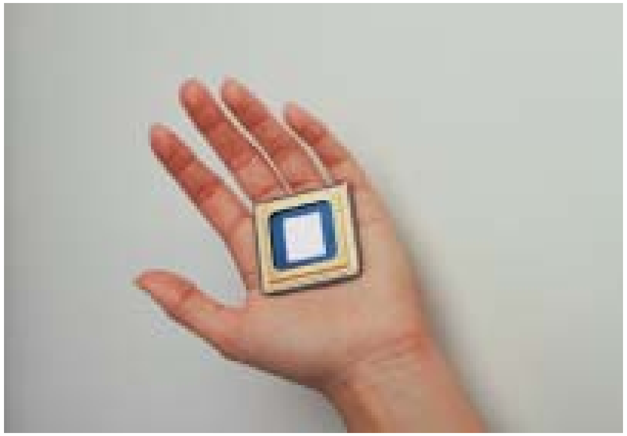
Electrostatically actuated mirror-array

- Array of micro mirrors
  (e.g. 1280x1024 SXGA)
- Produced by Texas Instruments



- “Digital Micro mirror Device”
- “Digital Light Processing”

- Used in projectors, TVs, movie theaters

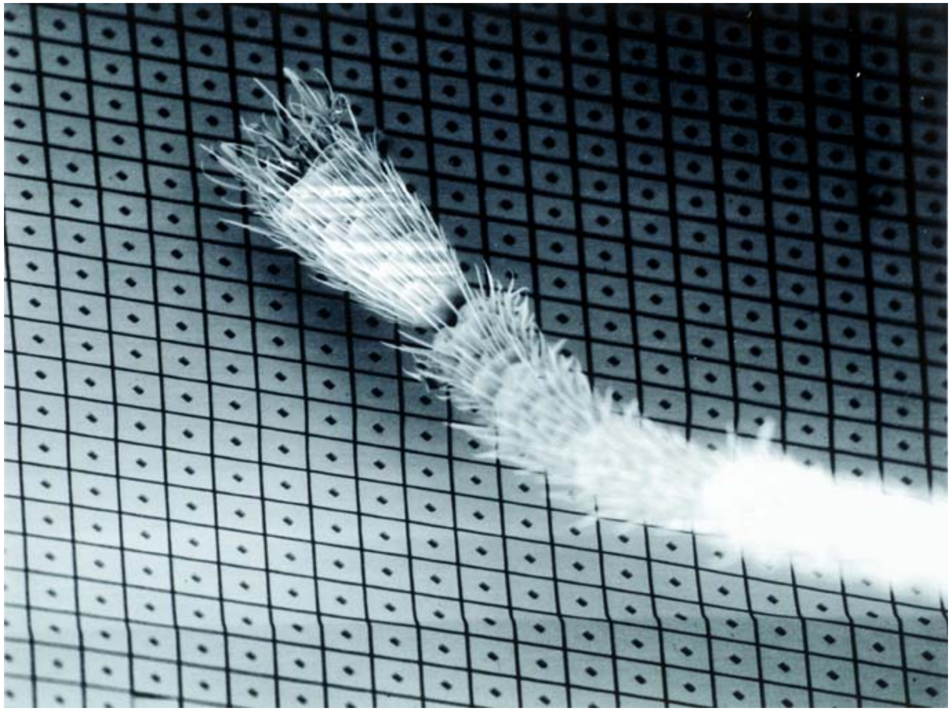
DLP
A TEXAS INSTRUMENTS TECHNOLOGY

- Digital images

Dimensions of micro mirror array

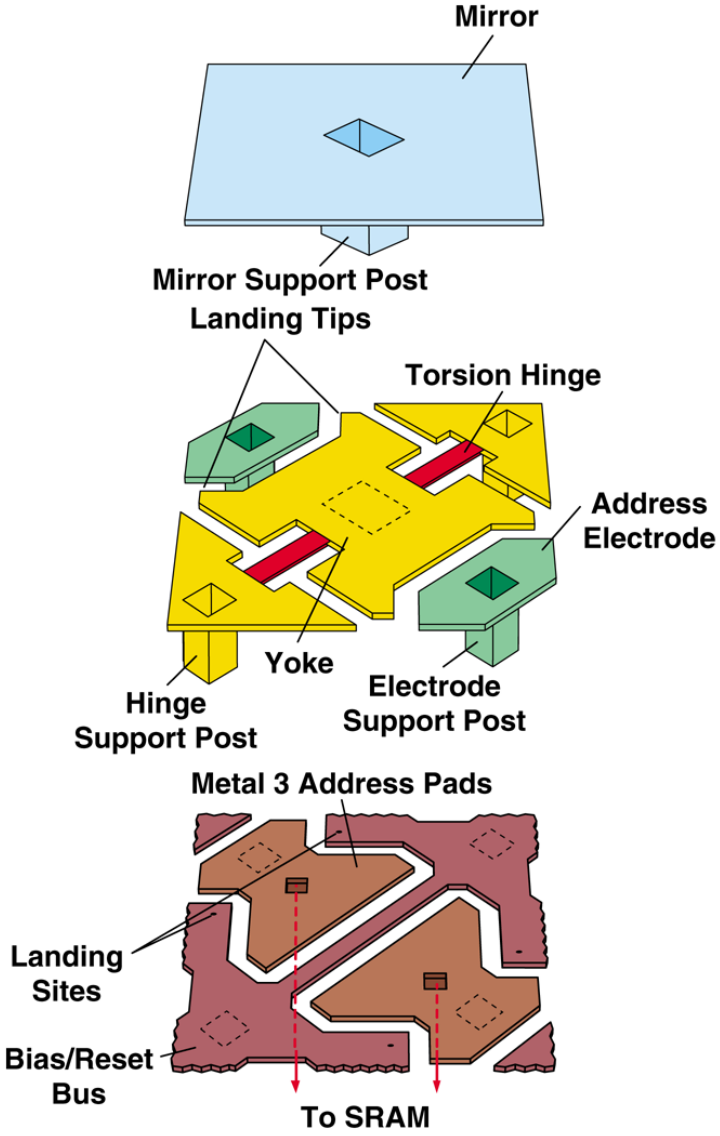
- Size of mirror: 16μmx16μm, 14μmx14μm, smaller and smaller
- Gap between mirrors 1 μm

- Switch more than 50000 times pr second
- Vacuum packed



Mirror tilting

- Mirror made in reflective aluminium
- Mirror sits on top of Yoke
- Yoke can tilt, supported by torsion hinges
- Yoke can be attracted to left or right electrode by electrostatic forces
- Every electrode for every mirror at the silicon surface can be accessed separately
- The voltage between mirror and electrode is large enough to cause pull-in
- Yoke is mechanically stopped by landing tips (no electric contact)



Fabrication of DMD (from Maluf)

