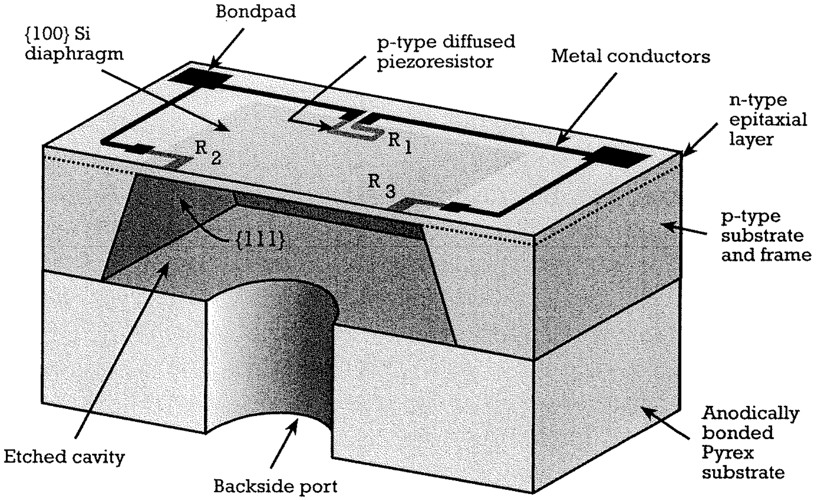
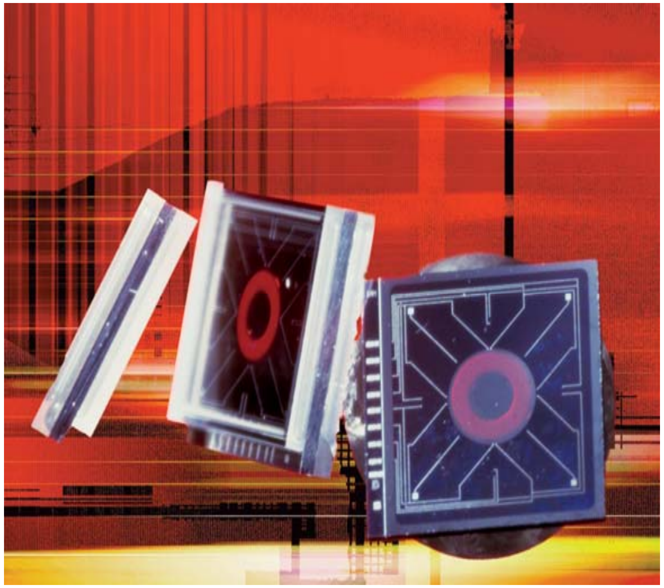
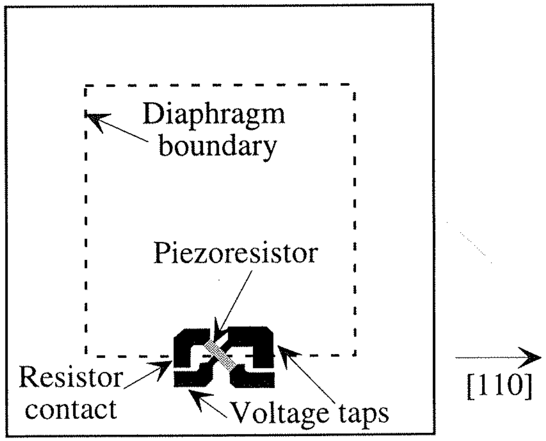
Piezoresistive pressure sensors



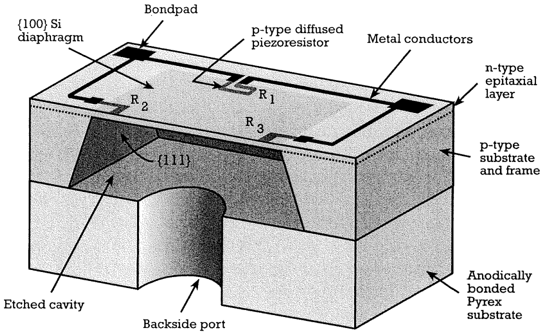


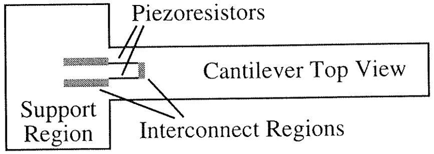


Two sensing principles

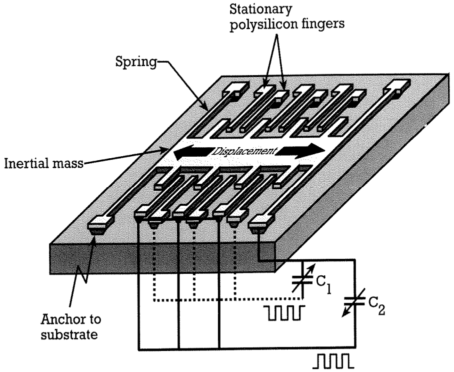
- Piezoresistive
- measure mechanical stress in doped resistor-area
- diaphragm pressure sensor
- bending beam due to
  - volume forces (e.g. acceleration)
  - end force (e.g. protein attached)

- capacitive
- measure deflection (distance to other capacitor plate)
- diaphragm pressure sensor
- bending beam due to
  - volume forces (e.g. acceleration)
  - end force (e.g. protein attached)

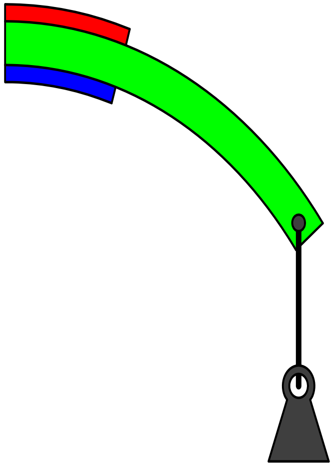




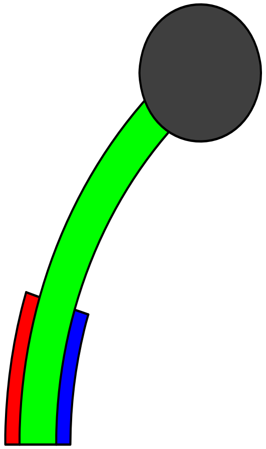
8.4. An example using piezoresistance to measure the deflection of a cantilever.



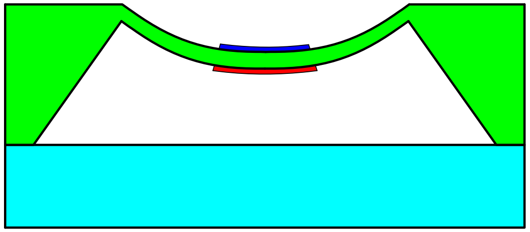
Piezoresistive sensing applications



Veieceller

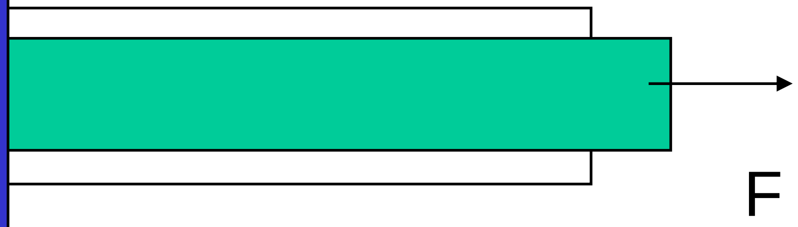


Akselerometer



Trykksensorer

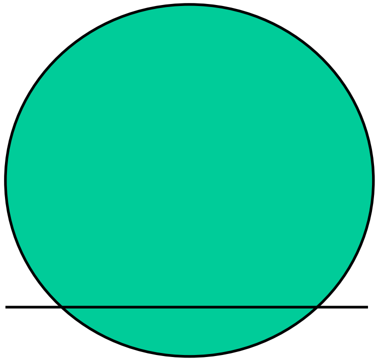
Relation between stress and strain in a coordinate system with axes equivalent to the axes of the unit cell

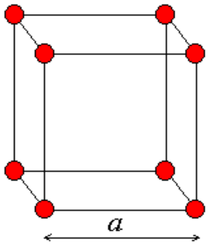


$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yx} \\ \tau_{zx} \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{12} & C_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{44} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{zx} \\ \gamma_{xy} \end{bmatrix}$$

