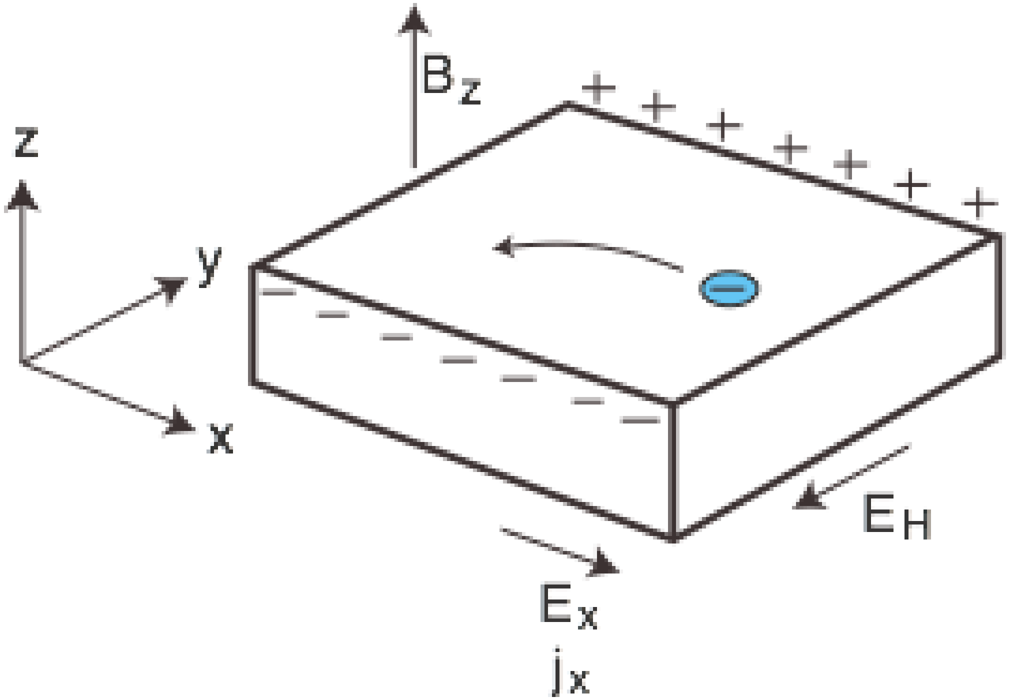
$$\lambda \approx 1\text{nm}$$

$$\tau = \lambda / v_t$$

$$\frac{1}{2}mv_t^2 = \frac{3}{2}k_BT$$

$$v_t = \sqrt{\frac{3k_BT}{m}}$$

FEG explaining Hall effect



- **Accumulation of charge leads to Hall field E_H.**
- **Hall field proportional to current density and B field**

$E_H = E_y = R_H j_x B_z$ R_H is called Hall coefficient

FEG explaining Hall effect

$$E_H = E_y = R_H j_x B_z$$

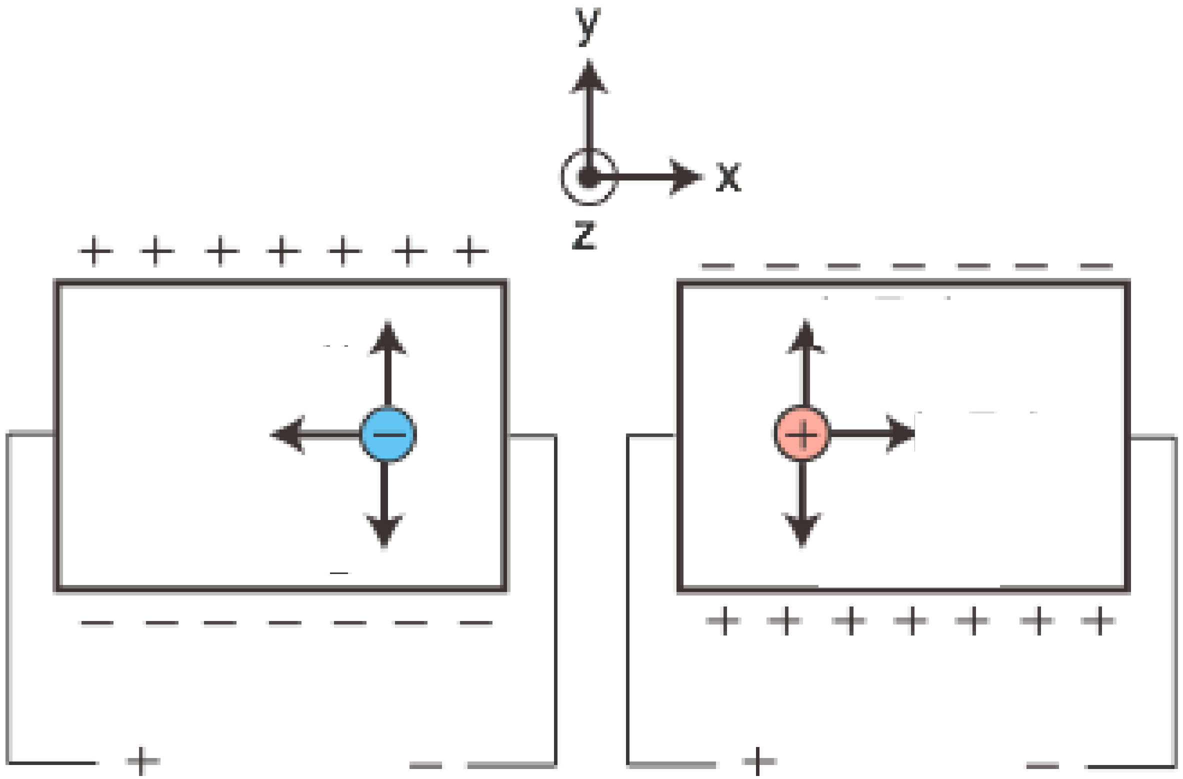
$$\mathbf{F} = -e\mathbf{v} \times \mathbf{B}$$
$$\mathbf{F} = -e\mathbf{E}_H$$

