Recent Advances in Permafrost Modelling

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ABSTRACT

This paper provides a review of permafrost modelling advances, primarily since the 2003 permafrost conference in Zürich, Switzerland, with an emphasis on spatial permafrost models, in both arctic and high mountain environments. Models are categorised according to temporal, thermal and spatial criteria, and their approach to defining the relationship between climate, site surface conditions and permafrost status. The most significant recent advances include the expanding application of permafrost thermal models within spatial models, application of transient numerical thermal models within spatial models and incorporation of permafrost directly within global circulation model (GCM) land surface schemes. Future challenges for permafrost modelling will include establishing the appropriate level of integration required for accurate simulation of permafrost-climate interaction within GCMs, the integration of environmental change such as treeline migration into permafrost response to climate change projections, and parameterising the effects of sub-grid scale variability in surface processes and properties on small-scale (large area) spatial models. Copyright © 2008 John Wiley & Sons, Ltd.

KEY WORDS: permafrost; models; geothermal; mountains; spatial models

INTRODUCTION

Permafrost (defined as ground where temperatures have remained at or below 0°C for a period of least two consecutive years) is a key component of the cryosphere through its influence on energy exchanges, hydrological processes, natural hazards and carbon budgets — and hence the global climate system. The climate-permafrost relation has acquired added importance with the increasing awareness and concern that rising temperatures, widely expected throughout the next century, may particularly affect permafrost environments. The Intergovernmental Panel on Climate Change (1990) has advocated that research should be directed towards addressing the climate-permafrost relation, including the effects of temperature forcing from climatic variation, local environmental factors such as snow and vegetation, and surficial sediments or bedrock types. Permafrost has been identified as one of six cryospheric indicators of global climate change within the international framework of the World Meteorological Organization (WMO) Global Climate Observing System (Brown et al., 2008).

This review gives an overview of permafrost modelling advances, primarily since the 2003 permafrost conference in Zurich, Switzerland, with an emphasis on spatial permafrost models, in both arctic and high mountain environments. Many models have been developed to predict the spatial variation of permafrost thermal response to changing climate

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conditions at different scales, including conceptual, empirical and process-based models. The permafrost models considered in this paper are those that define the thermal condition of the ground, based on some combination of climatic conditions and the properties of earth surface and subsurface properties. Particular emphasis is given to models used in spatial applications.

MODELLING BACKGROUND

A model is a conceptual or mathematical representation of a phenomenon, usually conceptualised as a system. It provides an idealised framework for logical reasoning, mathematical or computational evaluation as well as hypothesis testing. Explicit assumptions about or simplifications of the system may be part of model development; how these decisions affect the utility of the model will depend on whether the model serves its intended purpose with satisfactory accuracy. Rykiel (1996) suggests that model ‘validation’ can be separated into evaluations of theory, implementation and data; while some statistical tests can be performed to measure differences between model behaviour and the behaviour of the system, ‘validation’ ultimately depends on whether the model and its behaviour are reasonable in the judgement of knowledgeable people.

The value of a model depends on its usefulness for a given purpose and not its sophistication. Simple models can be more useful than models which incorporate many processes, especially when data are limited. Choosing the appropriate point along the continuum of model complexity depends on data availability, modelling (usually computer) resources and the questions under investigation. The data needed to obtain credible results usually increase with increasing model complexity and with the number of processes that are represented. In mountainous environments, lateral variations of surface and subsurface conditions as well as micro-climatology are far greater than in lowland environments.

Process-based permafrost models determine the thermal state of the ground based on principles of heat transfer, and can be categorised using temporal, thermal and spatial criteria. Temporally, models may define equilibrium permafrost conditions for a given annual regime (equilibrium models), or they may capture the transient evolution of permafrost conditions from some initial state to a modelled current or future state (transient models).

Thermally, simple models may define the presence or absence of permafrost, active-layer depth, or mean annual ground temperature, based on empirical and statistical relations or application of so-called equilibrium models utilising transfer functions between atmosphere and ground. Numerical models (finite-element or finite-difference models) may define the annual and longer term progression of a deep-ground temperature profile (transient models). The influence of surface energy exchange on subsurface conditions may be defined by simplified equations using a few parameters (such as freezing/thawing indices and n factors, or air temperature and snow cover in equilibrium models), or as a fully explicit energy balance, requiring data for atmospheric conditions.

Spatially, models may define conditions at a single location (either as an index or a one-dimensional vertical temperature profile), along a two-dimensional transect, or over a geographic region. Most geographic/spatial permafrost models today are collections of modelled point locations, with no lateral heat flow between adjacent points. Points in such spatial models behave as one-dimensional models, so that small-scale spatial variability in soil properties and snow cover is not considered. This is a central problem for mountain areas where the effects of slope, aspect and elevation must be considered at regional or local scales. Thus, in mountainous regions the scale of variation in these factors requires alternative approaches to spatial modelling.

Heat Flow Theory

Basic heat flow theory is described in some detail in textbooks, such as Carislaw and Jaeger (1959), Williams and Smith (1989) and Lunardini (1981). The equation for heat flow under transient conditions forms the basis of all geothermal models.

$$C \frac{dT}{dt} = k \frac{\partial^2 T}{\partial z^2}$$  \hspace{1cm} (1)

Definitions of equation symbols are given in Table 1.