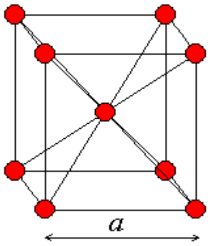
$\sigma_x = E\,\varepsilon_x$

1D hooke's law

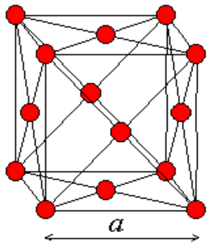




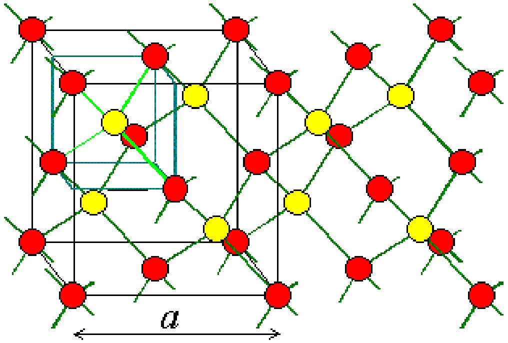
(a)



(b)



(c)



Small deflections
Beam equation, plate equation

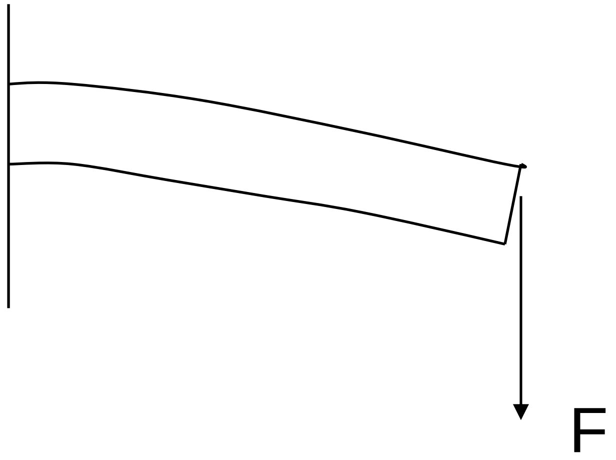
- Beam equation:

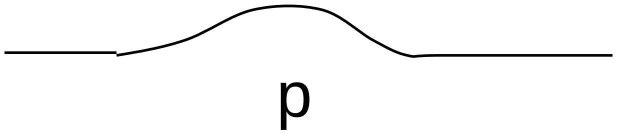
$$EI\frac{d^4w}{dx^4}=q$$

- Plate equation:

$$D(\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial^2 x\partial^2 y}+\frac{\partial^4 w}{\partial y^4})=q(x,y)$$

- Find displacement and stress





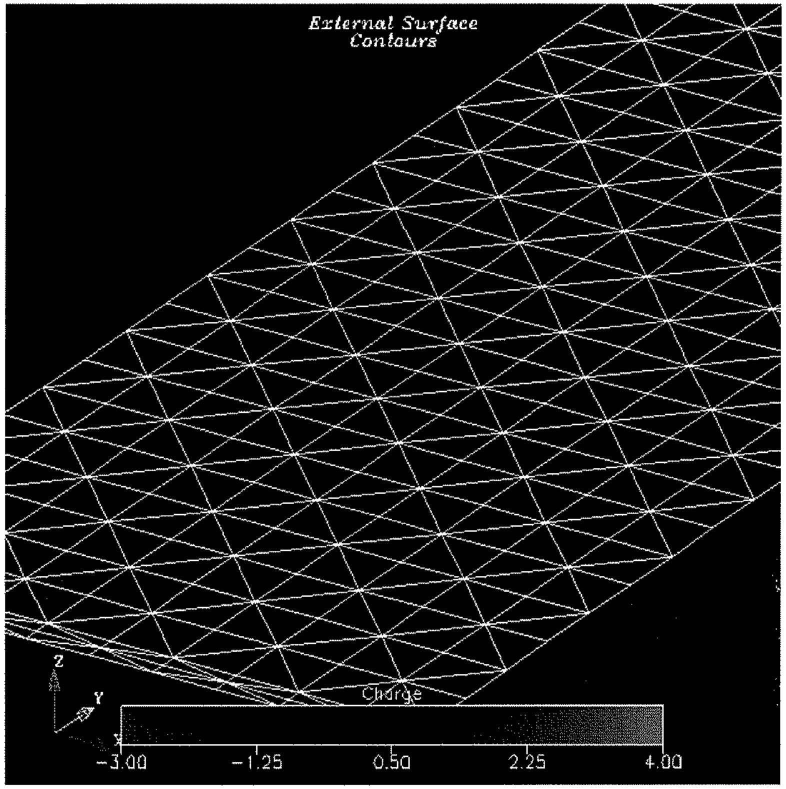
General: Finite Element Analysis

- Solve Navier's equation (partial differential equation)

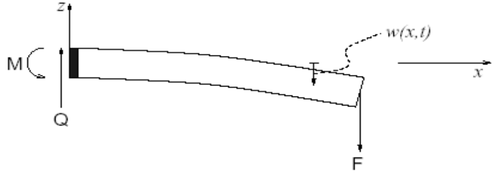
$$(\lambda + \mu)\nabla(\nabla \cdot \vec{u}) + \mu\nabla^2\vec{u} = 0$$

- Divide domain into elements
- Approximation of function (solution to partial differential equation) over domain
- Simple function over each element (linear, parabolic)
- Connect elements at nodes

Figure T1-20 Viewing the mechanical mesh



Example: beam with end load



- Assumption: constant rectangular cross section, width a and height b
- By integration or from tables: $I = ab^3/12$
- Loads: $q = 0$, end load D prescribed
- Governing equation:

$$EI\frac{d^4w}{dx^4} = 0$$

- Clamped left end $x = 0$: $w(0) = w'(0) = 0$
- Right end with load: $EIw'''(L) = F$, $s'' = 0$
- Integrate differential equation four times:

$$w(x) = C_1x^3 + C_2x^2 + C_3x + C_4$$

- Determine $C_1, \ldots, C_4$ from end conditions:

$$w(x) = \frac{FL^3}{6EI}\left(\frac{x}{L}\right)^2(3 - x/L)$$

$$w(x) = \frac{FL^3}{6EI}\left(\frac{x}{L}\right)^2(3 - x/L)$$

- Moment:

$$M(x) = F(L - x)$$

- Shear force (constant here):

$$Q(x) = F$$

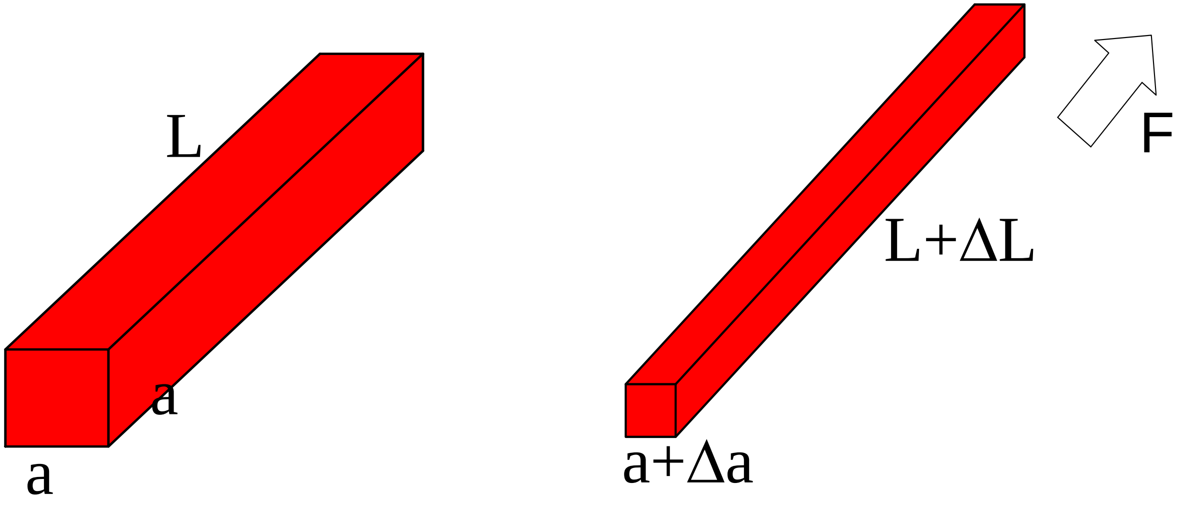
- Normal stress in a cross section:

$$\sigma_{xx} = z\frac{F}{I}(L - x)$$

- Largest stress at $x = 0$ and for $z = \pm b/2$

Resistor change in metal strain gauges

Mainly due to change of resistor FORM (geometric effect)