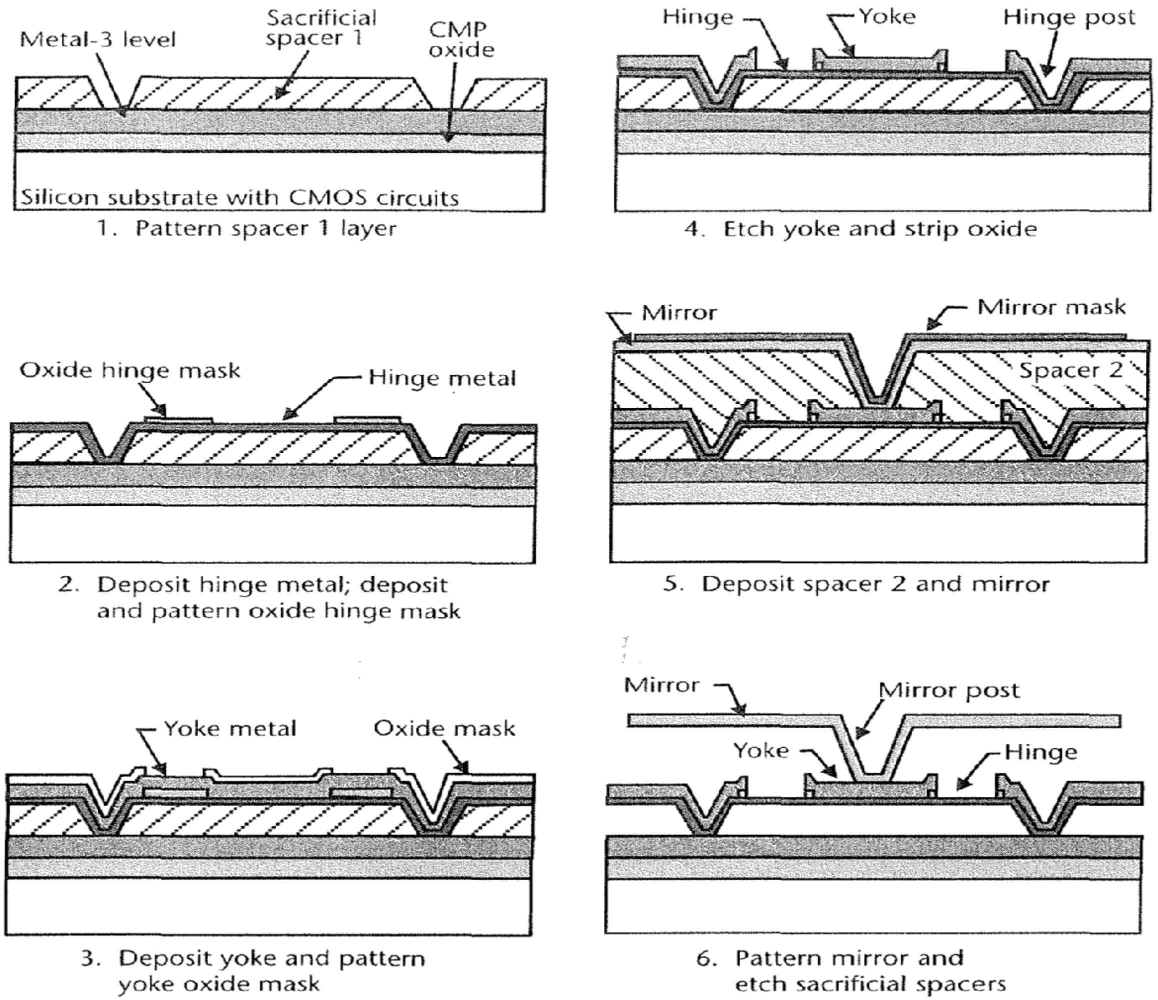
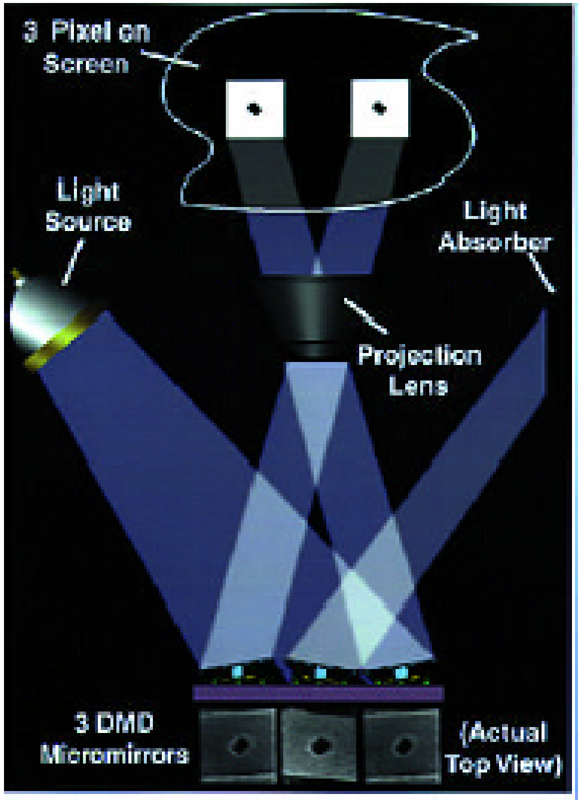
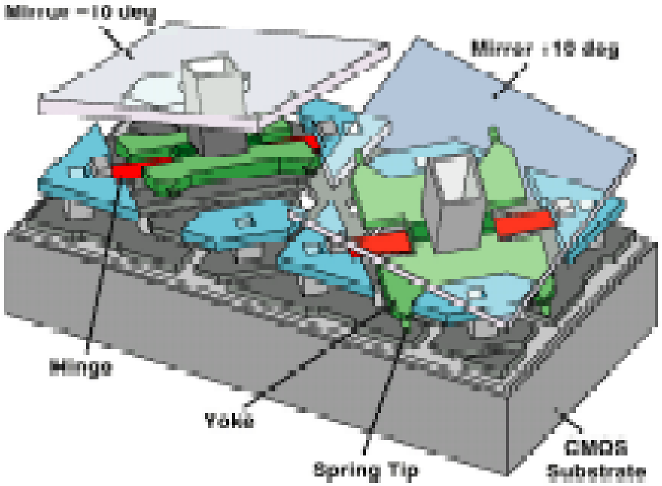


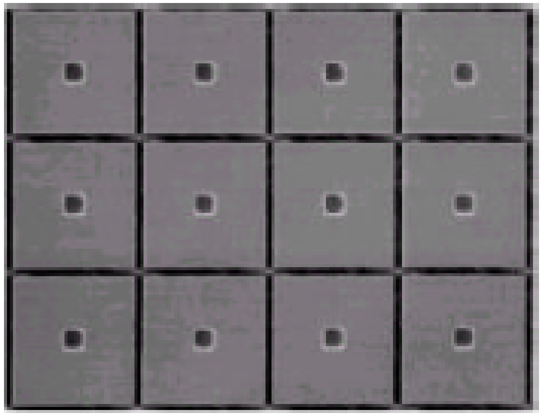
Figure 5.4 Fabrication steps of the Texas Instruments' DMD [2].

Principle of Image Projection

Electrostatic On-Off Control of Mirror Array

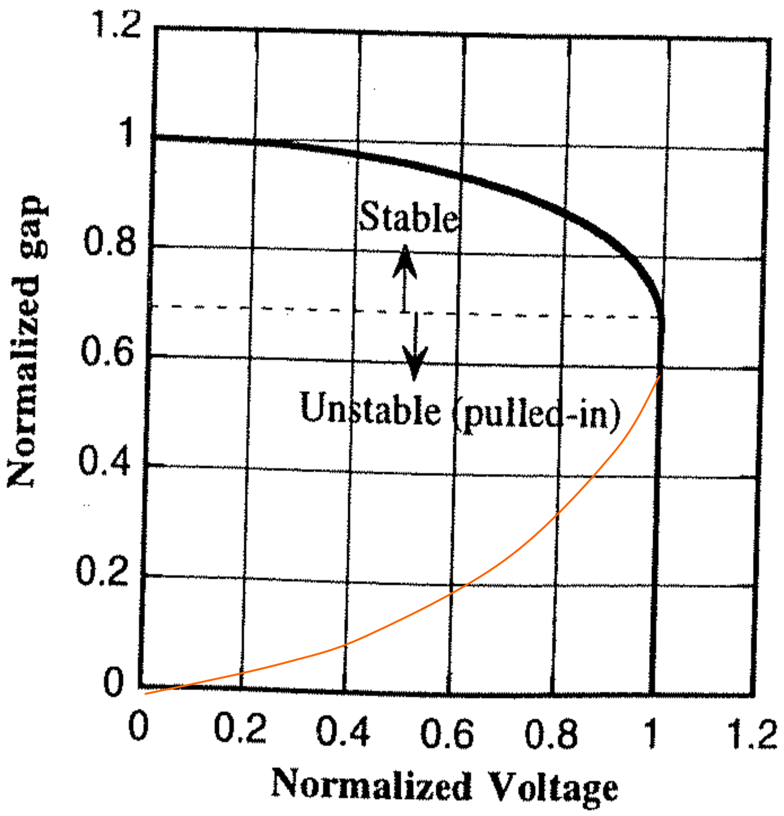






Hiroshi Toshiyoshi Hiroshi@ee.ucla.edu

Gap vs. voltage



- Parallell plates, linear spring elastic force

- Normalized gap g/g_0
- Normalized voltage V/V_{PI}

Electrostatic forces

- Forces between charges
- Electric potential Φ
- Laplace equation + boundary conditions (dirichlet)