Two exact analytical models derived for a semi-infinite and isotropic half-space using equation 1 are: the harmonic solution, describing ground temperatures at any time \(t\) and depth \(z\) below a ground surface experiencing sinusoidal temperature variations:

$$T_{z,t} = T + A_x \cdot e^{-z/\sqrt{\pi/\alpha P}} \cdot \sin \left( \frac{2\pi t}{P} \right)$$  \hspace{1cm} (2)

and the step change solution, describing changes to ground temperatures at any time \(t\) and depth \(z\) below a ground surface following a step change in ground
For ground that undergoes freezing and thawing, the release and absorption of the latent heat of fusion of the soil water dominate heat flow, although the temperature-dependence of thermal conductivity is also important. Accounting for latent heat is usually achieved by subsuming its effect in the heat capacity term in equation 1.

\[
C_a = \sum x_i \rho_i c_i + L \left( \frac{\partial \theta_a}{\partial T} \right) \tag{4}
\]

The Stefan Model

The analytical equation most widely employed in the formulation of permafrost models is the Stefan solution to the moving freezing (or thaw) front. When diffusive effects are small relative to the rate of frost front motion and the initial temperature of the ground is close to 0 °C, the exact equation for the moving phase change boundary can be simplified to a form of the Stefan solution using accumulated ground surface degree-day total \( I \) (either the freezing index \( I_F \) or thawing index \( I_T \)) (Lunardini, 1981):

\[
X = \sqrt{\frac{2 \lambda I}{L}} \tag{5}
\]

The form of Stefan solution represented by equation (5) is widely used for spatial active-layer characterisation by estimating soil properties (‘edaphic parameters’) empirically, using summer air temperature records and active-layer data obtained from representative locations (e.g. Nelson et al., 1997; Shiklomanov and Nelson, 2003; Zhang T. et al., 2005).

Carlson (1952, based on Sumgin et al., 1940) used equation 5 as the origin of a simple model for presence of permafrost, based on the notion that permafrost will

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Subscripts for \( \alpha, \lambda, \rho, c, C \) and \( n \):
- \( a \) = apparent
- \( i \) = subscript identifying component (mineral, ice, water, etc.)
- \( T \) = thawed or thawing
- \( F \) = frozen or freezing

Surface temperature:

\[
\Delta T_{z,t} = \Delta T_S \cdot \text{erfc} \left( \frac{z}{2 \sqrt{\alpha t}} \right) \tag{3}
\]

(\text{erfc} is the complementary error function)

Permafrost models are a subset of a more general class of geothermal models. In permafrost models, ground freezing and thawing are central in determining the important variables and parameters of which the model is comprised. Equations 2 and 3 form the basis of analyses of ground temperatures outside of permafrost regions, and are most useful where freezing and thawing of significant amounts of soil moisture do not occur. Exact analytical models of the thermal behaviour of the ground when freezing or thawing occurs are limited to a few idealised conditions (Lunardini, 1981). Real-world conditions such as seasonal variation in the ground surface temperature, accumulation and ablation of snow cover, and temperature dependent thermal properties, are beyond the capacity of these exact models. Two paths move beyond this impasse: approximate analytical models developed by making simplifying assumptions, or numerical techniques employed to solve more complex problems with the acceptance of limited error.
be present where predicted winter season freezing exceeds predicted summer season thaw, so that permafrost exists where $k_F I_{FS} > k_T I_{TS}$. Nelson and Outcalt (1987) derived the Frost Index model on this relationship. While the Frost Index model has been used extensively (e.g. Anisimov and Nelson, 1996), the Kudryavstev model has become more common in recent studies.

The Kudryavstev Model

An alternative solution to the Stefan problem was proposed by Kudryavstev et al. (1974) (Figure 1A) for estimating maximum annual depth of thaw propagation and the mean annual temperature at the base of the active layer $T_z$ (equivalent to the temperature at the top of permafrost, or $T_{TOP}$, described below):

$$Z_{thaw} = \frac{2(A_s - T_z) \cdot \sqrt{\frac{\lambda_T \cdot P_{sn} \cdot C_T}{\pi}} + \frac{(2A_s \cdot C_T \cdot Z_s + L \cdot Z_c) \cdot L \sqrt{\frac{P_{sn}}{\pi C_T}}}{2A_s \cdot C_T + L + (2A_s \cdot C_T + L) \cdot \sqrt{\frac{P_{sn}}{\pi C_T}}}}{2A_s \cdot C_T + L}$$

where

$$A_s = \frac{A_s - T_z}{\ln \left(\frac{A_s + L/2C_T}{T_z + L/2C_T}\right) - L} \frac{2C_T}{2C_T}$$

and

$$Z_s = \frac{2(A_s - T_z) \cdot \sqrt{\frac{\lambda_p \cdot P_{sn} \cdot C_T}{\pi}}}{2A_s \cdot C_T + L}$$

The mean annual temperature at the depth of seasonal thaw (permafrost surface) can be calculated as:

$$T_z = \frac{0.5 T_s \cdot (\lambda_F + \lambda_T) + A_s \cdot \frac{\lambda_F - \lambda_T}{\pi} \left[\frac{T_s}{A_s} \arcsin \frac{T_s}{A_s} + \sqrt{1 - \frac{T_s^2}{A_s^2}}\right]}{\lambda^*}$$

(7)

$$\lambda^* = \left\{ \begin{array}{ll}
\lambda_F, & \text{if numerator < 0} \\
\lambda_T, & \text{if numerator > 0}
\end{array} \right.$$

Kudryavstev’s equations were derived assuming a periodic steady state with phase change ($L > 0$) (Kudryavstev et al., 1974). Romanovsky and Osterkamp (1997) indicate that equations 6 and 7 can be applied on an annual basis. Extensive validation of Kudryavstev’s equations using empirical data from the North Slope of Alaska indicate that they provide more accurate estimates of annual maximum seasonal thaw depth than the Stefan equation (Romanovsky and Osterkamp, 1997; Shiklomanov and Nelson, 1999). Kudryavstev’s model is used extensively at regional (Shiklomanov and Nelson, 1999; Sazonova and Romanovsky, 2003; Stendel et al., 2007) and circum-arctic (Anisimov et al., 1997) scales.