$R_0 = \rho\, L/a^2$

$R = R_0 + R_0\, \Delta L/L + 3R_0 \Delta a/a$

$\Delta a/a = -\nu\, \Delta L/L$

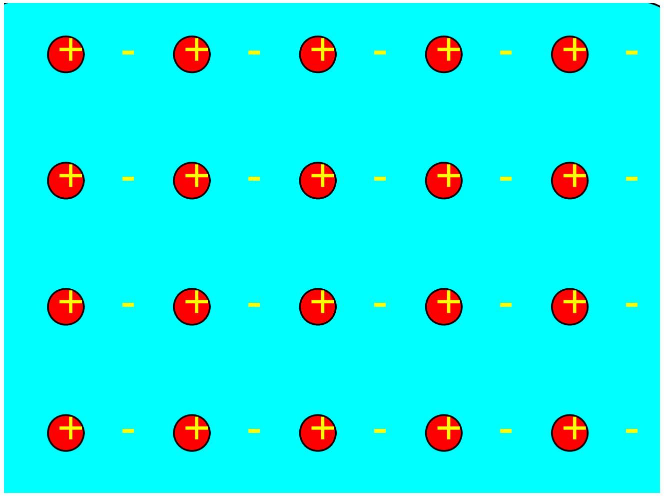
$\Delta R/R \approx 2\, \Delta L/L$ $\Delta R/R \approx 2\, \varepsilon$

$\nu = 0.3$

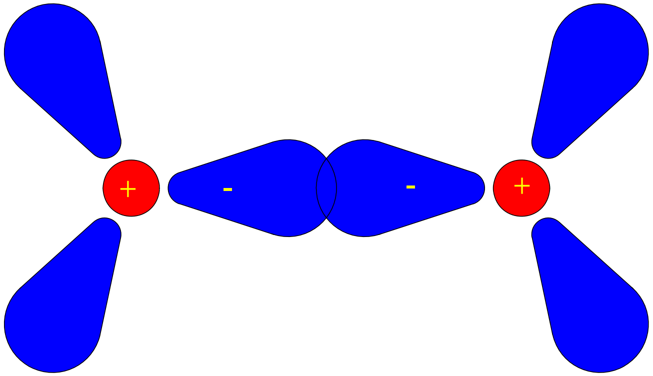
For silicon: large RESISTIVITY change with stress

(not mainly a geometric resistor-change factor)

Metal:



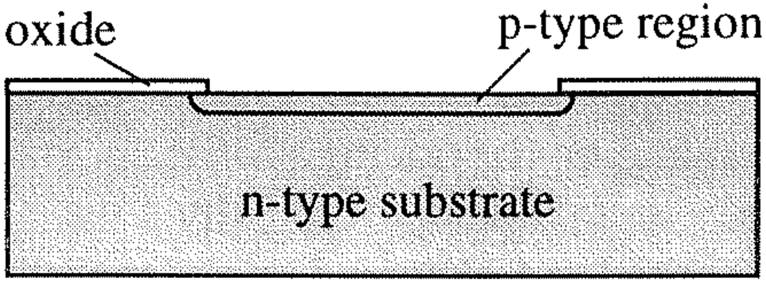
Silicon:



$\Delta R/R \approx 2\,\varepsilon$

$\Delta R/R \approx 90\,\varepsilon$

Electronics (Chapter 14.1 - 14.4) Doped resistors



Define a p-type circuit
in a n-type wafer

n-type wafer must be at positive potential relative to the p-type circuit

Reverse biased diode → no current between circuit and wafer/substrate

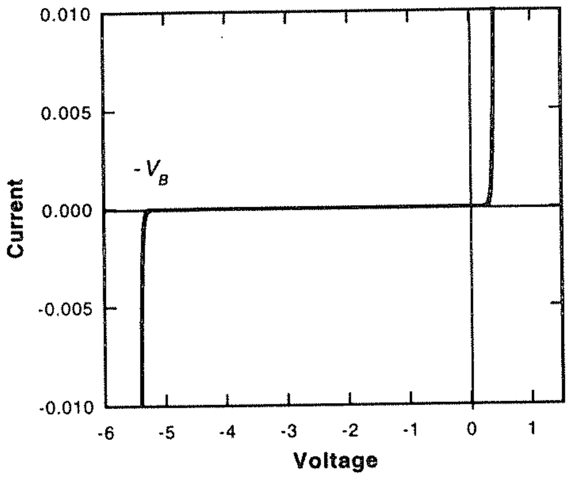
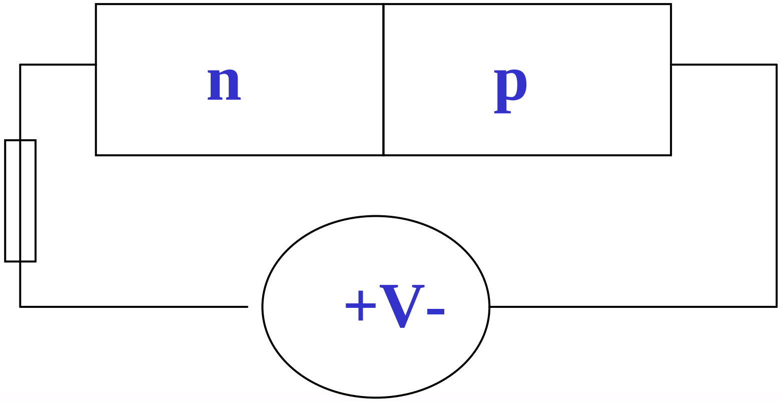


Figure 14.3. A typical diode current-voltage characteristic.



The resistivity changes with the mechanical stress

- E - electric field, three components
- j - current density, three components
- ρ_0 – homogeneous resistivity, unstressed silicon

- When mechanical stress is applied, the resistivity changes depending on the stress in different directions and the piezo coefficients

$$\begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} = \rho_0 \begin{bmatrix} j_1 \\ j_2 \\ j_3 \end{bmatrix} + \rho_0 \begin{bmatrix} d_1 & d_6 & d_5 \\ d_6 & d_2 & d_4 \\ d_5 & d_4 & d_3 \end{bmatrix} \begin{bmatrix} j_1 \\ j_2 \\ j_3 \end{bmatrix}$$

Silicon: Three independent piezoresistive coefficients

- Example of piezoresistive coefficients:

- doping: p-type
- sheet resistivity: 7.8 Ωcm

- value of Π_{11}= 6.6 10^{-11} Pa^{-1}
- value of Π_{12}=-1.1 10^{-11} Pa^{-1}
- value of Π_{44}= 138 10^{-11} Pa^{-1}

  - Equations 18.3, 18.4, 18.5 in Senturia

$$\begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} = \rho_0 \begin{bmatrix} j_1 \\ j_2 \\ j_3 \end{bmatrix} + \rho_0 \begin{bmatrix} d_1 & d_6 & d_5 \\ d_6 & d_2 & d_4 \\ d_5 & d_4 & d_3 \end{bmatrix} \begin{bmatrix} j_1 \\ j_2 \\ j_3 \end{bmatrix}$$

$$\begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \\ d_6 \end{bmatrix} = \begin{bmatrix} \Pi_{11} & \Pi_{12} & \Pi_{12} & 0 & 0 & 0 \\ \Pi_{12} & \Pi_{11} & \Pi_{12} & 0 & 0 & 0 \\ \Pi_{12} & \Pi_{12} & \Pi_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & \Pi_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & \Pi_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & \Pi_{44} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yx} \\ \tau_{zx} \\ \tau_{xy} \end{bmatrix}$$