for the steady state

$$R_H = \frac{E_H}{j_x B_z} = \frac{E_H}{(-e) n v_x B_z}$$

$$R_H = \frac{v_x B_z}{-e n v_x B_z} = \frac{-1}{ne}$$

FEG explaining Hall effect



$$R_H = \frac{-1}{ne}$$

$$R_H = \frac{1}{pe}$$

FEG explaining Wiedemann-Franz law

Wiedemann and Franz found in 1853 that the ratio of thermal and electrical conductivity for ALL METALS is constant at a given temperature (for room temperature and above).

$$\frac{\kappa}{\sigma} = constant$$

Later it was found by L. Lorenz that this constant is proportional to temperature

$$\frac{\kappa}{\sigma} = LT$$

FEG explaining Wiedemann-Franz law

estimated thermal conductivity
(from a classical ideal gas)

$$\kappa = \frac{1}{3} v_t^2 \tau c_v$$

$$\sigma = \frac{ne^2\tau}{m_e}$$

$$\frac{\kappa}{\sigma} = \frac{3}{2}\frac{k_B^2}{e^2}T = LT.$$

the actual quantum mechanical result is

$$\frac{\kappa}{\sigma} = \frac{\pi^2}{3}\frac{k_B^2}{e^2}T = LT.$$

this is 3, more or less....

FEG explaining Wiedemann-Franz law

at 273 K

metal	10^{-8} Watt Ω K^{-2}
Ag	2.31
Au	2.35
Cd	2.42
Cu	2.23
Mo	2.61
Pb	2.47
Pt	2.51
Sn	2.52
W	3.04
Zn	2.31

$$\frac{\kappa}{\sigma} = \frac{\pi^2}{3}\frac{k_B^2}{e^2}T = LT$$

L = 2.45 10^{-8} Watt Ω K^{-2}

FEG heat capacity

- **an average thermal energy of an ideal gas particle, e.g. an electron from FEG, moving in 3D at some temperature *T* is:**

$$E = \tfrac{3}{2} k_B T$$

- **Then for a total nr of N electrons the total energy is:**

$$U = \tfrac{3}{2} N k_B T$$

- **And the electronic heat capacity would then be:**

$$C_{el} = \frac{dU}{dT} = \tfrac{3}{2} N k_B$$

- **If FEG approximation is correct this C_{el} should be added to phonon-related heat capacitance, however, if we go out and measure, we find the electronic contribution is only around one percent of this, specifically at high temperature is still Dulong-Petit value, $3N_\mathrm{A}k_\mathrm{B}$, that is valid.**

Free electron gas (FEG) - Drude model

- **Why does the Drude model work so relatively well when many of its assumptions seem so wrong?**
- **In particular, the electrons don't seem to be scattered by each other. Why?**
- **How do the electrons sneak by the atoms of the lattice?**
- **What are mysterious ”positive” charges realed by Hall effect measuremt in semiconductors?**
- **Why do the electrons not seem to contribute to the heat capacity?**

Lecture 14: Free electron gas and free electron Fermi gas

- Free electron gas (FEG) - Drude model
  - FEG explaing Ohm's law
  - FEG expaling Hall effect
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- **Free electron Fermi gas (FEFG) – a gas of electrons subject to Pauli principle**

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Free electron Fermi gas – a gas of electrons subject to Pauli principle