N Factors

Freezing and thawing $n$ factors relate ground surface temperature to air temperature as an empirical alternative to the energy balance (Lunardini, 1978). $N$ factors are applied to seasonal degree-day totals (FDD or $I_F$ for freezing; TDD or $I_T$ for thawing), calculated as the accumulated departure of mean daily temperature above (or below) 0°C, so that equation 5 can be applied using air temperature data.

$$n_F = \frac{I_{FS}}{I_{FA}}; \quad n_T = \frac{I_{TS}}{I_{TA}}$$

(8)

$N$ factors account in a lumped form for the complex processes within the atmosphere-soil system and, as such, $n$ factors will vary for a given location. Shur and Slavin-Borovskiy (1993) found that site-specific $n$ factors are stable, with interannual changes less than 10 per cent in continental arctic areas. In mountainous regions, however, $n$ factor variations can be much higher, especially during winter due to interannual variation of snow cover (e.g. Juliussen and Humlum, 2007). This variability might be even more accentuated in maritime mountainous areas (e.g. Etzel-müller et al., 2007, 2008). In low-topography, continental areas $n$ factors can be correlated with surface cover and extrapolated over larger areas (e.g. Duchesne et al., 2008).

The TTOP Model

The TTOP model (Smith and Riseborough, 1996) (Figure 1B) estimates the mean annual temperature at the top of perennial frozen/unfrozen soil by combining a model (Romanovsky and Osterkamp, 1995) of the thermal offset effect (in which the mean annual
temperature shifts to lower values in the active layer because of the difference between frozen and thawed thermal conductivities of the soil) with freezing and thawing indices in the lower atmosphere linked to values at the ground surface using n factors (equation 8):

\[
T_{\text{TOP}} = \frac{n_F \lambda_F I_{FA} - n_F \lambda_F I_{FA}}{k_F P}, \quad T_{\text{TOP}} < 0
\]

\[
T_{\text{TOP}} = \frac{n_F \lambda_F I_{FA}}{\lambda_F P}, \quad T_{\text{TOP}} > 0
\]

Applications of the TTOP model have depended on empirically derived n factors (Wright et al., 2003; Juliussen and Humlum, 2007), although Riseborough (2004) attempted to develop a physically based n-factor parameterisation of the effect of snow cover.

**Statistical-Empirical Models and Multiple-Criteria Analysis**

Statistical-empirical permafrost models relate permafrost occurrences to topoclimatic factors (such as altitude, slope and aspect, mean air temperature, or solar radiation), which can easily be measured or computed. This type of model is widely used in mountain permafrost studies (see also Hoelzle et al., 2001), assumes equilibrium conditions and usually relies on basal temperature of the snow cover (BTS), temperatures measured with miniature data loggers, geophysical investigations, or the existence of specific landforms (rock glaciers) as evidence of permafrost occurrence. In the early 1990s, the availability of Geographical Information Systems (GIS) allowed the first estimation and visualisation of the spatial distribution of permafrost in steep mountains. PERMAKART (Keller, 1992) uses an empirical topographic key for permafrost distribution and in the PERMAMAP approach of Hoelzle and Haeberli (1995), BTS measurements, which are associated with the presence or absence of permafrost, are statistically related to mean annual air temperature (MAAT, estimated using nearby climate station records) and potential direct solar radiation during the snow-free season, calculated using digital elevation models (DEMs) (Funk and Hoelzle, 1992). The challenges with these approaches include the often indirect nature of the driving information on permafrost, the strong interannual variability of BTS, and the need to recalibrate for different environments. Statistical-empirical models usually estimate mean ground temperatures or provide measures of permafrost probability.

For small-scale mapping of large mountain areas, MAAT is often used as the sole predictor of permafrost occurrence, using thresholds based on field measurements. This approach involves the spatial interpolation of air temperature patterns from a common reference elevation (usually sea level) and the subsequent calculation of the near-surface MAAT using a DEM and a (sometimes spatially variable) lapse rate. Studies in southern Norway and Iceland have shown that a MAAT of 

\[ -3 \text{ to } -4^\circ \text{C} \]

is a good estimate for the regional limit of the lower mountain permafrost boundary (Etzelmüller et al., 2003, 2007). These threshold values are normally estimated based on ground surface temperature data from several locations, or other proxy information such as landforms. The resulting mountain permafrost distributions are then compared with more local-scale
observations or maps, confirming the overall spatial pattern for Scandinavia (e.g. Isaksen et al., 2002; Heggem et al., 2005; Etzelmüller et al., 2007; Farbrot et al., 2008).

For more remote areas or regions with sparse data on permafrost indicators, multi-criteria approaches within a GIS framework have been applied to generate maps of ‘permafrost favorability’. Here, scores were derived for single factors (elevation, topographic wetness, potential solar radiation, vegetation) based on simple logistic regression or basic process understanding, with the sum of the derived probabilities used as a measure of permafrost favourability in a given location (Etzelmüller et al., 2006).

**Numerical Models**

The limitations of equilibrium models have spurred the recent adaptation of transient numerical simulation models in spatial applications. Numerical models are flexible enough to accommodate highly variable materials, geometries and boundary conditions. Most thermal models for geoscience applications are implemented by simulating vertical ground temperature profiles in one dimension employing a finite-difference or finite-element form of equation 1, usually following a standard procedure:

1. Define the modelled space: set a starting point in time, and upper and lower boundaries.
2. Divide continuous space into finite pieces (a grid of nodes or elements) and continuous time into finite time steps.
3. Specify the thermal properties of the soil materials.
4. Specify the temperature or heat flow conditions as a function of time (i.e. for each time step) for the upper and lower boundaries.
5. Specify an initial temperature for every point in the profile.
6. For each time step after the starting time, the new temperature profile is calculated, based on the combination of thermal properties, antecedent and boundary conditions.

While the use of numerical models allows the accommodation of heterogeneity in both space and time, it also gives rise to the problem of actually supplying spatial data fields of material properties and initial conditions.

