FYS 3610

Exercise Week 36

Questions from the (midterm)exam:

Draw the motion of electrons and protons for

1) $B = B_o$ (uniform B), $F_\perp = 0$ (no external force)

2) $\nabla B \perp B_o$ (B gradient perpendicular to B), $F_\perp = 0$ (no external force)

3) $B = B_o$ (uniform B), $F_\perp \neq 0$ (external force)

4) $\nabla B \parallel B_o$ (B gradient along B), $F_\perp = 0$ (no external force)

Exercises

In a static magnetic field $\vec{B}_0$ the equation of motion of a single charged particle is given by:

$$m\frac{d\vec{v}}{dt} = q(\vec{v} \times \vec{B}_0).$$

The position vector $\vec{r}$ is then found from

$$\frac{d\vec{r}}{dt} = \vec{v}.$$

To study the motion of the particle in the plane perpendicular to the magnetic field, we will assume that is oriented along the z-axis, i.e., $\vec{B}_0 = (0,0,B_0)$. The equation of motion then simplifies to

$$\frac{dv_x}{dt} = \frac{q}{m} v_y B_0$$

$$\frac{dv_y}{dt} = -\frac{q}{m} v_x B_0$$

and

$$\frac{dr_x}{dt} = v_x$$

$$\frac{dr_y}{dt} = v_y.$$

To solve these equations numerically we use Euler's method to approximate the time derivative as the difference of the function value between time t and the next time step $t+h$, where h is the step size:

$$\frac{df(t)}{dt} \approx \frac{f(t+h) - f(t)}{h}.$$

Applying this to the equations of motion gives

$$\frac{v_x(t+h) - v_x(t)}{h} = \frac{q}{m} v_y(t) B_0$$

$$\frac{v_y(t+h) - v_y(t)}{h} = -\frac{q}{m} v_x(t) B_0$$

and

$$\frac{r_x(t+h) - r_x(t)}{h} = v_x$$

$$\frac{r_y(t+h) - r_y(t)}{h} = v_y.$$

We can rearrange these equations such that we find an expression for v_x, v_y, r_x, and r_y at the next time step, i.e., at $t+h$, depending on the function values from the previous time step t:

$$v_x(t+h) = v_x(t) + h\left(\frac{q}{m}v_y(t)B_0\right)$$

$$v_y(t+h) = v_y(t) - h\left(\frac{q}{m}v_x(t)B_0\right)$$

$$r_x(t+h) = r_x(t) + hv_x(t)$$

$$r_y(t+h) = r_y(t) + hv_y(t).$$

Exercise 1: Write a computer program in the language of your choice (for example Python) that numerically solves the equation of motion for an electron and an oxygen ion (O^+) in a static magnetic field of 50,000 nT. Let the initial position vector be $\vec{r} = (0,0,0)$ and the initial velocity vector $\vec{v} = (500,0,0)$ m/s, for both particles. Choose your time step h such that the gyro motion is properly resolved!

Exercise 2: Plot the particle's trajectories and check the theoretical predictions of the gyro radius and gyro frequency!

Exercice 3: Modify your computer program to include a gradient in the magnetic field such that $\nabla B \perp B$ and $B = B(y)$. You can for instance choose a step function

$$B(y) = 1\ B_0 \text{ for } y < r_e$$

$$B(y) = 2\ B_0 \quad \text{for } y \geq r_e$$

where r_e corresponds the gyro radius the electron.

Plot the particle's trajectories and discuss the results.