Dependence of piezoresistivity on doping

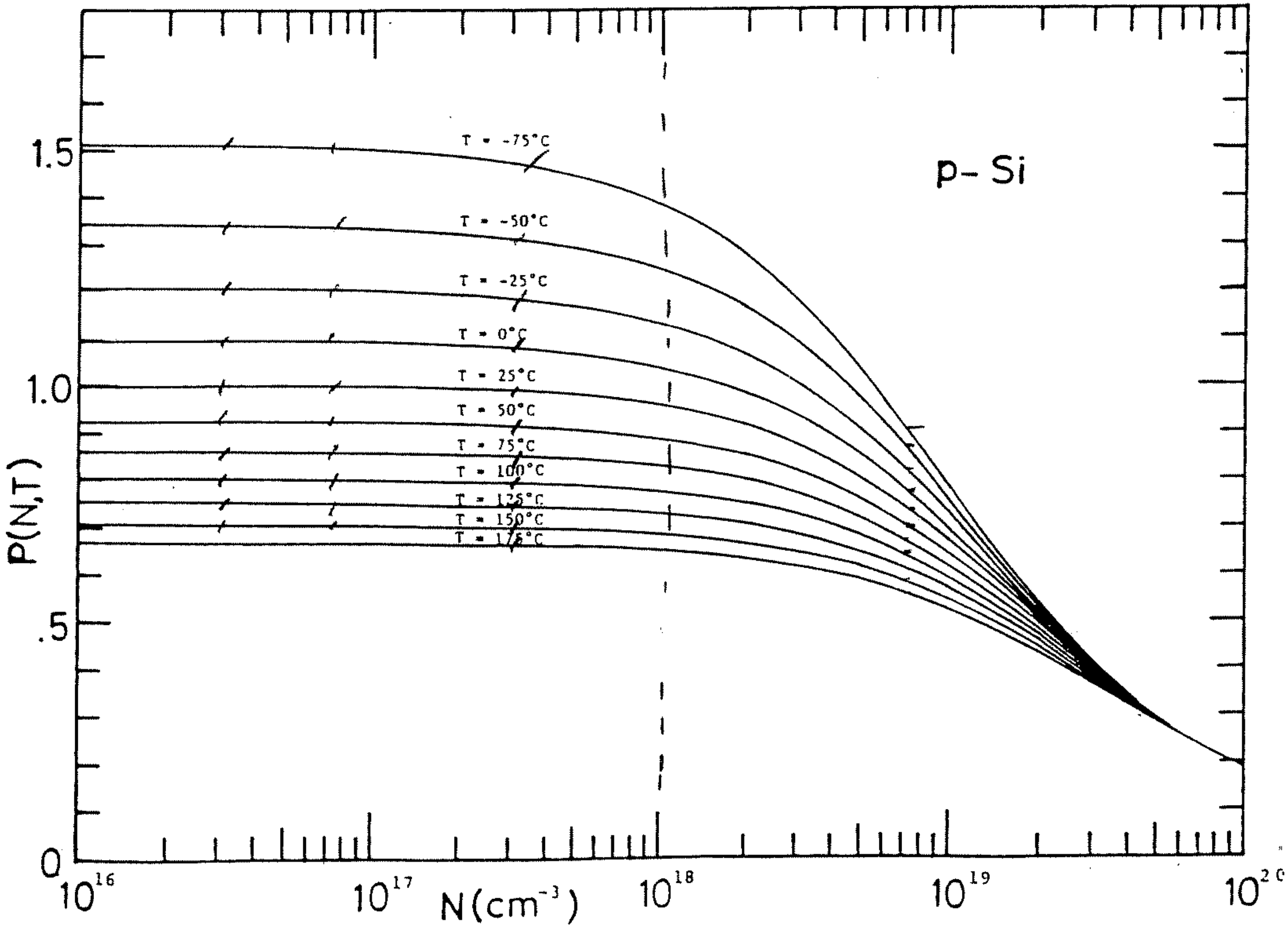
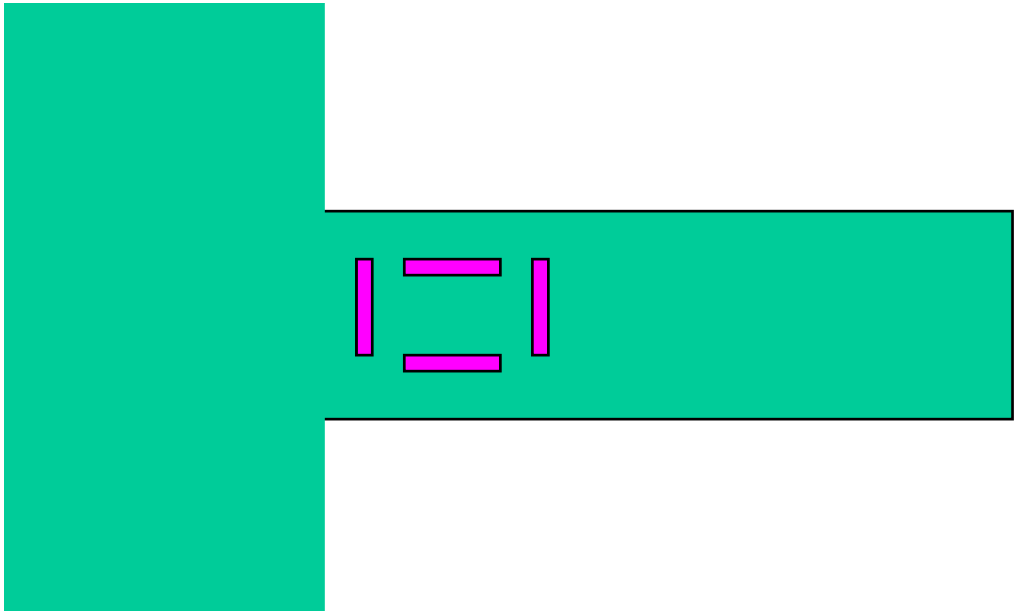


Fig. 9. Piezoresistance factor $P(N, T)$ as a function of impurity concentration and temperature for p-Si.

Long, narrow resistors

- Pre-calculated “piezocoefficients” that enables the designer to calculate the resistivity change for a long, narrow resistor
- Often given for a resistor that is placed in the <110> direction
- Transverse and longitudinal coefficients



$$\frac{\Delta R}{R} = \pi_l \sigma_l + \pi_t \sigma_t$$

Resistors in <110> direction

- Much used direction for piezoresistors, bulk micromachining
- Pre-calculated longitudinal and transverse piezo-coefficients

$$\pi_l = 1/2(\pi_{11} + \pi_{12} + \pi_{44})$$

$$\pi_t = 1/2(\pi_{11} + \pi_{12} - \pi_{44})$$

- σ positive: tensile stress
- σ negative: compressible stress
- π positive: increased resistivity with tensile stress
- π negative: decreased resistivity with tensile stress

$$\frac{\Delta R}{R} \approx \frac{\pi_{44}}{2}(\sigma_l - \sigma_t)$$

- p-type silicon: π_{44} dominates

Beam accelerometer

- Long resistors in <110> direction

$$\pi_{l,110} = \frac{1}{2}(\pi_{11} + \pi_{12} + \pi_{44})$$

$$\pi_{t,110} = \frac{1}{2}(\pi_{11} + \pi_{12} - \pi_{44})$$

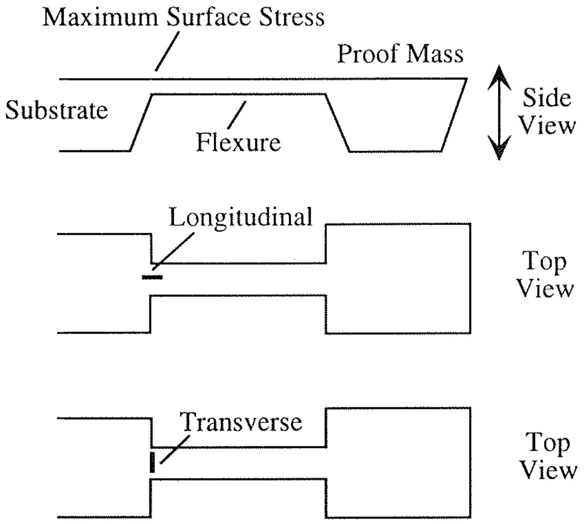
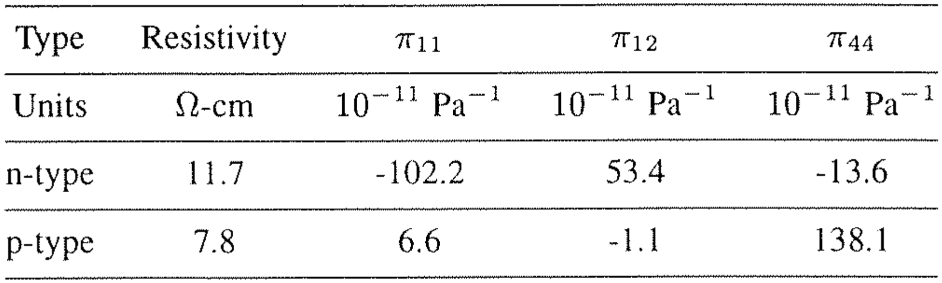


Figure 18.1. Illustrating lateral and transverse piezoresistor placements using an accelerometer flexure as an example.