**Upper Boundary Conditions**

Upper boundary temperature conditions in numerical models can be specified in various ways. Temperatures may be specified for the ground surface directly or using n factors (e.g. Duchesne et al., 2008); alternately, when snow cover is present the temperature at the snow surface may be specified so that ground surface temperature is determined by heat flow between the snow and the ground (e.g. Oelke and Zhang, 2004). The most elaborate method of establishing the surface boundary temperature is by calculation of the surface energy balance to determine the equilibrium temperature (Budyko, 1958) at the snow or ground surface (e.g. Zhang et al., 2003). Surface energy-balance models generally employ a radiation balance with partitioning of atmospheric sensible and latent heat using aerodynamic theory. The earlier generation of permafrost models generally employed data that were collected locally for site-specific application, including incoming solar radiation, wind speed and air temperature (e.g. Outcalt et al., 1975; Ng and Miller, 1977; Mittaz et al., 2000). Spatial permafrost modelling at regional (Hinzman et al., 1998; Chen et al., 2003) and continental (Anisimov, 1989; Zhang et al., 2007) scales has required a less site-specific approach to the energy balance, with short-wave radiation attenuated through the atmosphere, wind fields modified by local conditions based on satellite-derived leaf area indices, etc., often with climatic conditions obtained from GCM output (e.g. Sushama et al., 2007). In mountain areas, extreme spatial variability requires the parameterisation of the influence of topography on surface micro-climatology for the derivation of spatially distributed information about temperature and snow cover (cf. Stocker-Mittaz et al., 2002; Gruber, 2005).

**SPATIAL MODELS**

Application of the types of models described in the previous sections to the simulation and prediction of permafrost distribution at continental, regional and local scales is discussed in the following sections. Mountain permafrost models are discussed in a separate section, as is recent work incorporating permafrost into General Circulation Model (GCM) land surface schemes.

**Continental/circumpolar**

Due to the strong dependence of permafrost conditions on regional climate, geocryological spatial modelling has been dominated by national- to circumpolar-scale (i.e. small scale) studies in which broad spatial patterns can be related to a few readily available
climatic parameters. The empirical methods frequently used to assess permafrost distribution at these scales implicitly assume that the thermal regime of near-surface permafrost is determined by modern climatic conditions. The use of macro-scale patterns of climatic parameters to infer the configuration of permafrost zones was first proposed by G. Wild (Shiklomanov, 2005), who was able to correlate the southern boundary of permafrost with the −2°C isotherm of MAAT (Wild, 1882). Such simple empirical relations were frequently used throughout the twentieth century for permafrost regionalisation (Heginbottom, 2002), although they do not account for the thermal inertia of permafrost.

Equilibrium Models.

The Frost Number model was applied successfully to central Canada (Nelson, 1986) and continental Europe (Nelson and Anisimov, 1993) for modern climatic conditions. Calculated boundaries of continuous and discontinuous permafrost were in satisfactory agreement with existing geocryological maps. The Frost Number was used with an empirical scenario derived from palaeoanalogues and with output from several GCMs to predict the future distribution of ‘climatic’ permafrost in the northern hemisphere (Anisimov and Nelson, 1996; Anisimov and Nelson, 1997). An alternative approach was used by Smith and Riseborough (2002) to evaluate the conditions controlling the limits and continuity of permafrost in the Canadian Arctic by means of the TTOP model. Using a model derived from the TTOP model, based primarily on the roles of snow and soil thermal properties on the thermal effect of the winter snow cover, Riseborough (2004) produced maps of the southern boundary of permafrost in Canada for a range of substrate conditions.

Several variations of the Kudryavtsev model have been used with GIS technology to calculate both active-layer thickness and mean annual ground temperatures at circum-arctic scales (Anisimov et al., 1997). The thermal and physical properties of snow, vegetation, and organic and mineral soils were fixed both spatially and temporally. These properties were varied stochastically within a range of published data for each grid cell, producing a range of geographically varying active-layer estimates. The final active-layer field was produced by averaging of intermediate results. The model was extensively used to evaluate the potential changes in active-layer thickness under different climate change scenarios at circumpolar (Anisimov et al., 1997) and continental (Anisimov and Reneva, 2006) scales, to assess the hazard potential associated with progressive deepening of the active layer (Nelson et al., 2001, 2002; Anisimov and Reneva, 2006) and to evaluate the emission of greenhouse gases from the Arctic wetlands under global warming conditions for Russian territory (Anisimov et al., 2005; Anisimov and Reneva, 2006).

Numerical Models.

A one-dimensional finite-difference model for heat conduction with phase change and a snow routine (Goodrich, 1978, 1982) has been adapted at the National Snow and Ice Data Center (NSIDC) to simulate soil freeze/thaw processes at regional or hemispheric scales (Oelke et al., 2003, 2004; Zhang et al., 2005). The NSIDC permafrost model requires gridded fields of daily meteorological parameters (air temperature, precipitation), snow depth and soil moisture content, as well as spatial representation of soil properties, land cover categories, and DEMs to evaluate the daily progression of freeze/thaw cycles and soil temperature at specified depth(s) over the modelling domain. Initial soil temperatures are prescribed from available empirical observations associated with permafrost classification from the International Permafrost Association’s Circum-Arctic Permafrost Map (Brown, 1997; Zhang et al., 1999), with a grid resolution of 25 km × 25 km. The numerical model has been shown to provide excellent results for active-layer depth and permafrost temperatures (Zhang et al., 1996; Zhang and Stammes, 1998) when driven with well-known boundary conditions and forcing parameters at specific locations.

Several numerical models have been developed recently for simulating permafrost evolution at continental scales. The Main Geophysical Observatory (St Petersburg) model was developed for Russian territory, using principles similar to those applied at the NSIDC but differing in computational details and parameterisations. In particular, the model was designed to be driven by GCM output to provide a substitute for unavailable empirical observations (Malevsky-Malevich et al., 2001; Molkentin et al., 2001).

Zhang et al. (2006) developed the Northern Ecosystem Soil Temperature (NEST) numeric model to simulate the evolution of the ground thermal regime of the Canadian landmass since the Little Ice Age (1850) (Figure 2). The model explicitly considered the effects of differing ground conditions, including vegetation, snow, forest floor or moss layers, peat layers, mineral soils and bedrock. Soil temperature dynamics were simulated with the upper boundary condition (the ground surface or snow surface when snow is present) determined by the surface energy balance and the lower boundary condition (at a depth of 120 m) defined by the geothermal heat flux (Zhang et al., 2003). The NEST
A model operates at half-degree latitude/longitude resolution and requires inputs relating to vegetation (vegetation type and leaf area index), ground conditions (thickness of forest floor and peat, mineral soil texture and organic carbon content, ground ice content, thermal conductivity of bedrock and geothermal heat flux) and atmospheric climate (air temperature, precipitation, solar radiation, vapour pressure and wind speed).