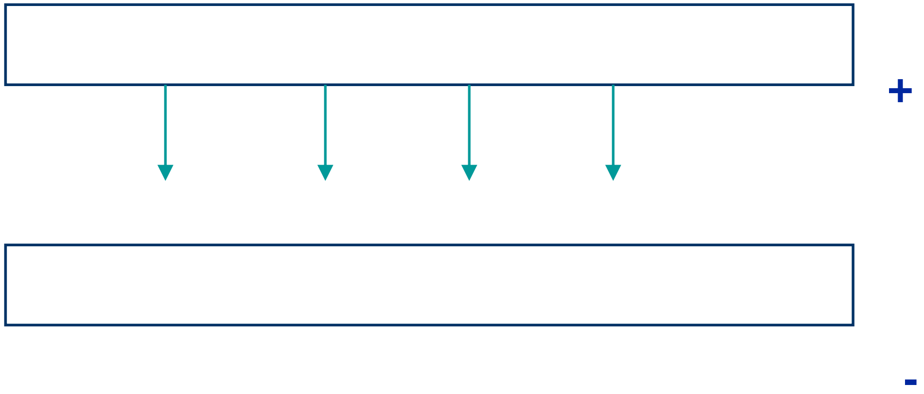
$$\nabla^2 \Phi = 0$$

- Electrostatic field: gradient of potential

$$\vec{\varepsilon} = -\nabla \Phi$$

- Electric force normal to conductor surface
- Charge distribution on surface conductors related to field

$$q(r) = \left|\vec{\varepsilon}(r)\right| \varepsilon$$



- Force proportional to electric field

$$\vec{F} = q\vec{E}$$

- Forces between parallel plates in capacitor:

$$F = \frac{-\varepsilon A V^2}{2g^2}$$

Pull-in of mirror

- Electrostatic force depends on tilt
- Elastic torque: $M=k_{\Theta}\,\Theta$
- Electrostatic torque:

$$\tau = \int_A \left(\frac{\varepsilon V^2}{2\left[g_0 - w(x)\right]^2} \right) x\,dx$$

- Electrostatic torque varies as

$\tau \sim \Theta^2$:

- Pull-in effect also for mirror

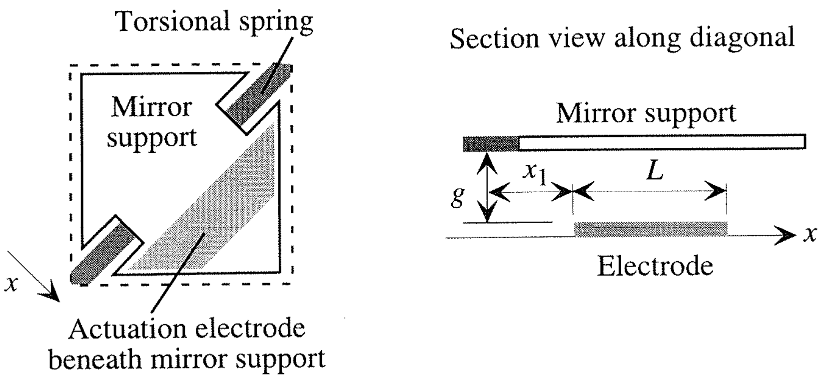


Figure 20.9. Geometry of the DMD mirror.

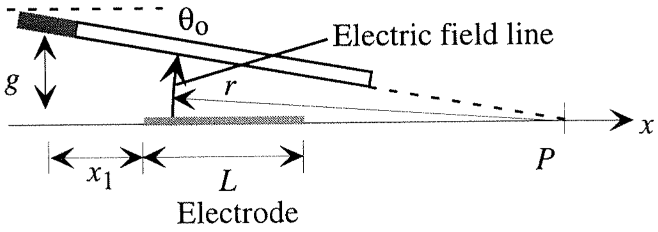


Figure 20.10. Illustrating the field and capacitance calculation for the titled DMD.

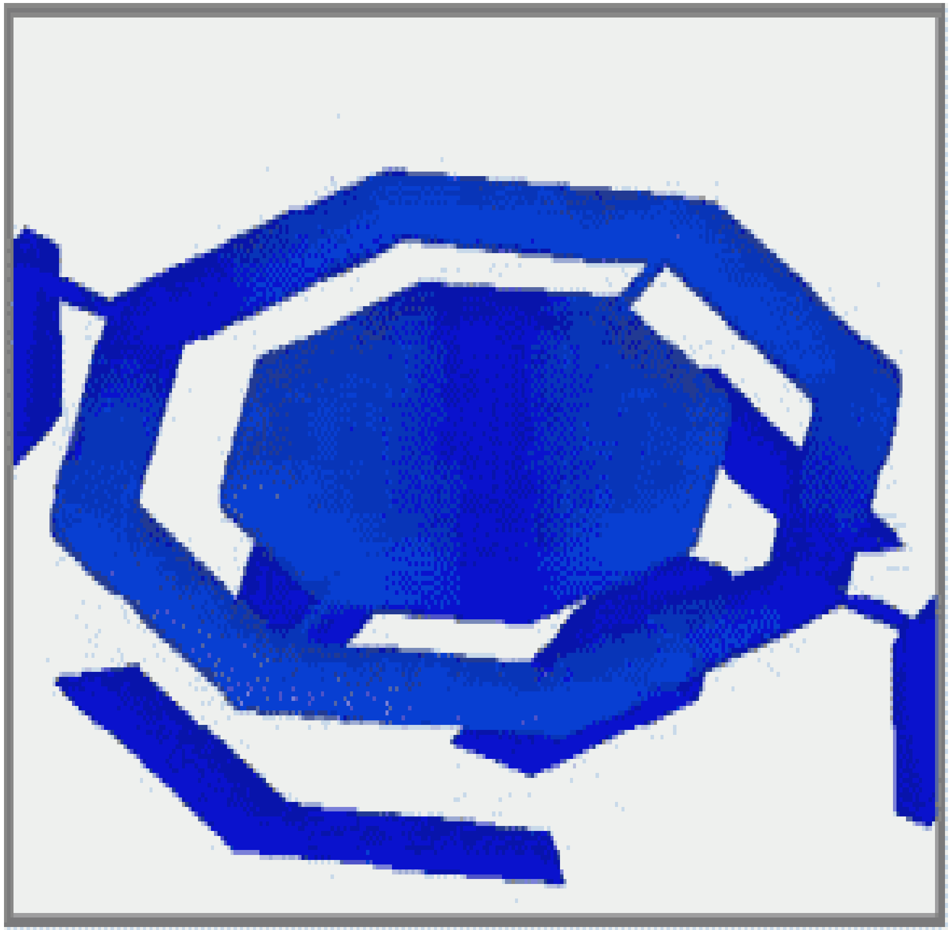
Mechanical and electrostatic equations

- Naviers equation for elastic forces: (isotropic version)