- Piezoresistance coefficients

Table 18.1. Typical room-temperature piezoresistance coefficients for n- and p-type silicon [98].



Type	Resistivity	π_{11}	π_{12}	π_{44}
Units	Ω-cm	10^{-11} Pa^{-1}	10^{-11} Pa^{-1}	10^{-11} Pa^{-1}
n-type	11.7	-102.2	53.4	-13.6
p-type	7.8	6.6	-1.1	138.1

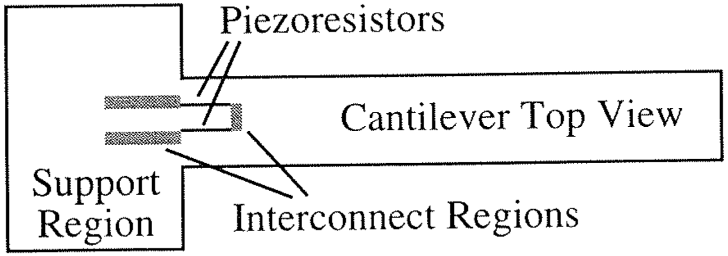
Cantilever with piezoresistors

- length 200 μm
- width 20 μm
- thickness 5 μm

- point load at free end

- p-type piezoresistor
- length 20 μm
- width 2 μm
- depth 0.2 μm

- $\Delta R/R = \pi_l \, \sigma_l = 0.02$



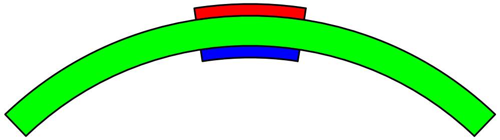
8.4. An example using piezoresistance to measure the deflection of a cantilever.

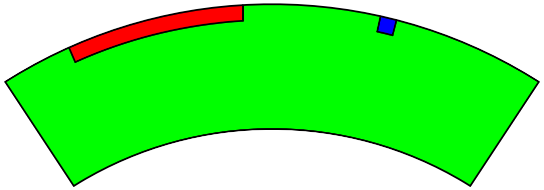
$$w = \frac{3}{2} w_{max} \left(\frac{x}{L_c}\right)^2 \left(1 - \frac{x}{3L_c}\right)$$

$$\frac{1}{\rho} = \left|\frac{d^2 w}{dx^2}\right| = \frac{3 w_{max}(L_c - x)}{L_c^3}$$

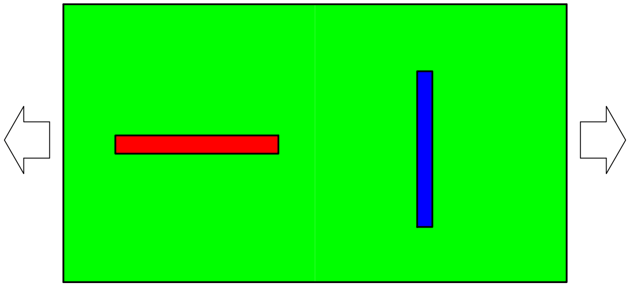
$$\sigma_l = \frac{EH}{2\rho} = \frac{3 E w_{max}(L_c - x)}{2 L_c^3}$$

Placement of piezoresistors on diaphragm

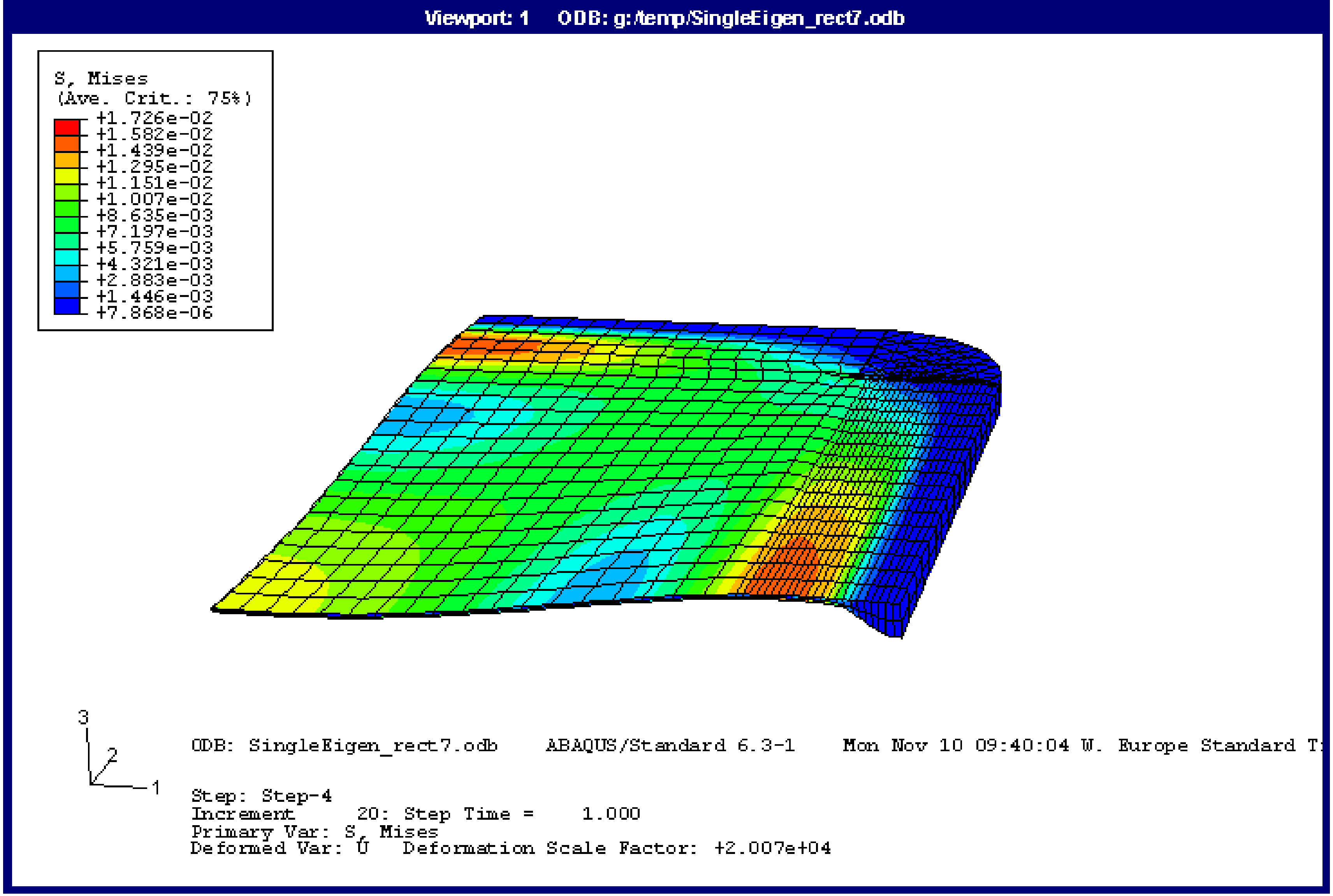




Over/under konfigurasjon
er ikke praktisk i silisium.