Marchenko et al. (2008) developed University of Alaska Fairbanks-Geophysical Institute Permafrost Lab model Version 2 (UAF-GIPL 2.0), an Alaska-specific, implicit finite-difference, numerical model (Tipenko et al., 2004). The formulation of the one-dimensional Stefan problem (Alexiades and Solomon, 1993; Verdi, 1994) makes it possible to use coarse vertical resolution without loss of latent-heat effects in the phase transition zone, even under conditions of rapid or abrupt changes in the temperature fields. Soil freezing and thawing follow the unfrozen water content curve in the model, specified for each grid point and soil layer, down to the depth of constant geothermal heat flux (typically 500 to 1000 m). The model uses gridded fields of monthly air temperature, snow depth, soil moisture, and thermal properties of snow, vegetation and soil at 0.5° × 0.5° resolution. Extensive field observations from representative locations characteristic of the major physiographic units of Alaska were used to develop gridded fields of soil thermal properties and moisture conditions.

Non-linearity of sub-grid scale processes may lead to biased estimation of permafrost parameters when input data and computational results are averaged at the grid scale. While coarse spatial resolution is sufficient for many small-scale geocryological applications such as permafrost regionalisation, evaluation of spatial variations in near surface permafrost temperature and the active layer should account for more localised spatial variability. This problem is addressed to some extent by regional, spatial permafrost models.

Local/regional

In general the approaches to permafrost modelling at regional scale are similar to those at small geographical scale. However, regional, spatial permafrost models are applied in areas for which significant amounts of data are available, with the underlying principle to ‘stay close to the data’. Available observations allow comprehensive analysis of landscape-specific (vegetation, topography, soil properties and composition) and climatic (air temperature and its amplitude, snow cover, precipitation) characteristics that influence the ground thermal regime and provide realistic parameterisations for high-resolution modelling.

Empirical Models.

Using empirically derived landscape-specific edaphic parameters representing the response of the active layer and permafrost to both climatic forcing and local factors (soil properties, moisture conditions and vegetation), empirical spatial modelling was successfully applied to high-resolution spatial modelling of the annual thaw depth propagation over the 29,000 km² Kuparuk region in north-central Alaska (Nelson et al., 1997; Shiklomanov and Nelson, 2002) and the northern portion of west Siberia (Shiklomanov et al., 2008). Zhang T. et al. (2005) used a landscape-sensitivity approach in conjunction with Russian historic ground temperature measurements to evaluate spatial and temporal variability in active-layer thickness over several Russian arctic drainage basins.

Equilibrium Models (Kudryavtsev, Stefan and TTOP).

The Kudryavtsev approach was applied successfully over the Kuparuk region of north-central Alaska by Shiklomanov and Nelson (1999) for high-resolution (1 km²) characterisation of active-layer thickness and was used as the basis for the GIPL 1.0, an interactive GIS model designed to estimate the long-term response of permafrost to changes in climate. The GIPL 1.0 model has been applied to detailed analysis of permafrost conditions over two regional transects in Alaska and eastern Siberia (Sazonova and Romanovsky, 2003). In particular it
was used to simulate the dynamics of active-layer thickness and ground temperature, both retrospectively and prognostically, using climate forcing from six GCMs (Sazonova et al., 2004). To refine its spatial resolution, the GIPL 1.0 model was used in conjunction with a regional climate model (RCM) to provide more realistic trends of permafrost dynamics over the east-Siberian transect (Stendel et al., 2007).

Stefan-based methods range from straightforward computational algorithms requiring deterministic specification of input parameters (gridded fields of thawing indices, thermal properties of soil, moisture/ice content and land cover characteristics, and terrain models) (Klene et al., 2001) to establishing empirical and semi-empirical relationships between variables based on comprehensive field sampling (Nelson et al., 1997; Shiklomanov and Nelson, 2002). These methods were used for high-resolution mapping of the active layer in the Kuparuk region (Klene et al., 2001) and for detailed spatial characterisation of active-layer thickness in an urbanised area in the Arctic (Klene et al., 2003).

Wright et al. (2003) used the TTOP formulation for high-resolution (1 km²) characterisation of permafrost distribution and thickness in the broader Mackenzie River valley, north of 60°N. The TTOP model calibrated for the Mackenzie region using observations of permafrost occurrence and thickness at 154 geotechnical borehole sites along the Norman Wells Pipeline provided good general agreement with currently available information. The results indicate that given adequate empirically derived parameterisations, the simple TTOP model is well suited for regional-scale GIS-based mapping applications and investigations of the potential impacts of climate change on permafrost.

Numerical Models.

Although empirical and equilibrium approaches currently dominate spatial permafrost modelling at a regional scale, several numerical models were adopted for regional applications to provide spatial-temporal evolution of permafrost parameters.

A regional, spatial, numerical thermal model was developed by Hinzman et al. (1998) and applied to simulate active layer and permafrost processes over the Kuparuk region, the North Slope of Alaska, at 1 km² resolution. The model utilises a surface energy balance and subsurface finite-element formulations to calculate the temperature profile and the depth of thaw. Meteorological data were provided by a local high-density observational network and surface and subsurface characteristics were spatially interpolated based on measurements collected in typical landform and vegetation units.

Duchesne et al. (2008) use a one-dimensional finite-element heat conduction model integrated with a GIS to investigate the transient impact of climate change on permafrost over three areas of intensive human activity in the Mackenzie valley (Figure 3). The model uses extensive field survey data and existing regional maps of surface and subsurface characteristics as input parameters to predict permafrost distribution and temperature characteristics at high resolution (1 km²), and facilitates transient modelling of permafrost evolution over selected time frames. Statistical validation of modelling results indicates a reasonable level of confidence in model performance for applications specific to the Mackenzie River valley.