$$(\lambda + \mu)\nabla\nabla \cdot u + \mu\nabla^2 u = 0$$

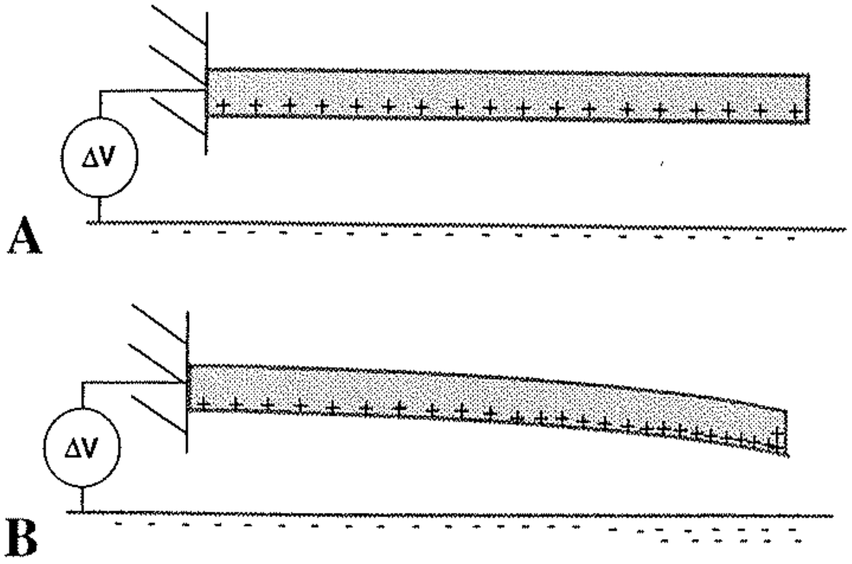
- Poisson equation for electrostatic field:

$$\nabla^2\Phi = -\frac{\rho}{\varepsilon}$$



Electrostatic bending of beam

- Set up voltage ΔV between beam and substrate
- Beam bend due to electrostatic forces
- Elastic forces tend to pull beam back



- Total force, parallel plates:

$$F_{net} = \frac{-\varepsilon A V^2}{2g^2} + k(g_0 - g)$$

- Equilibrium: $F_{net} = 0$

Pull-in of parallel plates with linear spring

- Increase ΔV, reach pull-in distance and voltage

$$g_{PI} = 2/3 g_0$$

$$V_{PI} = \sqrt{\frac{8kg_0^2}{27\varepsilon A}}$$

- If voltage is larger than pull-in volta
  => no stable solution except *g*=0

- $\zeta = 1 - g/g_0$

- Senturia section 6.4.3

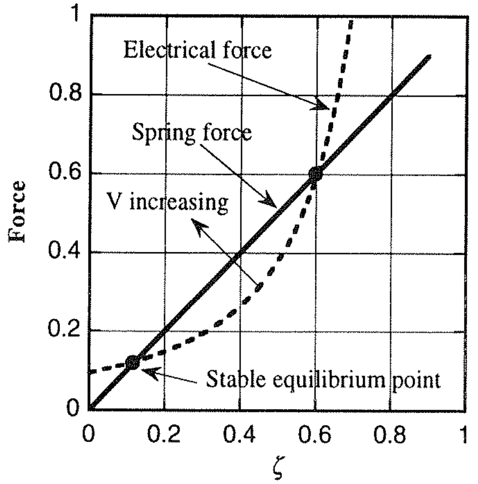
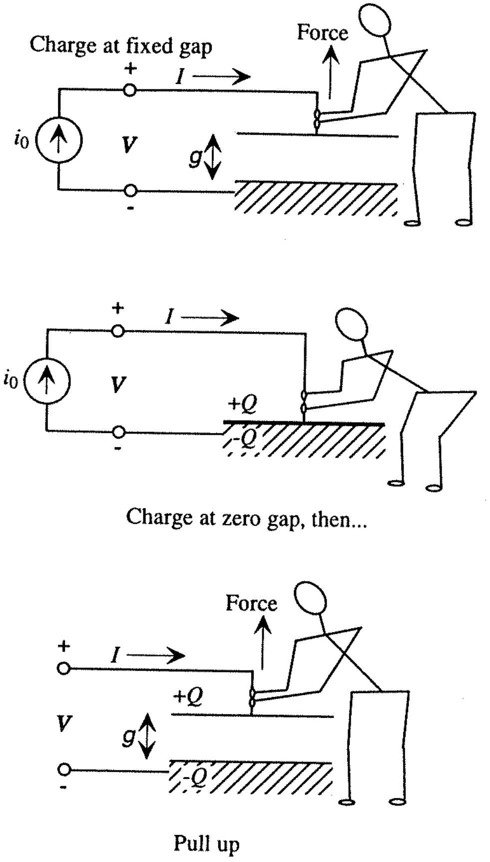


Figure 6.7. Electrical and spring forces for the voltage-controlled parallel-plate electrostatic actuator, plotted for $V/V_{PI} = 0.8$.

Potential energy of parallel plate capacitor



- Senturia p 127
- Stored energy

$$W(Q)=\int_0^Q V(Q)dQ$$

$$Q=CV$$

$$W(Q)=Q^2/2C=Q^2g/2\varepsilon A$$

$$\mathrm{E}=Q/\varepsilon A$$

$$F=Q^2g/2\varepsilon A$$

$$W(g)=Fg=Q^2g/2\varepsilon A$$

Pull-in voltage of tilting mirror

- Energy

$$W(\theta_0)=\frac{1}{2}CV^2$$

- Torque: negative gradient with respect to θ_0
- Find charge Q on electrode
- Find Capacitance

$$\tau=-\frac{1}{2}C(0)V^2\left[a_1+3a_3\theta_0^2\right]$$

- Restoring torque, torsional spring

$$\tau_{elastic}=k_\theta\theta_0$$

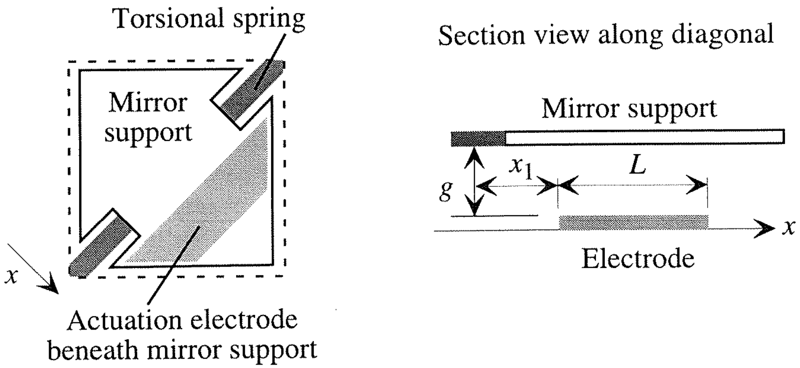


Figure 20.9. Geometry of the DMD mirror.

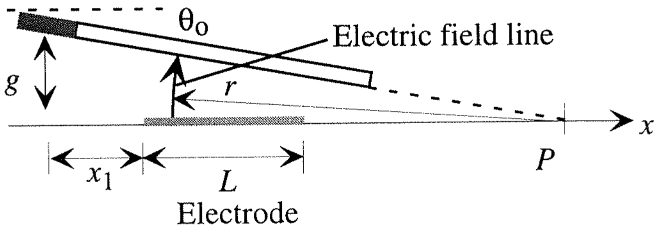


Figure 20.10. Illustrating the field and capacitance calculation for the titled DMD.

