UiO : Department of Physics
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FYS 3610 Exam 2016 Solution

PROBLEM 1 (14 points) In Magnetohydrodynamics, the equation of motion is given by $$\rho\left(\frac{\partial\vec{v}}{\partial t} + (\vec{v}\cdot\nabla)\vec{v}\right) = -\nabla p + \vec{j}\times\vec{B}$$ a) Name all variables and give their units. b) Using the MHD form of Ampere's Law, show that $\vec{j}\times\vec{B}$ can be split into two terms. What is the physical meaning of these terms? c) How is the plasma beta defined? Why do we call a plasma with $\beta \ll 1$ cold and a plasma with $\beta \geq 1$ warm? d) What are typical values for solar wind particle density, temperature, and magnetic field strength at 1 AU? Is the solar wind a warm or a cold plasma? e) E. N. Parker, when looking for a static, gas dynamic solution to the equation of motion for the solar wind, disregarded two terms and added another force. What equation of motion did he solve?	a) Density, velocity, pressure, current density, magnetic field (2 points) b) $\mu_0\vec{j} = \nabla\times\vec{B}$, $\left(\frac{1}{\mu_0}\nabla\times\vec{B}\right)\times\vec{B} = -\frac{1}{\mu_0}\left(\frac{\nabla B^2}{2} - (\vec{B}\cdot\nabla)\vec{B}\right) = -\nabla\left(\frac{\nabla B^2}{2\mu_0}\right) + (\vec{B}\cdot\nabla)\vec{B}$, magnetic pressure, magnetic tension force (3 points) c) $\beta = p_t/p_B$, $p_t = nk_BT$ so hot (high temperature) plasmas have large p_t and hence large betas (2 points) d) 1 < n < 20 cm^-3, 10^5 < T < 10^6 K, 1 < B < 20 nT, for n=5cm^-5, T=10^5K, B=5nT, beta=5x10^6*1.38x10^-23*10^5*2*4x10^-7*pi/(5x10^-9)^2=0.7, i.e., the solar wind is a warm plasma (4 points) e) Static -> $\partial/\partial t = 0$, gas-dynamic -> $\vec{j}\times\vec{B} = 0$, adding gravity $\rho\vec{g}$ (3 points)
a) Draw a sketch of the magnetosphere. Name different regions and boundaries both on the dayside and the nightside. b) Indicate typical distances for the dayside boundaries as well as average sizes of the nightside regions. c) Assume a positively charged particle that moves along the x direction with a constant velocity in a region of no magnetic field. Sketch the trajectory of that particle as it enters a region of positive magnetic field in the z direction. Also indicate the trajectory of a	a) Bow shock, magnetopause magnetosheath, lobes, plasma sheet (4 points) b) BS: 15 Re, MP: 10 Re, tail: 30x110 Re (3 points) c) Half circle, right turning direction, same radius (3 points) d) Magnetopause, current (2 points)

negatively charged particle of the same mass having the same initial velocity along the x direction. d) Where in the magnetosphere does a situation like that drawn in c) occur? What is the expected result?	
a) Sketch the plasma density profile as a function of altitude both for the dayside and the nightside. Label you axes and indicate the altitude at which the plasma density peaks. b) Explain why the two profiles are different. c) At 300 km altitude the ions are embedded in a much larger population of neutral particles; the ratio of neutral to ion is about 10 000:1. Assume that initially both ions and neutrals are at rest; then, an external electric field is suddenly switched on and the ions are forced to move with a constant speed v_i through the neutrals. As time progresses the ions impart momentum on the neutral through collisions, until both ions and neutral move side by side at the same speed. The equation of motion of the neutrals can then be written as $m_n n_n \frac{\partial u}{\partial t} = -m_n n_n \gamma_{ni}(u - v_i)$, where m_m and n_n are the neutral mass and density, respectively; u is the neutral velocity, v_i is the (constant) ion velocity, and γ_ni is the neutral-ion collision frequency. At 300 km altitude γ_ni is typically 5×10^(-5) Hz. Solve the equation of motion for the neutrals (separation of variables!) and calculate the time it takes the neutrals to reach 90% of the velocity of the ions.	a) Dayside: production greater than on nightside, hence higher densities; E-region present; 300 km, 10^11 m^-3 particles (3 points) b) in E-region there is dissociative recombination is dominant, which is fast -> no E region at night. In F region, radiative recombination dominates, which is slow -> F region is present on nightside (2 points) c) $\frac{\partial u}{\partial t} = -\gamma_{ni}(u - v_i)$ $\int_0^{u(t)} \frac{du}{(u - v_i)} = -\int_0^t \gamma_{ni} dt$ $[\ln u - v_i]_0^{u(t)} = [-\gamma_{ni} t']_0^t$ $\ln u(t) - v_i - \ln -v_i = -\gamma_{ni} t$ $\frac{u(t) - v_i}{-v_i} = e^{-\gamma_{ni} t}$ $u(t) = v_i(1 - e^{-\gamma_{ni} t})$ For $u(t) = 0.9 v_i$, t is about 46 000 s, 12 hours. (4 points) d) Solar wind is dynamic on time scales of a few minutes -> No (1 point)
a) In the equatorial plane the Earth's	a) Easy (1 point)

dipole field is given by B(r)=(μ_o m)/(4πr^3), where m is the Earth's magnetic dipole moment (see appendix) and r is the distance from the center of the Earth. Calculate the gradient ∂B/∂r. b) For a perpendicular kinetic energy E_⊥ of 1 keV, what is the perpendicular velocity of a proton? c) The gradient drift velocity is given by u_∇=1/2 m v^2 (B ∂B/∂r)/(qB^3). Calculate the drift velocity at r = 5 Re of a proton with a perpendicular energy E_⊥ of 1 keV. d) How long does it take for that electron to drift once around the Earth?	$\frac{\partial B}{\partial r} = -\frac{3\mu_0 M}{4\pi r^4}$ b) Easy: $\frac{1}{2}mv_\perp^2 = 1.6 \times 10^{-16}$, $v_\perp =$ 447km/s. No: $\frac{v_\perp}{c} \ll 1$ (2 points) c) Easy: $E_\perp = 1.6 \times 10^{-16}$J. $r =$ 3.186×10^7m. $B(r) = 2.44 \times 10^{-7}$T. $\frac{\partial B}{\partial r} = 2.30 \times 10^{-14}$T/m. $u_\nabla =$ -386.3m/s (3 points) d) Easy: $\frac{2\pi 5R_e}{u_\nabla} = 143$h. No, the dynamics of the system happen on much shorter timescales. No, an electron drifts at the same speeds (3 points).