Mountains

The lateral variability of surface micro-climate and subsurface conditions is far greater in mountains than in lowland environments. The processes that govern the existence and evolution of mountain permafrost can be categorised into the scales and process domains of climate, topography and ground conditions (Figure 4). At the global scale, latitude and global circulation patterns determine the distribution of cold mountain climates. These climatic conditions are modified locally by topography, influencing microclimate and surface temperature due to differences in ambient air temperature caused by elevation, differences in solar radiation caused by terrain shape and orientation, and differences in snow cover due to transport by wind and avalanches. The influence of topographically altered climate conditions on ground temperatures is further modified locally by the physical and thermal properties of the ground. Substrate materials with high ice content can significantly retard warming and permafrost degradation at depth, while, especially in mountainous terrain, coarse blocky layers promote ground cooling relative to bedrock or fine-grained substrates (Hanson and Hoelzle, 2004; Juliussen and Humlum, 2008). Conceptually, these three scales and process domains are useful in understanding the diverse influences on mountain permafrost characteristics and the differences between modelling approaches, although divisions between scales are not sharply defined. The overall magnitude of the effect of topography on ground temperature conditions can be as high as 15°C within a horizontal distance of 1 km — comparable to the effect of a latitudinal distance of 1000 km in polar lowland areas.

Lewkowicz and Ednie (2004) designed a BTS- and radiation-based statistical model in the Yukon Territory in Canada, using an extensive set of observations in pits for calibration and validation. Lewkowicz and Bonnave-venture (2008) examined strategies for transferring BTS-based statistical models to other regions, with the aim of modelling permafrost probabilities over the extensive mountain areas of western Canada. Brenning

Figure 3  Near-surface permafrost temperature in the Fort Simpson region of Northwest Territories, Canada, simulated using a finite-element model with surface boundary conditions (n factors, subsurface properties) derived from satellite and map-based vegetation characteristics (Duchesne et al., 2008).

Figure 4  Conceptual hierarchy of scales and process domains that influence ground temperature and permafrost conditions in mountain areas. The white disk in the two leftmost images refers to a location that is then depicted in the image to the right — and has its conditions further overwritten by the respective conditions at that scale.
et al. (2005) have outlined statistical techniques for improving the definition of such models, as well as for designing effective measurement campaigns.

A very simple alternative approach is to represent the influence of solar radiation on ground temperatures using a model of the form: \( \text{MAGT} = \text{MAAT} + a + b \cdot \text{PR} \), where PR is the potential direct short-wave radiation, MAGT is the mean annual ground temperature, and \( a \), \( b \) are model constants. This approach is usually limited by the available data for MAGT but can be a valuable first guess in remote locations (cf. Abramov et al., 2008).

DEM-derived topographic parameters and information derived from satellite images offer the potential for more accurate permafrost distribution modelling in remote mountainous areas. Heggem et al. (2006) estimated the spatial distribution of mean annual ground surface temperature and active-layer thickness based on measured ground surface temperatures in different landscape classes defined by topographic parameters (elevation, potential solar radiation, wetness index) and satellite image-derived factors (forest and grass cover). A sine function was fitted to the surface temperature measurements (parameterised as the mean annual temperature and amplitude), providing input for the Stefan equation. Unlike the statistical approaches, this allowed the spatial mapping of simulated ground surface temperature and active-layer thickness fields for changing temperature or snow cover.

Juliusen and Humlum (2007) proposed a TTOP-type model adapted for mountain areas. Elevation change is described through air temperatures, while slope dependencies are parameterised by potential solar radiation as a multiplicative summer-thaw \( n \) factor, including parameterisation for convective flow in blocky material. While there are benefits to this approach (such as a broad base of experience in lowland areas), the multiplicative treatment of solar radiation in the \( n \) factors will lead to problems when used over large ranges in elevation and needs further study. For Iceland, Etzelmüller et al. (2008) used the TTOP-modelling approach to estimate the influence of snow cover on permafrost existence at four mountain sites. The results were compared against transient heat flow modelling of borehole temperatures. Both studies show a high variability of winter \( n \) factors through the years of monitoring.

Transient Models, Energy-balance Modelling and RCM-Coupling.

In Scandinavia, one-dimensional transient models were applied to ground temperatures measured in permafrost boreholes in Iceland (Farbrot et al., 2007; Etzelmüller et al., 2008).

The slope instability associated with the rapid thermal response of permafrost in steep bedrock slopes (cf. Gruber and Haeberli, 2007) has led to increased interest in measurements (Gruber et al., 2003) and models (Gruber et al., 2004a, 2004b) of near-surface rock temperatures in steep terrain. The physics-based modelling (and validation) of temperatures in steep bedrock is an interesting subset of modelling the whole mountain cryosphere, because the influence of topography on micro-climate is maximised, while the influence of all other factors is minimal. The mostly thin snow cover on steep rock walls implies gravitational transport of snow and deposition at the foot of the slope. This is manifested in the occurrence of low-elevation permafrost as well as small glaciers that are entirely below the glacier equilibrium line altitude. A simple and fast algorithm for gravitational redistribution of snow (Gruber, 2007) is currently being tested in distributed energy-balance models (e.g. Strasser et al., 2007).

Terrain geometry and highly variable upper boundary conditions determine the shape of the permafrost body, borehole temperature profiles (cf. Gruber et al., 2004c) and potential rates of permafrost degradation. Noetzli et al. (2007) have conducted detailed experiments with two- and three-dimensional thermal models, demonstrating that zones of very high lateral heat flux exist in ridges and peaks, and that these zones are subject to accelerated degradation as warming takes place from several sides (Noetzli et al., 2008).