Pull-in voltage, torsional mirror

- Force equilibruim

$$\theta_0 = -\frac{k_\theta}{3a_3C_0V^2} \pm \sqrt{(\frac{k_\theta}{3a_3C_0V^2})^2 - \frac{a_1}{3a_3}}$$

- Real solution if

$$\left(\frac{k_\theta}{3a_3C_0V^2}\right)^2 \geq \frac{a_1}{3a_3}$$

- Pull-in voltage

$$V_{PI} = \left(\frac{{k_\theta}^2}{3a_1a_3C_0^2}\right)^{\frac{1}{4}}$$

Mirror design with analyzer (tutorial 2)

Figure T3-1 Mirror Model

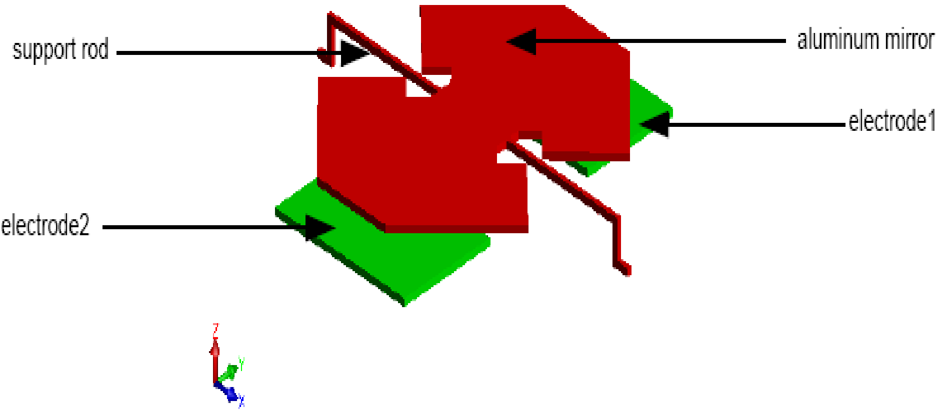
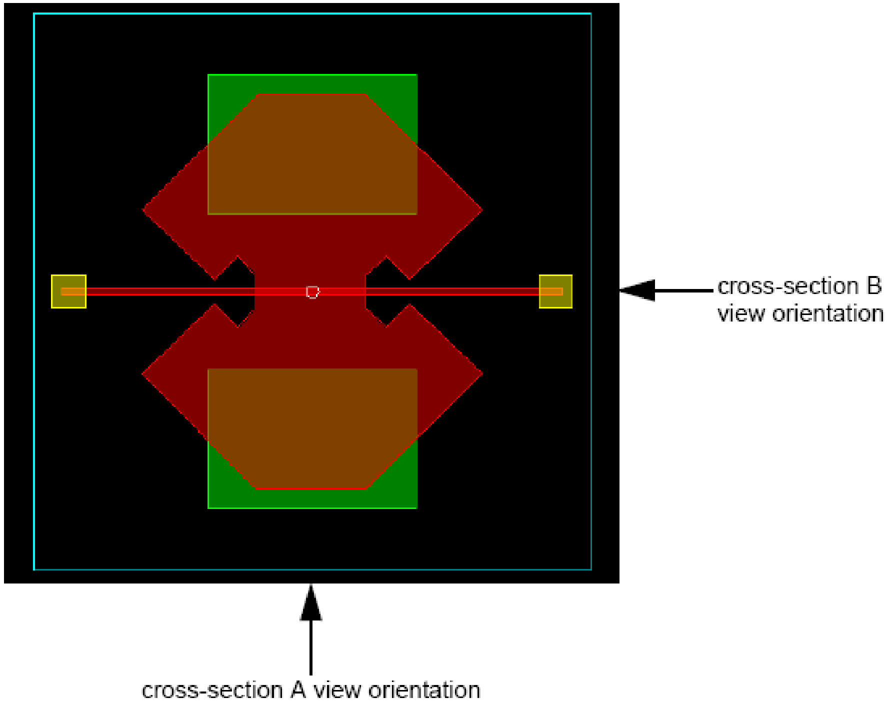
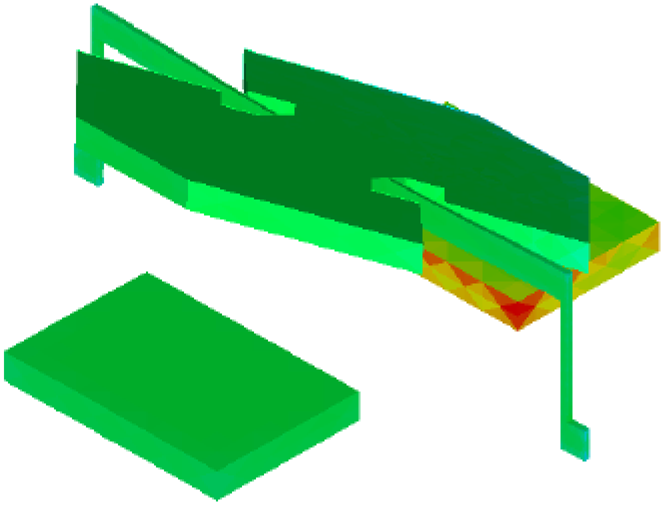