Men vi kan snu retningen
på motstandene



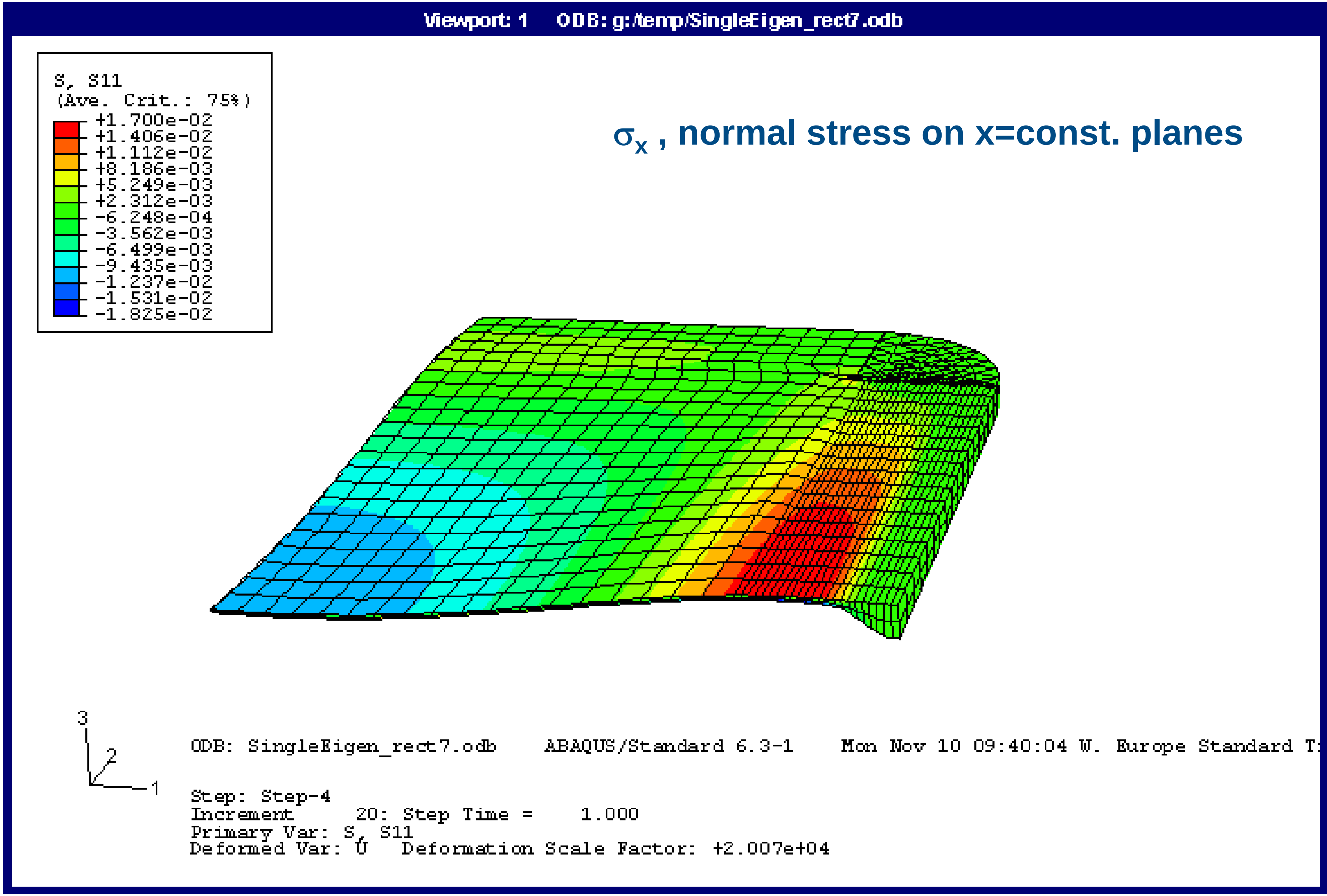
```
ODB: SingleEigen_rect7.odb    ABAQUS/Standard 6.3-1    Mon Nov 10 09:40:04 W. Europe Standard T

Step: Step-4
Increment     20: Step Time =    1.000
Primary Var: S, Mises
Deformed Var: U   Deformation Scale Factor: +2.007e+04
```

Stress level at point in structure: Mises yield criterion

$$\sigma_{mieses} = \sqrt{1/2\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]}$$

σ_x , normal stress on x=const. planes



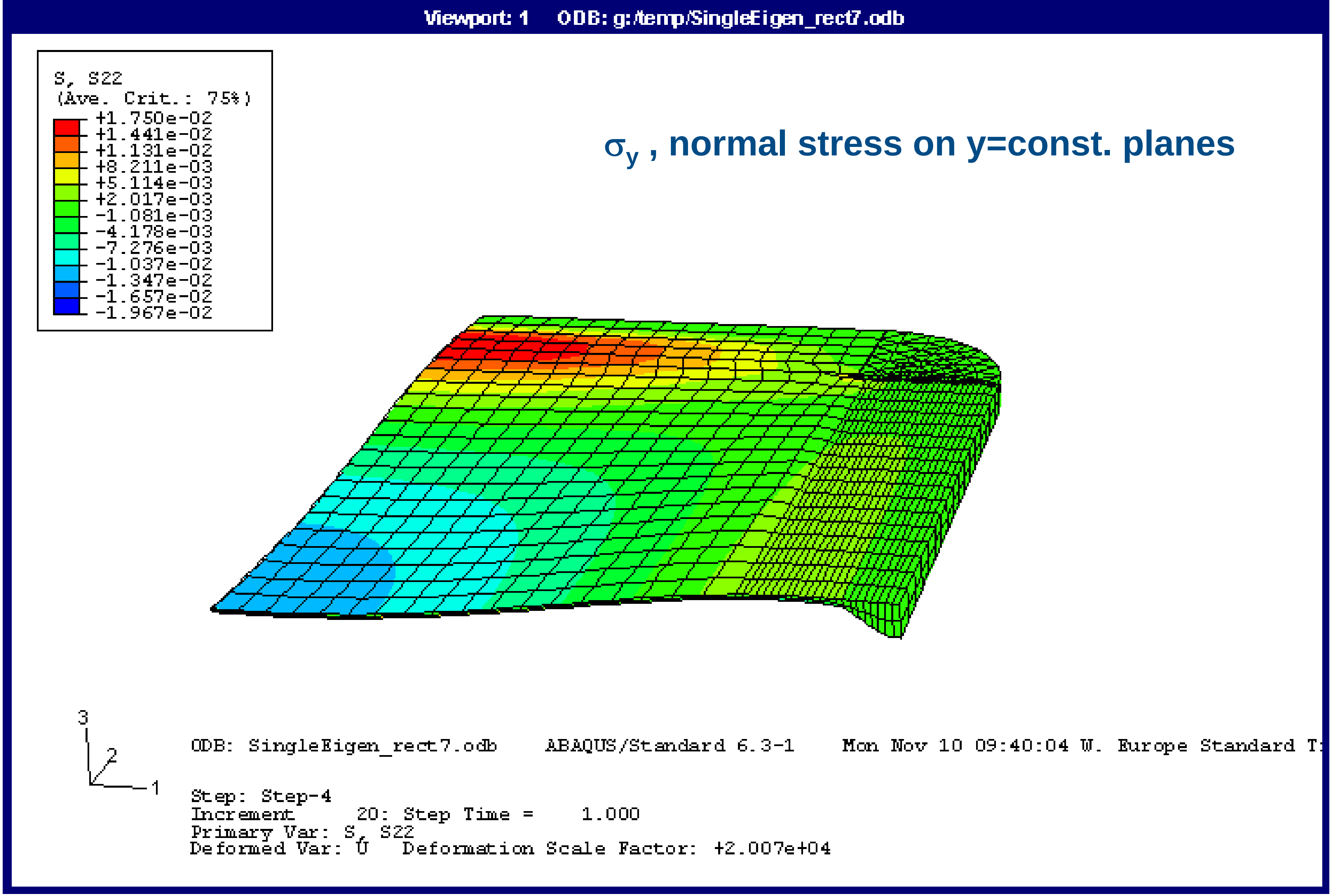
Viewport: 1 ODB: g:/temp/SingleEigen_rect7.odb

S, S11
(Ave. Crit.: 75%)
+1.700e-02
+1.406e-02
+1.112e-02
+8.186e-03
+5.249e-03
+2.312e-03
-6.248e-04
-3.562e-03
-6.499e-03
-9.435e-03
-1.237e-02
-1.531e-02
-1.825e-02

```
ODB: SingleEigen_rect7.odb    ABAQUS/Standard 6.3-1    Mon Nov 10 09:40:04 W. Europe Standard T

Step: Step-4
Increment     20: Step Time =    1.000
Primary Var: S, S11
Deformed Var: U   Deformation Scale Factor: +2.007e+04
```

σ_y , normal stress on y=const. planes



```
ODB: SingleEigen_rect7.odb    ABAQUS/Standard 6.3-1    Mon Nov 10 09:40:04 W. Europe Standard T

Step: Step-4
Increment     20: Step Time =    1.000
Primary Var: S, S22
Deformed Var: U   Deformation Scale Factor: +2.007e+04
```