- At low temperature, free mean path of a conduction electron in metal can be as long as 1 cm! Why is it not affected by ion cores or other conduction electrons? (30 seconds discussions)
  - Motion of electrons in crystal (matter wave) is not affected by periodic structure such as ion cores.
  - Electron is scattered infrequently by other conduction electrons due to the Pauli exclusion principle

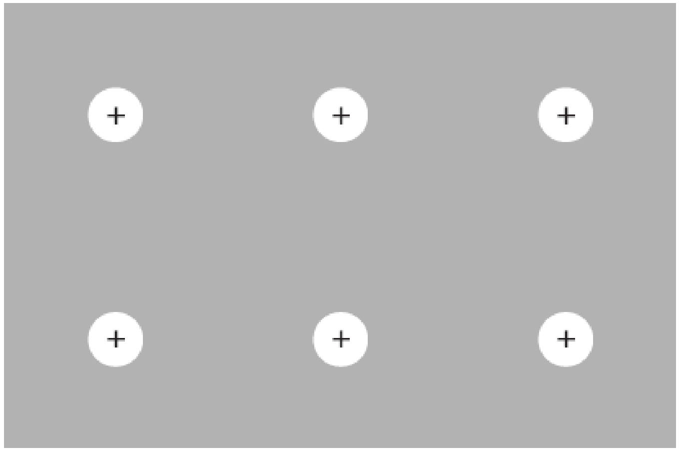

Figure 1 Schematic model of a crystal of sodium metal. The atomic cores are Na^+ ions: they are immersed in a sea of conduction electrons. The conduction electrons are derived from the 3*s* valence electrons of the free atoms. The atomic cores contain 10 electrons in the configuration $1s^22s^22p^6$. In an alkali metal the atomic cores occupy a relatively small part (~15 percent) of the total volume of the crystal, but in a noble metal (Cu, Ag, Au) the atomic cores are relatively larger and may be in contact with each other. The common crystal structure at room temperature is bcc for the alkali metals and fcc for the noble metals.

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- **One electron system – wave functions – orbits; FEFG in 1D in ground state**

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One electron system – wave functions - orbits

- Neglect electron-electron interaction, infinite potential well, simple QM solution

$$\psi_n = A\sin\left(\frac{2\pi}{\lambda_n}x\right) \; ; \qquad \tfrac{1}{2}n\lambda_n = L \; , \qquad \epsilon_n = \frac{\hbar^2}{2m}\left(\frac{n\pi}{L}\right)^2 \; .$$

Standing wave B. C.
n = 1, 2, …

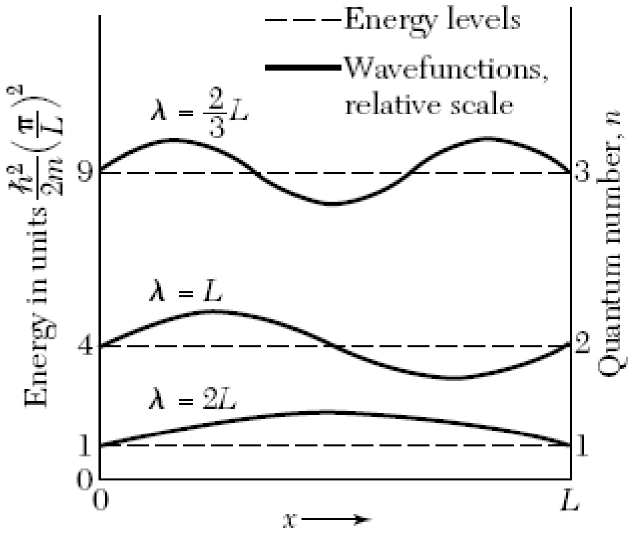

Figure 2 First three energy levels and wavefunctions of a free electron of mass m confined to a line of length L. The energy levels are labeled according to the quantum number n which gives the number of half-wavelengths in the wavefunction. The wavelengths are indicated on the wavefunctions. The energy ϵ_n of the level of quantum number n is equal to $(h^2/2m)(n/2L)^2$.

- The Pauli exclusion principle
– n: quantum number
– m(=1/2 and -1/2): magnetic quantum number
- degeneracy: # of orbitals with the same energy
- Fermi energy (E_F): energy of the topmost filled level in the ground state of the N electron system

In this simple system, every quantum state holds 2 electrons => n_F = N/2 ➔ Fermi energy:

$$\epsilon_F = \frac{\hbar^2}{2m}\left(\frac{n_F\pi}{L}\right)^2 = \frac{\hbar^2}{2m}\left(\frac{N\pi}{2L}\right)^2$$

Great, if we know the electron density, we know the Fermi energy!