Figure T3-2 2-D Layout View of Mirror

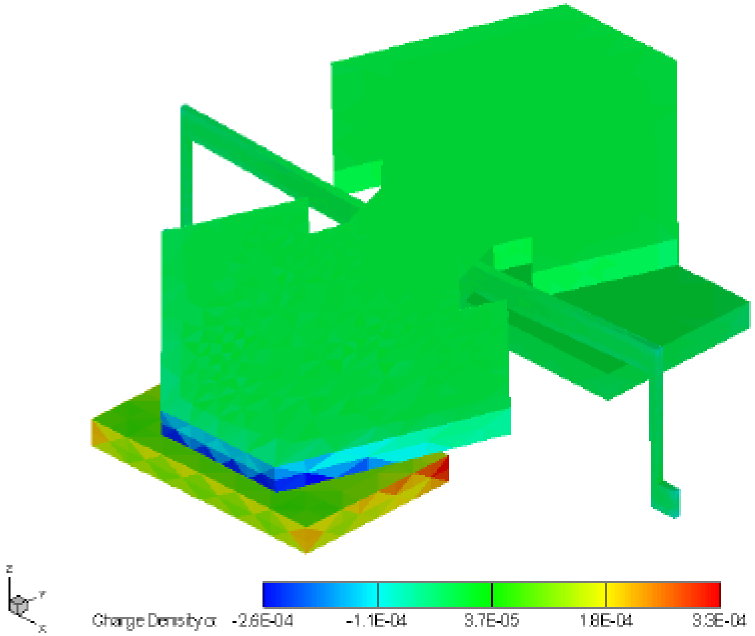


Charge distribution due to tilt

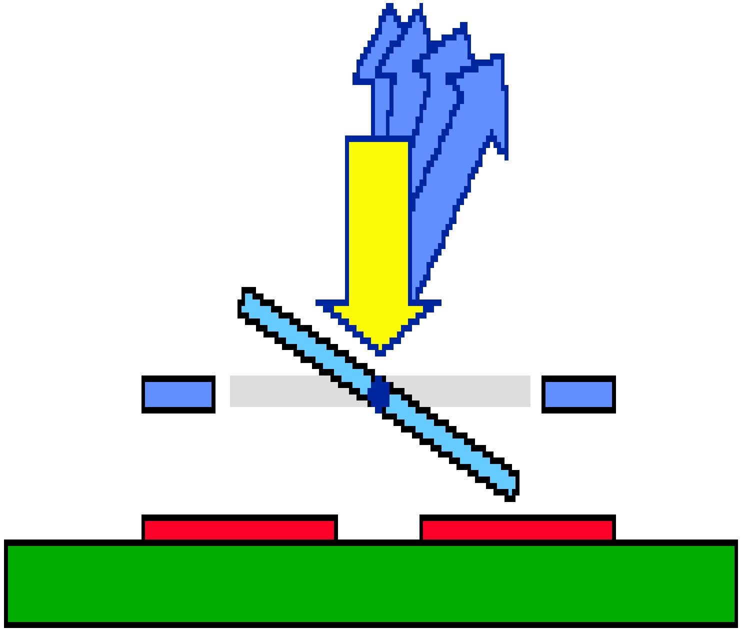
electrode1 = 20 volts



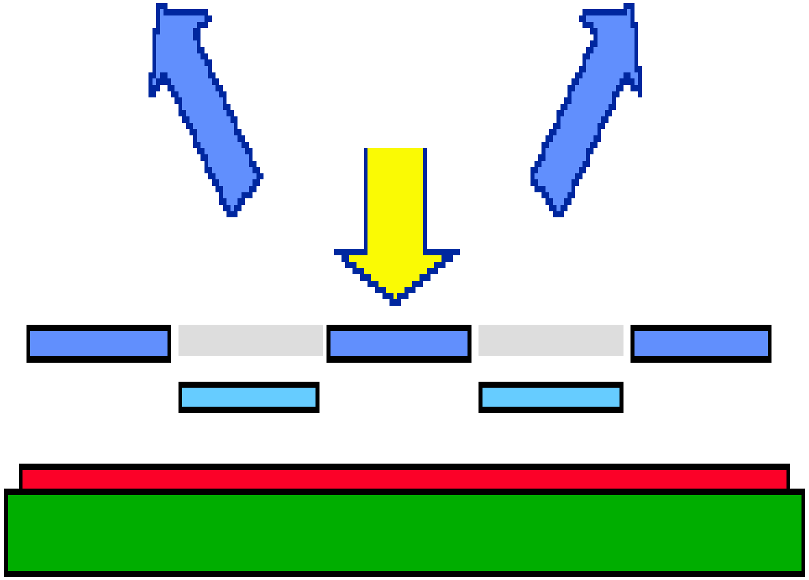
electrode2 = 20 volts



Two display principles



Tilting Mirror Optical MEMS



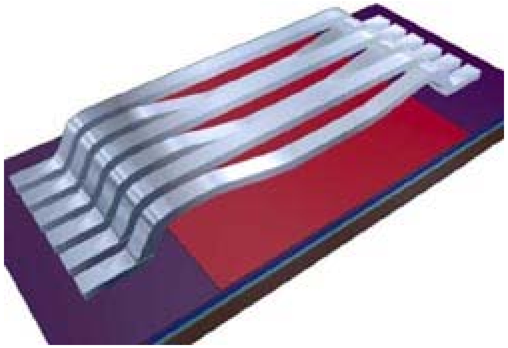
GLV Diffraction Grating MEMS

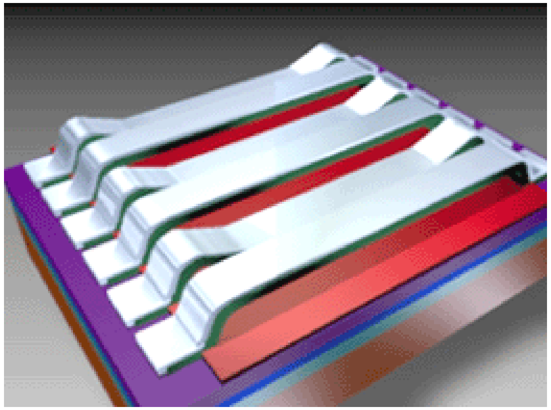
Silicon Light Machines

- Grating Light Valve

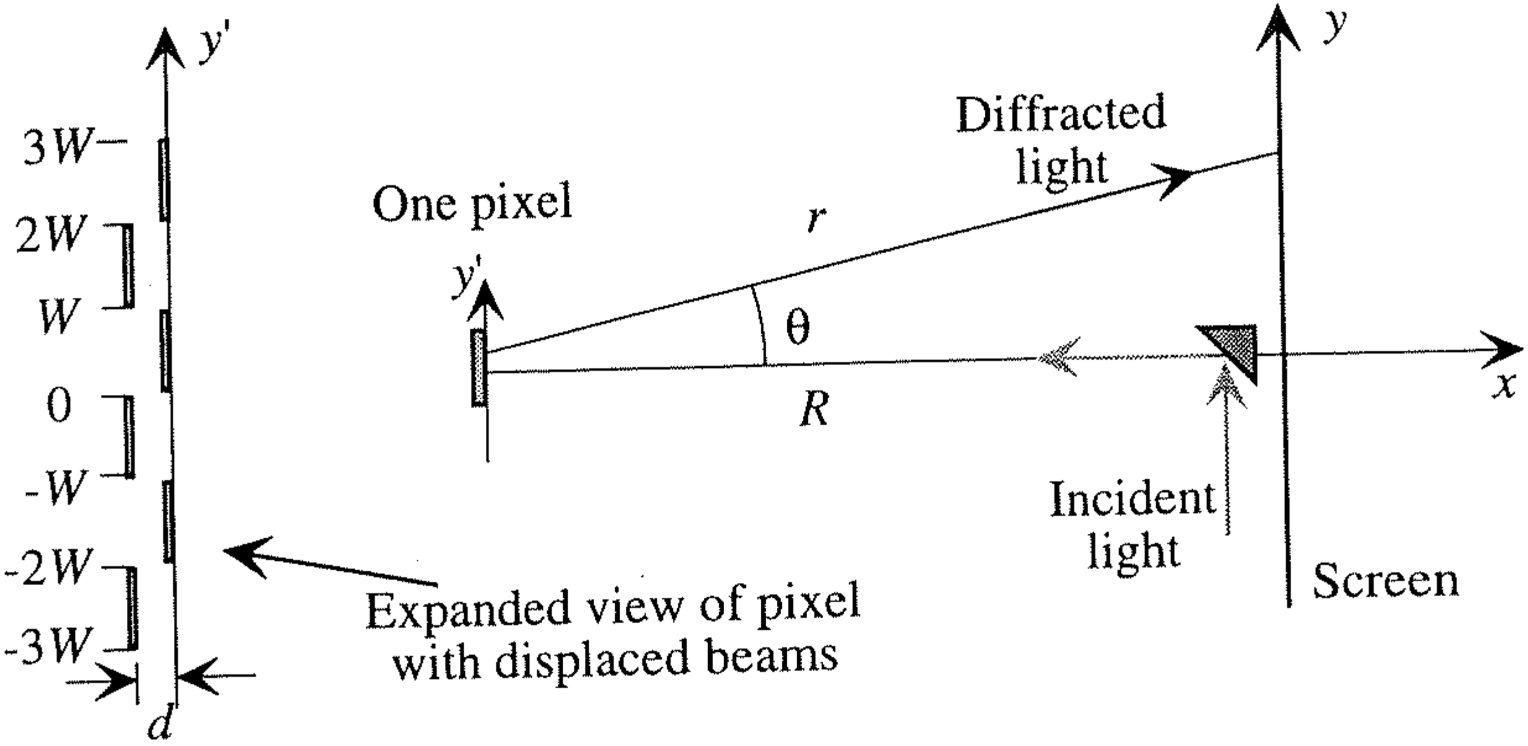
- Electrostatically deflect ribbons
- Distance to wafer $\lambda/4$
- Light is reflected or diffracted
- Diffracted light is projected to screen

- Possible to use pull-in or pull-control
- Possible to have one row of ribbon pixels only



