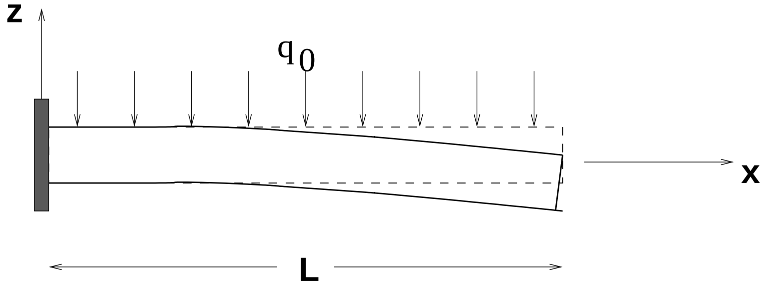
Exercise: Silicon Beam Accelerometer



For a clamped cantilever silicon beam the deflection function can be calculated by integrating the general differential equation for a beam and applying appropriate boundary conditions:

$$w(x) = \frac{q_0 x^2}{24EI}(x^2 + 6L^2 - 4Lx)$$

Here, q_0 is a uniformly distributed load (per unit length along the beam), E is Young's modulus, I is the moment of inertia, and L is the length of the beam. Note that the beam is clamped at $x = 0$, with a free end at $x = L$. The cross section of the beam has rectangular shape,

$$-b/2 \leq y \leq b/2, \quad -h/2 \leq z \leq h/2$$

where the z axis points in the opposite direction of the force q_0. I is then $bh^3/12$. An important stress measure is the normal stress on a surface perpendicular to the axis of the beam:

$$\sigma_{xx} = zEw''(x)$$

The beam is a part of a system that is subjected to an acceleration a.

Suitable data for the problem are $a = 50g$, $g = 9.81$ m/s^2, $L = 3$ mm, $b = 50$ μm, $h = 10$ μm, $E = 168$ GPa. The density can be taken as $\varrho = 2300$ kg/m^3.

(a) Draw the cross section in the y, z plane.

(b) What is the force q_0 acting on the beam per unit length when the external acceleration is a? (Hint: The acceleration can be treated as a fictitious force in an accelerated coordinate system in which the deflected beam is at rest.)

(c) Plot the deflection $w(x)$ along the beam ($0 \leq x \leq L$).

(d) Where is the maximum deflection, and how large is this deflection?

(e) Draw σ_{xx} as function of z for $a = 50g$ and $x = 0$.

(f) What is the physical interpretation of the quantity σ_{xx}? Plot σ_{xx} as a function of x for the z value which gives the largest stress in an arbitrary cross section. Find where σ_{xx} reaches its maximum value.

(g) Plot the maximum deflection as a function of a when $0 \leq a \leq 50$ and $x = L$.