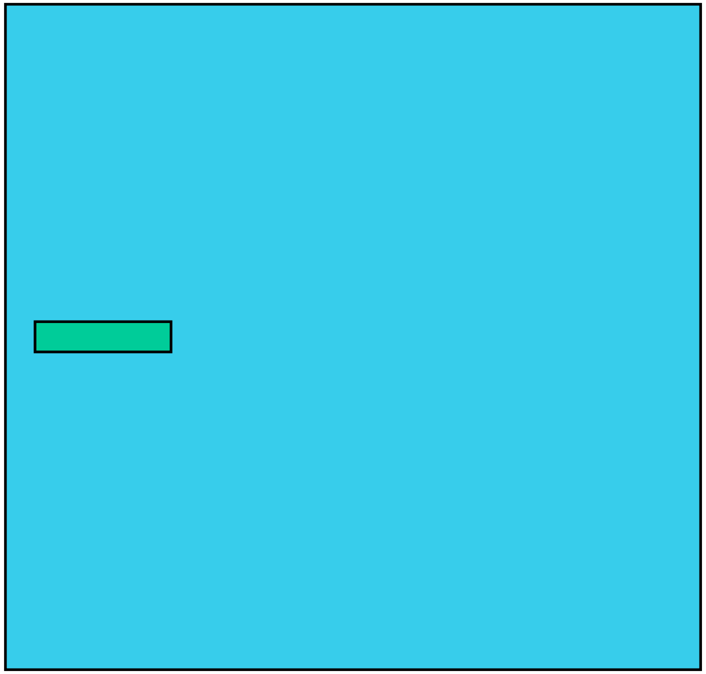
Piezoresistor placed normal to diaphragm edge

- Apply pressure from above
- Diaphragm bends down
- Piezoresistor is stretched longitudinally
- σ_l is positive, tensile stress
- Rough argument for mechanical stress in transversal direction: stress must avoid contraction: $\sigma_t = \sigma_l \nu$
- Transverse stress is tensile/positive
- Change in resistance:

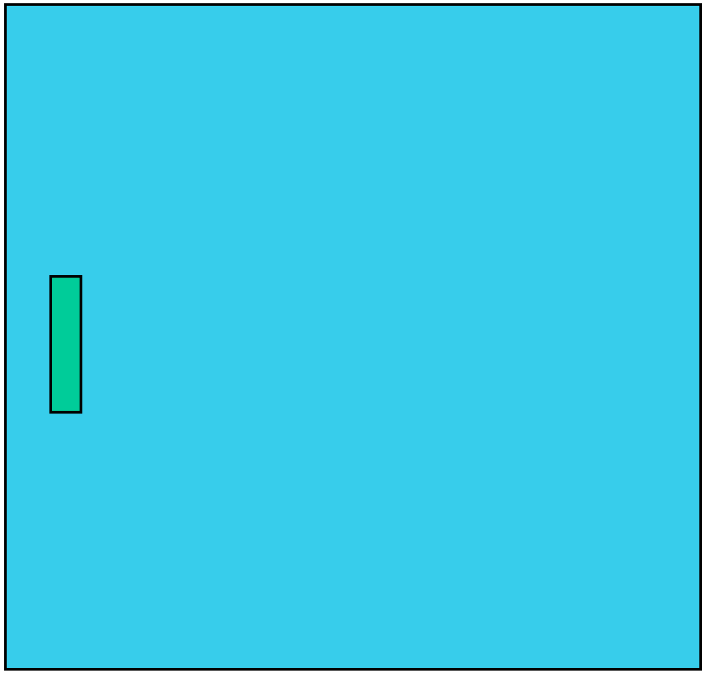
$$\frac{\Delta R}{R} = \pi_l \sigma_l + \pi_t \sigma_t$$

$$\Delta R_1 / R_1 = (\pi_l + \nu \pi_t) \sigma_l$$

- (π_t is negative)
- Resistance increases



Piezoresistor placed parallel to diaphragm edge



- Apply pressure from above
- Diaphragm bends down
- Piezoresistor is stretched transversally
- σ_t is positive
- Rough argument for mechanical stress in longitudinal direction: stress must avoid contraction: $\sigma_l = \sigma_t \nu$
- Tensile, positive stress in longitudinal dir.
- Change in resistance:

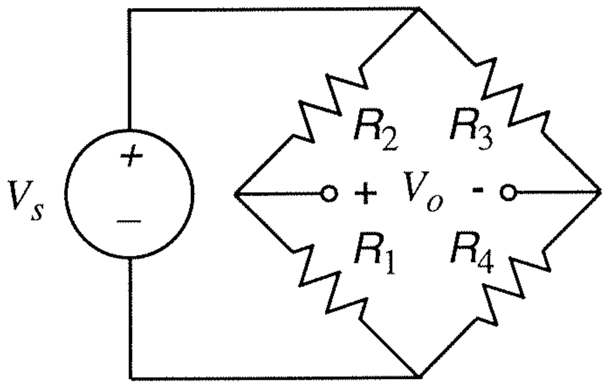
$$\Delta R_2 / R_2 = (\pi_t + \nu \pi_l)\sigma_t$$

- (π_t is negative)
- Resistance decreases

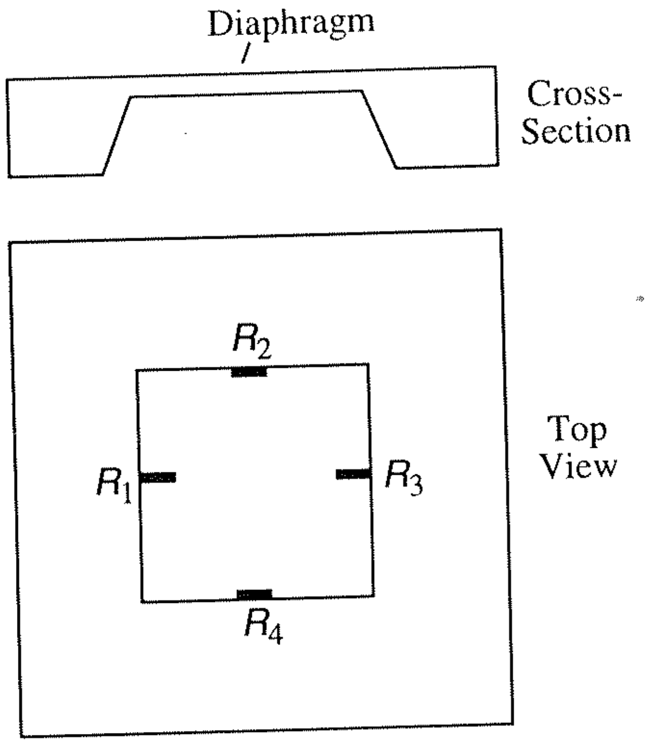
Membrane pressure sensor

$$\frac{\Delta R_1}{R_1} = \left(67.6 \times 10^{-11}\right) \sigma_l$$

$$\frac{\Delta R_2}{R_2} = -\left(61.7 \times 10^{-11}\right) \sigma_l$$



Wheatstone-bridge circuit constructed from the resistors in Fig. 18.2.