The one-dimensional model SNOWPACK (Bartelt and Lehning, 2002), originally developed to represent a highly differentiated seasonal snow cover, has been used in several pilot studies to investigate permafrost (Luetschg et al., 2004; Luetschg and Haeberli, 2005). In this model, mass and energy transport as well as phase change processes are treated with equal detail throughout all snow and soil layers, with water transported in a linear reservoir cascade. The distributed model Alpine3D (employing SNOWPACK) can calculate or parameterise wind transport of snow, terrain-reflected radiation and snow structure (Lehning et al., 2006).

Freeze-thaw processes combined with steep slopes produce significant fluctuations in water and ice content in the active layer of mountain permafrost. GEOTop (Zanotti et al., 2004; Bertoldi et al., 2006; Rigon et al., 2006; Endrizzi, 2007) is a distributed hydrological model with coupled water and energy budgets that is specifically designed for use in mountain areas and is currently being adapted to permafrost research. While the parameterisation, initialisation and validation of such complex models are demanding, the initial results of this research are promising.
In contrast to high-latitude mountains and lowlands, the central Asian mid-latitude permafrost of the Tien Shan Mountains can be attributed exclusively to elevation. In the inner and eastern Tien Shan region the high level of solar radiation and high winds in combination with low atmospheric pressure and low humidity promote very intensive evaporation/sublimation in the upper ground and can lead to the formation of unexpectedly thick permafrost, especially within blocky debris (Haebler et al., 1992; Harris, 1996; Humlum, 1997; Harris and Pedersen, 1998; Delaloye et al., 2003; Gorbunov et al., 2004; Delaloye and Lambiel, 2005; Juliussen and Humlum, 2008; Gruber and Hoelzle, 2008). Spatial modelling of altitudinal permafrost in the Tien Shan and Altai mountains examined both permafrost evolution over time and permafrost dynamics at the local scale for specific sites within selected river basins (Marchenko, 2001; Marchenko et al., 2007) and also at regional scale for the entire Tien Shan permafrost domain. Regional-scale simulation used a multi-layered numerical soil model, including the latent heat of fusion, with snow cover and vegetation having time-dependent thermal properties. The model uses soil extending down to a depth at least of 100 m (without horizontal fluxes), and takes into account convective cooling within coarse debris and underlying soils (Marchenko, 2001). An international team of experts is currently working on a unified permafrost map of Central Asia (Lai et al., 2006), including the Altai Mountain region. Figure 5 shows the first attempt to simulate the Altai’s spatially distributed altitudinal permafrost, with a 5-km grid size.

**Permafrost in Global/RCMs**

Until recently the transient response of permafrost to projected climate change has usually been modelled outside of GCMs, using GCM results only to force surface conditions, in what Nicolsky et al. (2007) call a ‘post-processing approach’, primarily because the coarse subsurface resolution within GCMs did not adequately represent permafrost processes. The representation of the soil column within early GCM land surface schemes did not include freezing and thawing processes, and most recent implementations include few soil layers and a soil column of less than 10 m (e.g. Li and Koike, 2003; Lawrence and Slater, 2005; Saha et al., 2006; Saito et al., 2007; Sushama et al., 2007). Christensen and Kuhry (2000) and Stendel and Christensen (2002) used RCM- and GCM-derived surface temperature indices to model active-layer thickness using the Stefan equation, and permafrost distribution using the Frost Number model, while Stendel et al. (2007) evaluated permafrost conditions by driving the Kudryavtsev model with GCM output downscaled to RCM resolution. Li and Koike (2003) and Yi et al. (2006) have developed schemes to improve the accuracy of frost line or active-layer thickness estimates with coarsely layered soils grids using modified forms of the Stefan equation.

Lawrence and Slater (2005) modelled transient permafrost evolution directly within the Community Climate System Model GCM (Collins et al., 2006) and Community Land Model Version 3 (CLM3) (Oleson et al., 2004). Their results generated significant discussion within the permafrost community (Burn and Nelson, 2006; Lawrence and Slater, 2006; Delisle, 2007; Yi et al., 2007) for reasons largely related to the shallow (3.43 m) soil profile (see following section on depth, memory and spin-up), although Yi et al. (2007) also demonstrate the importance of the surface organic layer. Subsequent work on the geothermal component of CLM3 (Mölders and Romanovsky, 2006; Alexeev et al., 2007; Nicolsky et al., 2007) has clarified many issues involved in accounting for the annual permafrost regime. While a modified scheme has been evaluated against long-term monitoring data (Mölders and Romanovsky, 2006), spatial results using the modified scheme at global or regional scale have not yet appeared.
At present the most realistic representation of transient permafrost dynamics within spatial models is within externally forced ‘post-processed’ models which incorporate high-resolution finite-element or finite-difference, numerical heat flow models (e.g. Oelke and Zhang, 2004; Zhang Y. et al., 2005, 2006). The application of RCM/GCM output to the physics-based modelling of permafrost is especially difficult in mountain areas. Because of the poor representation of steep topography on coarse grids, strong differences between simulated and measured climate in those areas exist. Salzmann et al. (2006, 2007) and Noetzli et al. (2007) have described a method that allows downscaling RCM/GCM results in mountain areas and their application to energy-balance and three-dimensional temperature modelling in steep bedrock.

DISCUSSION AND FUTURE DIRECTIONS

Model Uncertainty and Data Uncertainty

Permafrost models that operate on broad (circumpolar/continental) geographical scale are the most appropriate tools for providing descriptions of climate-permafrost interactions over the terrestrial Arctic. However, their spatial resolution and accuracy are limited by availability of data characterising the spatial heterogeneity of many important processes controlling the ground thermal regime. Shiklomanov et al. (2007) found large differences between spatially modelled active-layer fields produced by various small-scale permafrost models, due primarily to differences in the models’ approaches to characterisation of largely unknown spatial distributions of surface (vegetation, snow) and subsurface (soil properties, soil moisture) conditions. Further, Anisimov et al. (2007) found the differences in global baseline climate datasets (air temperature, precipitations), widely used for forcing small-scale permafrost models, can translate into uncertainty of up to 20 per cent in estimates of near-surface permafrost area, which is comparable to the extent of changes projected for the current century.

