FYS 3610: Midterm solutions 2017

Exercise 1 (14 points)

a) F region between 170 km and 1000 km altitude. E region between 90 km and 170 km altitude.

b) q =production, l =losses, d = transport.

c) $n_{O^+} = 10^{11}\ m^{-3}$, $n_{o_2} = 10^{15}\ m^{-3}$, $n_{N_2} = 10^{16}\ m^{-3}$, so $l = 1.75 \cdot 10^9\ m^{-3}s^{-1}$.

d) The same as in point c), i.e. $q = l$.

e) Use $p_i = nk_BT_i$ and $p_e = nk_BT_e$ to get $p_i + p_e = nk_B(T_i + T_e)$.

Thus

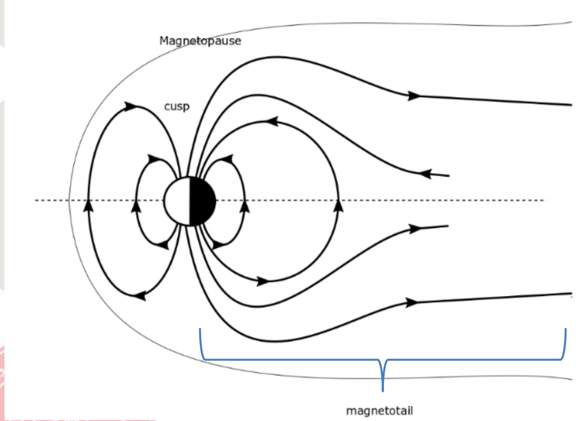
$$\frac{d(p_i+p_e)}{dz} = k_B(T_i + T_e)\frac{dn}{dz} = -n\ (m_i + m_e)g,$$

Separating variables, one gets $\frac{dn}{n} = -\frac{g(m_i+m_{e)}}{k_B(T_i+T_e)}dz = -\frac{1}{H}dz$, which can be integrated on both sides

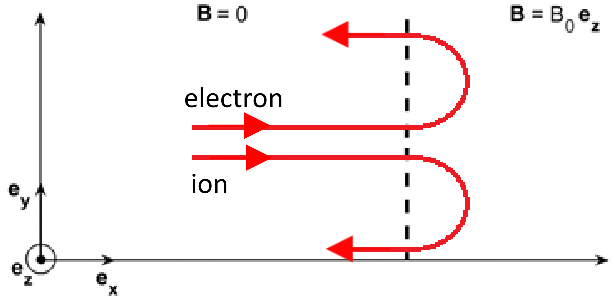
$$\int_{n(z_o)}^{n(z)} \frac{dn}{n} = \int_{z_o}^{z} \frac{-dz}{H},$$

to obtain $n(z) = n(z_0)\ \exp(-\frac{z-z_0}{H})$

Exercise 2 (14 points)



a)



b)

c) magnetopause

d) Chapman-Ferraro current (magnetopause current), Ring current, neutral sheet current (tail current)

e) Start with $K\, n_{sw}\, m_p v_{sw}^2 = 2B_{dip}^2/\mu_0$ to get $v_{sw} = \sqrt{\frac{2B_{dip}^2}{K n_{sw} m_p \mu_o}}$

Using $B_{dip} = B_{00}/L^3$ (from the appendix), one can get $v \approx 955\,\frac{km}{s}$.

This is a fast solar wind.

Exercise 3 (14 points)

a) Use the gyrofrequency $\Omega_0 = qB/m$, to find

$B = \frac{\Omega_0 m}{q} = 6 \cdot 10^{-6}\, T.$

Using the formula in the appendix $B_0 = B_{00}/L^3$, one can get $L = (B_{oo}/B_0)^{1/3} \approx 1.7\, R_E$

b) The gyroradius is given by $r_0 = v_\perp/\Omega_0$.

Using $E = \frac{1}{2} m\, v^2$ to find v, and $v_\perp = v \sin\alpha_0$ (from the scheme), one can find $r_0 \approx 1.7 \cdot 10^3\, m.$

c) $B_m = \frac{B_0}{\sin^2\alpha_0} = 2.4 \cdot 10^{-5}\, T$ so

$r_m = \frac{v}{\Omega_m} = \frac{vm}{qB_m} \approx 852\, m.$

d) $\frac{B_m L^3}{B_{00}} \approx 4$ so from the graph, $\varphi = 33$ degrees (± 5 degrees accepted)

The height can be found using $h = r - R_E = L\, R_E \cos^2 \varphi - R_E \approx 1291\ km$.

e) At lower altitude.

Exercise 4 (8 points)

a) ρ: mass density, $\vec{j}$: current density, ∇p: pressure gradient.

b) Parker model

c) $\beta = \frac{p_{th}}{p_{mag}} = \frac{n k_B T}{B^2/2\mu_o}$

$\beta \gg 1$ implies thermal pressure dominates, and thus the plasma is called warm.

$\beta \leq 1$ implies that magnetic pressure is larger or equal to thermal pressure, and thus the plasma is called cold.

$\beta \gg 1$ for the solar wind (warm)

d) From $\rho \left(\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\, \vec{v} \right) = -\nabla p + \rho\, \vec{g} + \vec{j} \times \vec{B}$.

Static equilibrium and no $\vec{B}$ gives

$0 = -\nabla p + \rho\, \vec{g}$

which is known as the aerostatic (or hydrostatic equation) (cf. p.33 of the book)