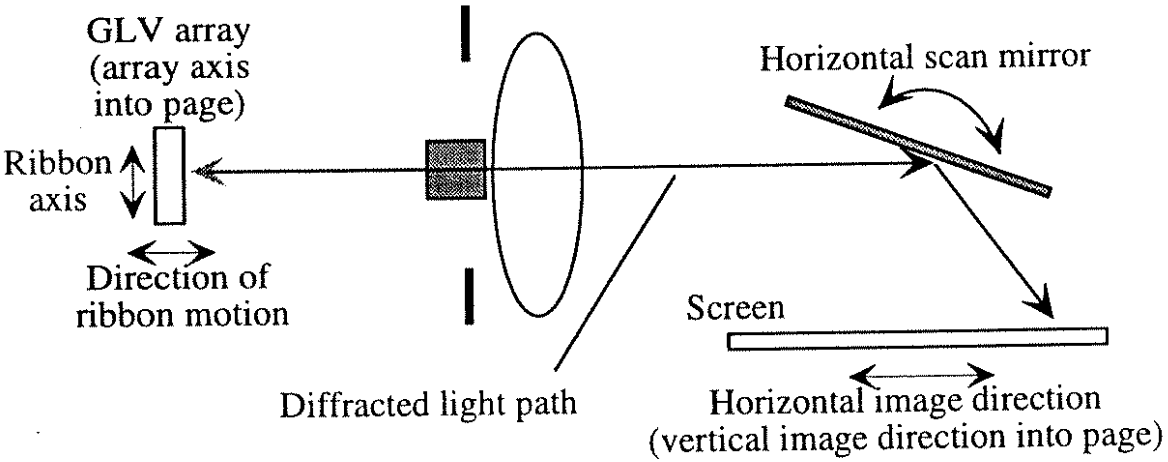


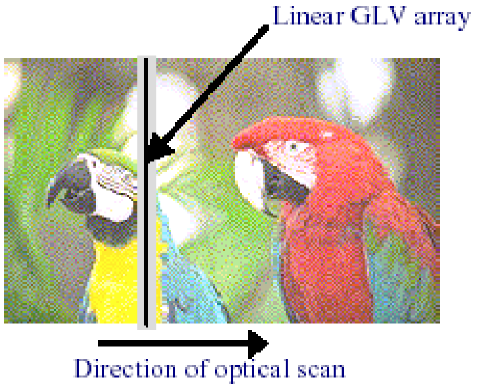
Diffraction from a 6-element DLV



GLV linear array of pixels

- One line of mirror elements only





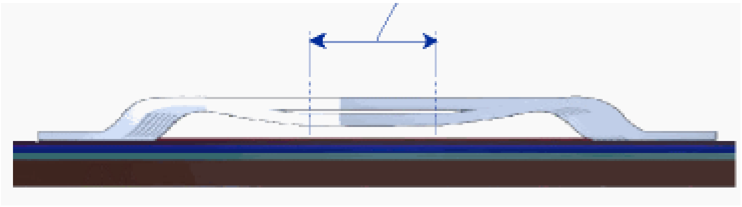
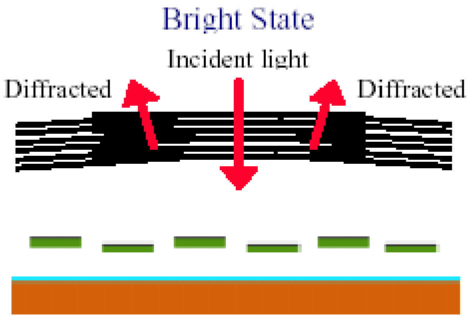
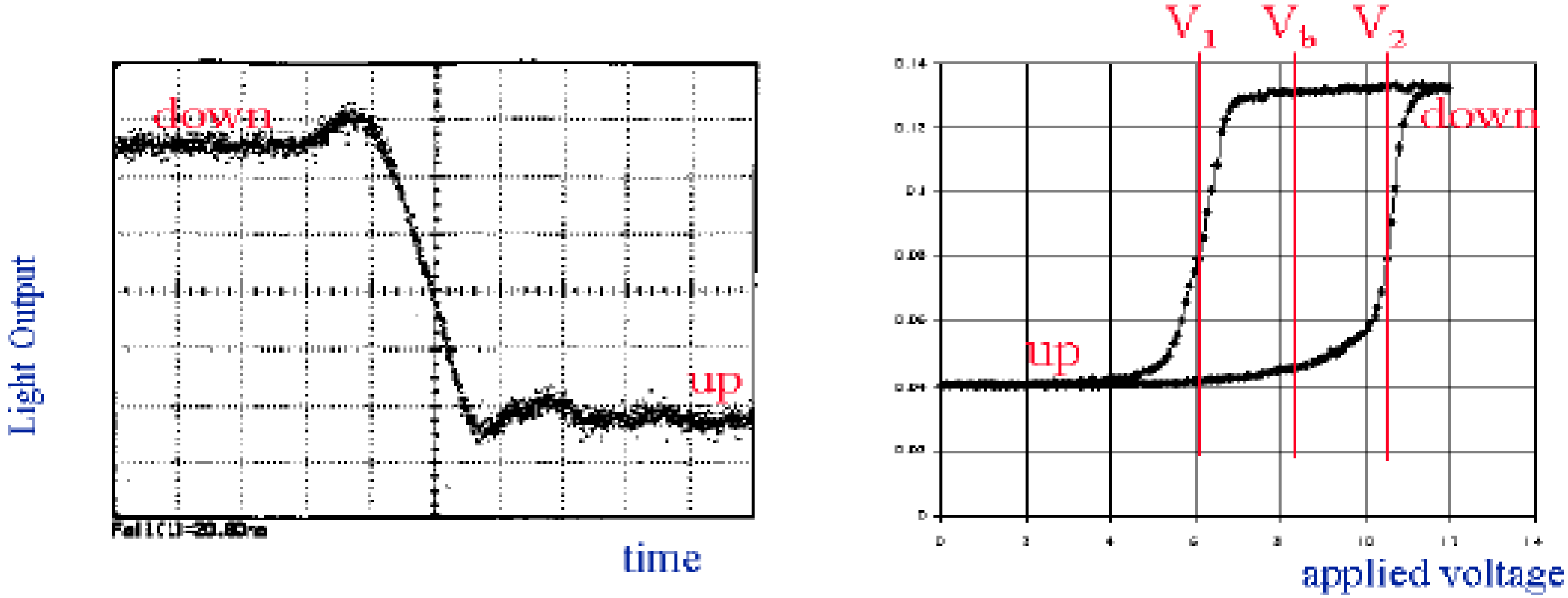


Figure 1: A GLV pixel with alternate reflecting ribbons electrostatically deflected to produce a square-well diffraction grating (vertical deflection greatly exaggerated)



Pull in hysteresis of mirrors



20 nsec Switching Speed

Pixel Hysteresis

Figure 5: To switch a ribbon down requires a voltage differential of V2 volts or more between the ribbon and a bottom electrode. The ribbon will remain down until the voltage differential falls below V1 volts. This ribbon hysteresis offers mechanical memory and zero-power pixel-state retention. Switching time is approximately 20 nanoseconds.

