UNIVERSITY OF OSLO

Faculty of Mathematics and Natural Sciences

Exam in: **FYS3610**
Day of exam: **October 12th, 10:00 am**
Exam hours: **3 hours**
This examination paper consists of 2 pages.
Appendices: **1, on page 2**
Permitted materials: **Calculator**

Make sure that your copy of this examination paper is complete before answering.

PROBLEM 1 (14 points)
In magnetohydrodynamics, the equation of motion is given by

$$\rho\left(\frac{\partial\vec{v}}{\partial t}+(\vec{v}\cdot\nabla)\vec{v}\right)=-\nabla p+\vec{j}\times\vec{B}.$$

a) What physical parameters do the variables stand for?
b) Using the MHD form of Ampere's Law, show that $\vec{j}\times\vec{B}$ can be split into two terms. What is the physical meaning of those two terms?
c) How is the plasma beta defined? Why do we call plasmas with $\beta \ll 1$ cold and plasmas with $\beta \geq 1$ warm?
d) What are typical values for solar wind particle density, temperature, and magnetic field strength at 1 AU? Is the solar wind a warm or a cold plasma?
e) E. N. Parker, when looking for a solution for the solar wind, disregarded two terms of the equation of motion shown above and added another force. What equation of motion did he solve?

PROBLEM 2 (12 points)

a) Draw a sketch of the magnetosphere. Name different regions and boundaries both on the dayside and the nightside.
b) Indicate typical distances for the dayside boundaries as well as average sizes of the nightside regions.
c) Assume a positively charged particle that moves along the x direction with a constant velocity in a region of no magnetic field. Sketch the trajectory of that particle as it enters a region of constant magnetic field pointing in the z direction. Also indicate the trajectory of a negatively charged particle of the same mass having the same initial velocity along the x direction.
d) Where in the magnetosphere does a situation like that drawn in c) occur? What is the expected result?

PROBLEM 3 (10 points)

a) Sketch the ionospheric plasma density profile as a function of altitude both for the dayside and the nightside. Label you axes and indicate the altitude at which the plasma density peaks.

b) Explain what causes the differences between the two profiles.

c) At 300 km altitude the ions are embedded in a much larger population of neutral particles; the ratio of neutral to ion is about 10 000:1. Assume that initially both ions and neutrals are at rest; then, an external electric field is suddenly switched on and the ions are forced to move with a constant speed v_i through the neutrals. As time progresses the ions impart momentum on the neutrals through collisions, accelerating them. The equation of motion of the neutrals can then be written as

$$m_n n_n \frac{\partial u(t)}{\partial t} = -m_n n_n \gamma_{ni}(u(t) - v_i),$$

where m_m and n_n are the neutral mass and density, respectively; u is the neutral velocity, v_i is the (constant) ion velocity, and γ_{ni} is the neutral-ion collision frequency. At 300 km altitude γ_{ni} is typically 5×10^{-5} Hz. Solve the equation of motion for the neutrals (separation of variables!) and calculate the time it takes the neutrals to reach 90% of the ion velocity.

d) The solar wind drives the electric fields that cause the ion motion. Compare the time you found in c) with typical time scales of changes in the solar wind. Do you think that the neutrals ever move with the same velocity as the ions at 300 km altitude?

PROBLEM 4 (9 points)

a) In the equatorial plane the Earth's dipole field is given by $B(r) = \frac{\mu_0 M}{4\pi r^3}$, where M is the Earth's magnetic dipole moment (see appendix) and r is the distance from the center of the Earth. Calculate the gradient $\frac{\partial B}{\partial r}$.

b) For a perpendicular kinetic energy $E_\perp$ of 1 keV, what is the perpendicular speed $v_\perp$ of a proton? At these speeds, would you need to take into account relativistic effects?

c) The gradient drift velocity is given by

$$u_\nabla = \frac{1}{2} m v_\perp^2 \frac{B \frac{\partial B}{\partial r}}{q B^3}.$$

Calculate the drift velocity at r = 5 R$_e$ of that proton ($E_\perp$ = 1 keV).

d) How long does it take for that proton to drift once around the Earth? Is a 1 keV proton likely to complete a full drift around Earth? Motivate your answer. How about a 1 keV electron?

APPENDIX

$$2\vec{A} \times (\nabla \times \vec{A}) = \nabla(\vec{A} \cdot \vec{A}) - 2(\vec{A} \cdot \nabla)\vec{A}$$

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

$$1 \text{ R}_e = 6372 \text{ km}$$

$$\text{Proton: } q = 1.6 \times 10^{-19} \text{ C}, m = 1.6 \times 10^{-27} \text{ kg}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ V s A}^{-1} \text{ m}^{-1}$$

$$M = 7.9 \times 10^{22} \text{ Am}^{-2}$$