Wheatstone bridge circuit

- <100> direction

$\pi_l = 71.8$ σ positive, tensile stress

$\pi_t = -66.3$ σ negative, compressible stress

$$V_0 = V_s\left(\frac{R_2}{R_2+R_1} - \frac{R_3}{R_3+R_4}\right)$$

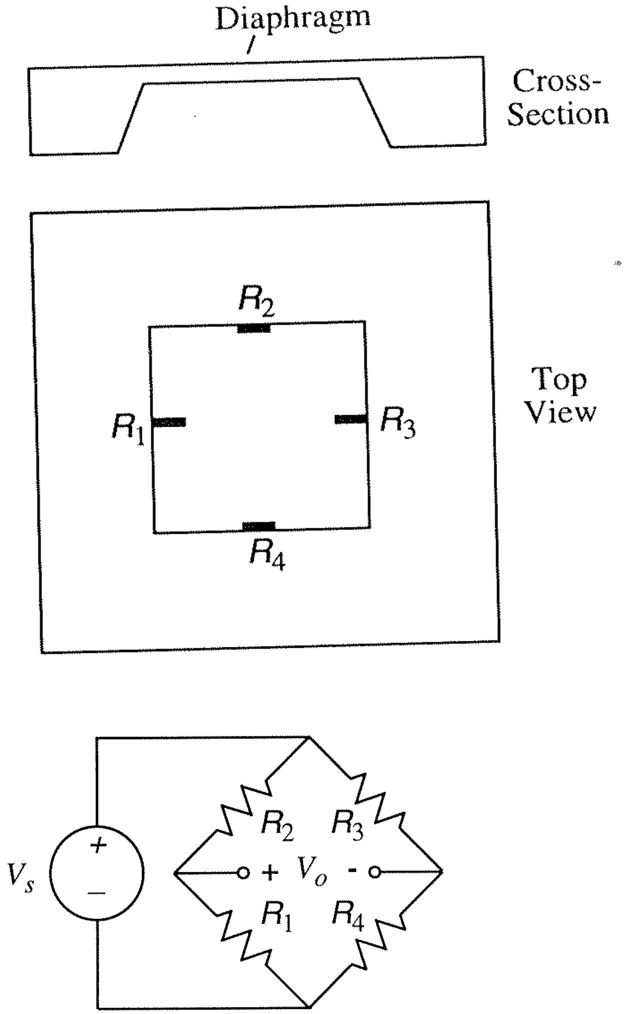
$$R_1 = R_1 - \Delta R$$

$$R_2 = R_2 + \Delta R$$

$$R_3 = R_3 - \Delta R$$

$$R_4 = R_4 + \Delta R$$

$$V_0 = V_s\left(\frac{\Delta R}{R}\right)$$



Wheatstone-bridge circuit constructed from the resistors in Fig. 18.2.

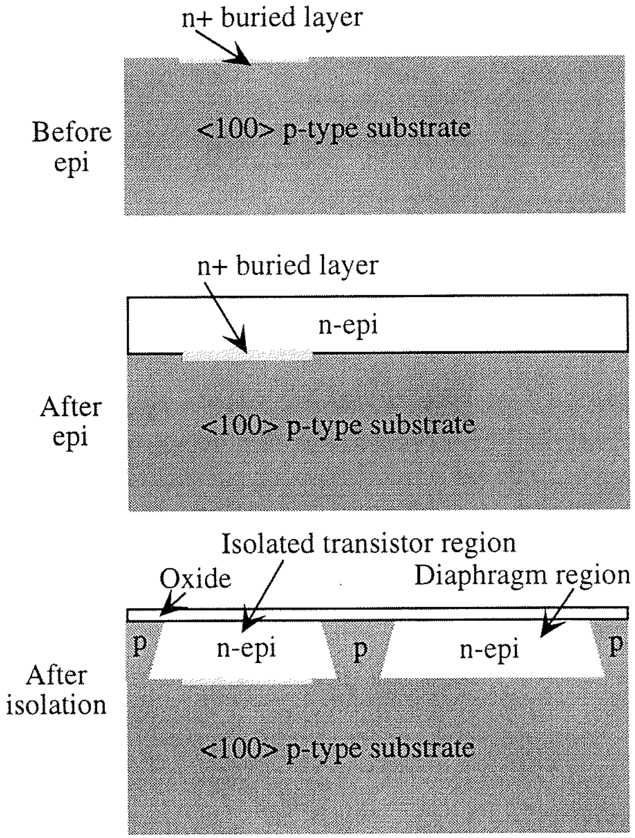
Averaging over stress and doping variations

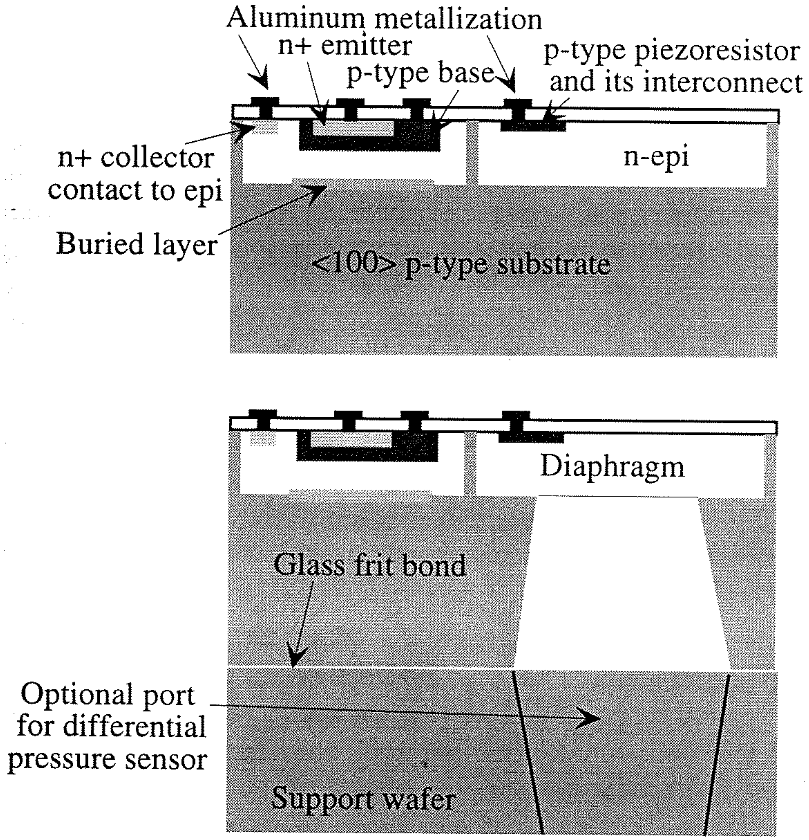
- Senturia 18.2.5

$$R = R_0\left[1 + R_0\int_0^{z_j}\frac{W}{L\rho_{e,o}(z)}\pi_l\sigma_l(z)dz\right]$$

$$\begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} = \rho_0\begin{bmatrix} j_1 \\ j_2 \\ j_3 \end{bmatrix} + \rho_0\begin{bmatrix} d_1 & d_6 & d_5 \\ d_6 & d_2 & d_4 \\ d_5 & d_4 & d_3 \end{bmatrix}\begin{bmatrix} j_1 \\ j_2 \\ j_3 \end{bmatrix}$$

Motorola MAP sensor



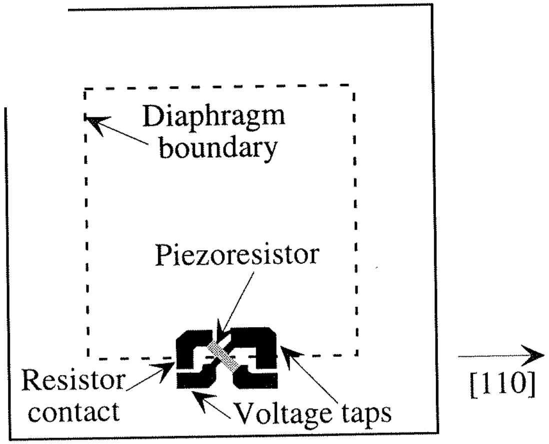


Potential across piezoresistor

Electric field in transverse direction

$$\begin{aligned}\mathcal{E}_1 &= \rho_e\left(1+\pi_{11}\sigma_1+\pi_{12}\sigma_2\right)J_1 && (18.31)\\ \mathcal{E}_2 &= \rho_e\pi_{44}\tau_{12}J_1 && (18.32)\\ \mathcal{E}_3 &= 0 && (18.33)\end{aligned}$$

$$\begin{bmatrix}E_1\\E_2\\E_3\end{bmatrix}=\rho_0\begin{bmatrix}j_1\\j_2\\j_3\end{bmatrix}+\rho_0\begin{bmatrix}d_1 & d_6 & d_5\\ d_6 & d_2 & d_4\\ d_5 & d_4 & d_3\end{bmatrix}\begin{bmatrix}j_1\\j_2\\j_3\end{bmatrix}$$



Illustrating the position and orientation of the Motorola Xducer piezoresistor. resistor contacts which provide the current in the piezoresistor and two voltage asure a transverse voltage set up by by the stresses in the resistor.

Coventor tutorials-choose one

- In files mems_analy.pdf and mems_supp.pdf (2004 version)
- In files MEMS Design and Analysis Tutorials vol1 +2 (2005 version)

- Mirror design (electrostatic forces)
- Beam simulation analysis (pull-in)
- Modal and harmonic analysis (resonant frequencies, modal shapes)
- Temperature analysis (temperature distribution, different coefficients)

- MemHenry (magnetic sensor)
- MemPackage (thermal and mechanical effects)
- MemPZR (piezoresistors)
- MemETherm (Joule heating, thermal expansion)
- AutoSpring (Extract spring constants)
- MemDamping (Squeeze film, slide damping)
- MemTrans (Transient analysis)