Different colors diffracted to same spot

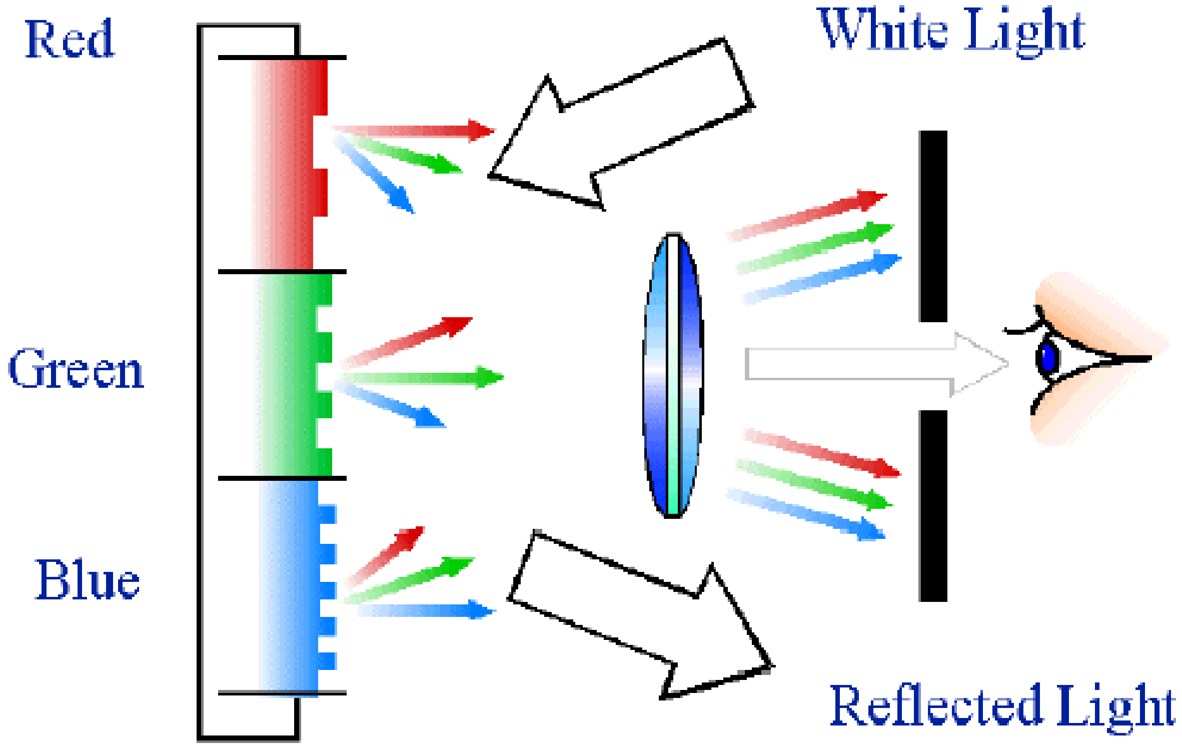


Figure 7: By using different spacing between ribbons, one can create color-oriented sub-pixels.

Linear elastic force

- Partial differential equation for force-elastic displacement (beam):

$$EI\frac{\partial^4 w}{\partial x^4} = q$$

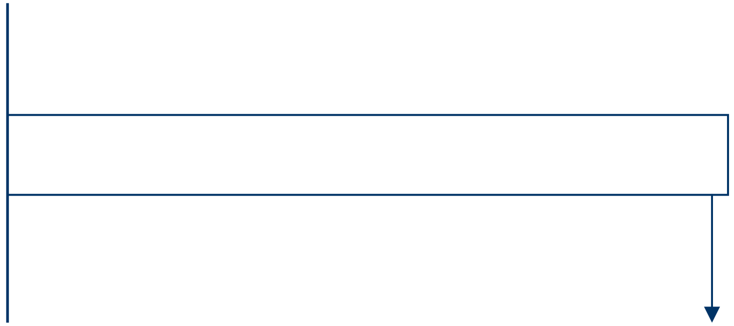
- Approximation e.g.

$$F = kw_{\max}$$

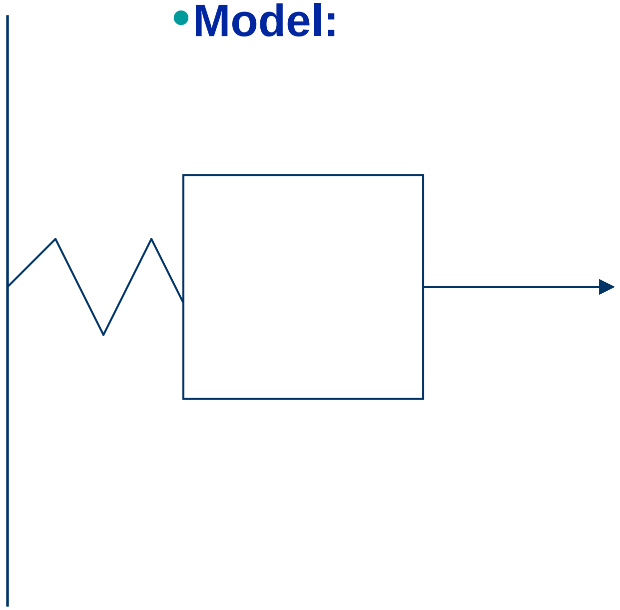
- Linear relation stress-strain (Always true for single-crystal silicon):

$$\sigma = E\varepsilon$$

- Strain is derivative of displacement. May give non-linear relation displacement-strain for large deflections (geometric effect)

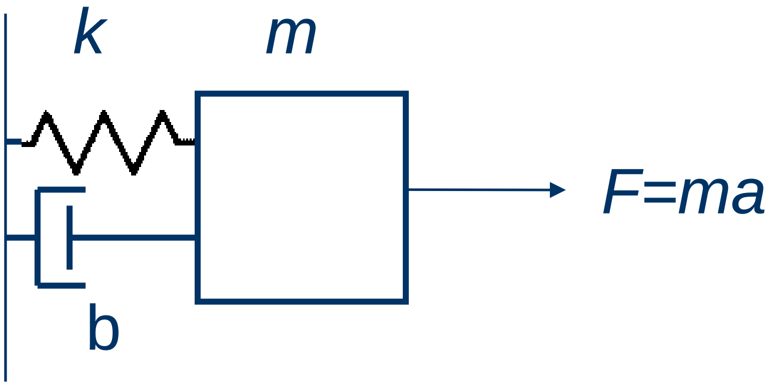


- **Model:**

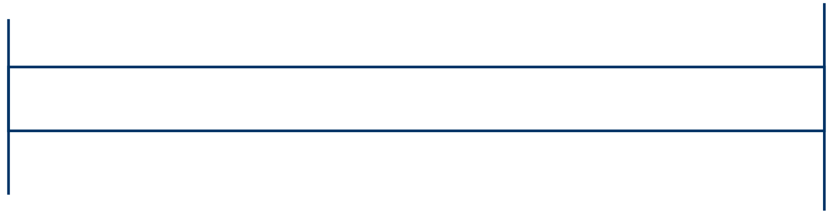


Resonant frequency of elastic structure

- Measure resonance frequency in vacuum
- Damping due to fluid flow around moving structure



- Simplified dynamical equation: ma + bv + kx = F



- Dynamic equation for beam

$$\rho \frac{\partial^2 u}{\partial t^2} + EI \frac{\partial^4 u}{\partial x^4} = f(x,t) + F/L$$

Couette flow

- Steady viscous flow between parallel plates
- One plate is moving parallel to the other
- Mass conservation, no x-dependence on flow velocity
- Streamlines parallel to the walls
- Stationary flow
- No-slip boundary conditions

$$U_x = \frac{y}{h} U$$

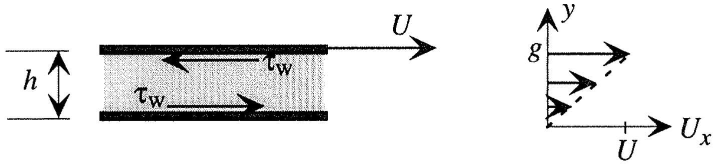


Figure 13.4. Illustrating Couette flow

- Shear stress acting on the plate as a result of motion:

$$\tau_w = -\eta \frac{U}{h}$$

- Damping coefficient b

$$b = \frac{\eta A}{h}$$

Squeezed film damping

- Displacement of beam, dynamic partial differential equation

$$\rho \frac{\partial^2 u}{\partial t^2} + EI \frac{\partial^4 u}{\partial x^4} = P(x,t) + \frac{F}{L}$$

- Flow of gas: Reynolds equation gives pressure in gap

$$12\eta \frac{\partial(Ph)}{\partial t} = \nabla\left[(1+6K_n)h^3 P \nabla P\right]$$

- Characteristic of solution:

$$P = b\frac{\partial u}{\partial t}$$

- Senturia gives the damping constant for a long beam (L)

$$b = \frac{96\eta L W^3}{\pi^4 g_0^{\ 3}}$$

- Squeeze film number σ
- Relative importance of viscous to spring forces

$$\sigma = \frac{12\eta W^2}{g_0^{\ 2} P_0}\omega$$