Lower Boundary and Initial Conditions — Depth, Memory and Spin-up

Ground temperature is largely controlled by changes at the surface, with change lagged and damped by diffusion at depth. The depth to which numerical models simulate temperature is limited by the relationship between the thermal properties of the ground and the size of time step and grid spacing (and as determined by the limitations of the model). One-dimensional permafrost model studies typically specify a modelled soil depth of 20 m or more, in order to capture the annual ground temperature cycle, with deeper profiles specified when the long-term evolution of the ground thermal regime is being evaluated. The medium- and long-term fate of permafrost is determined by changes at depth, such as the development of supra-permafrost taliks. Truncating the ground temperature profile at too shallow a depth will introduce errors as the deep profile influences the thermal regime of the near surface, with errors accumulating as the temperature change at the surface penetrates to the base of the profile.

Alexeev et al. (2007) suggest that in general the lower boundary should be specified so that the total soil depth is much greater than the damping length for the timescale of interest. They suggest that the timescale of maximum error is about two years for a 4 m-deep soil layer, or about 200 years for a 30 m-deep grid, although their analysis ignores the effect of latent heat. In a comparison with an equilibrium model (a situation with no memory at all) and a numerical model that accounts for latent heat, Riseborough (2007) showed that the long-term mean annual temperature at the base of the active layer could be accurately predicted under transient conditions except where a talik was present. Applying this result to the analysis of Alexeev et al. (2007), they likely underestimate the magnitude of errors due to a shallow base, as thaw to the base does not imply the disappearance of permafrost, but does imply the development of a talik, and that permafrost is no longer sustainable under the changing climate.

The inclusion of a deep temperature profile for the modelled space introduces the additional challenge of estimating a realistic initial condition. While this can be established using field temperature data, this approach is impractical in spatial modelling; where no data are available, an initial temperature profile is usually established by an equilibration or ‘spin-up’ procedure, running the model through repeated annual cycles with a stationary surface climate, until an equilibrium ground temperature profile develops.

Depending on the profile used to initiate the spin-up procedure, equilibration of deep profiles typically requires hundreds of cycles, although as Lawrence and Slater (2005) and others have noted, the initial profile has little effect if the profile is very shallow. In a regional permafrost model, with a 120 m-deep temperature profile Ednie et al. (2008) found that permafrost profiles equilibrated using twentieth century climate data did not adequately reproduce...
the essential characteristics of the current regime in the Mackenzie Valley, Northwest Territories, Canada, in particular the presence of deep, nearly isothermal (disequilibrium) permafrost. They initiated their model with a spin-up to equilibrium for the year 1721, followed by a surface history combining local palaeo-climatic reconstructions and the instrumental record. For a national scale model, Zhang Y. et al. (2005, 2006) began their long-term simulations assuming equilibrium in 1850, with climate data for the 1850–1900 period estimated by backward linear extrapolation from twentieth century climatic data.

Initial modelling of permafrost distribution in the Mackenzie Valley (Northwest Territories, Canada) used the TTOP model (Wright et al., 2003), predicting mean annual ground temperature and permafrost thickness. The TTOP model was used to improve efficiency when modelling the transient evolution of permafrost in this environment using a one-dimensional finite-element model (Duchesne et al., 2008). First, the accuracy of the TTOP model for estimating equilibrium conditions allowed for a reasonable first estimate of the initial ground temperature profile, minimising the time required for model equilibration. Second, the equilibrium permafrost thickness estimated using the TTOP model was used to establish the depth of the bottom of the grid.

Modelling Permafrost and Environmental Change

Future refinements to modelling of permafrost response to climate change projections will require consideration of the interaction between permafrost, snow cover, vegetation and other environmental factors at timescales ranging from decadal to millennial. To a significant degree, the position of treeline controls the geographic distribution of snow density (Riseborough, 2004), so that changes in permafrost distribution, migration of treeline and changes in snow cover properties in the forest-tundra transition zone will be highly inter-dependent. At much longer timescales, the distribution of peatlands depends on the relative rates of carbon accumulation and depletion in the soil, with the zone of peak soil carbon and peatland distribution close to the position of the 0°C mean annual ground temperature isotherm (Swanson et al., 2000). Changing ground temperatures under climate warming will alter the distribution of peatlands, thereby altering local permafrost distribution. These changes will influence surface and subsurface physical, thermal and hydraulic properties, which are often currently assumed to remain unchanged, even in a future climate.

N Factors and Parameterisation

N factors expressed as a ratio (equation 8) or the layers of the Kudryavtsev model (which function by diffusive extinction) may not adequately express the empirical relationships among the multitude of processes they subsume and the underlying system behaviour (as opposed to model behaviour). The functional form in which parameters encapsulate complex processes in simple models will influence the behaviour of the model, especially where models are used for predictions beyond the scope in which the parameters were derived. Unless there is a strong theoretical basis for the functional form of the parameterisation, it may be better to express the relationship in a form dictated by empirical results; alternate forms include differences or multi-parameter linear or non-linear relationships.

CONCLUSION

The fate of permafrost under a changing climate has been a concern since the earliest GCM results demonstrated the magnitude of the problem. Current permafrost transient modelling efforts generally follow two approaches: incorporation of permafrost dynamics directly into GCM surface schemes and increasingly sophisticated regional, national and global models forced by GCM output. Although computational cost is becoming less critical as computer technology advances, these two approaches will continue to evolve in tandem, and are unlikely to merge. While Stevens et al. (2007) demonstrate the importance of the deep geothermal regime on heat storage under a changing climate, the influence of permafrost conditions on the evolving GCM climate and the computational requirements of a multi-layer ground thermal scheme will likely be balanced with a level of complexity that is less than can be achieved in detailed regional models. Once this balance is achieved, the relationship between GCM-based and post-processed models will be equivalent to the relationship between global and regional climate models.

Historically, spatial models of permafrost in arctic lowlands and mountain terrain have developed following different principles. Permafrost was initially studied in Artic lowland areas, mainly because of human development in these regions. Both numerical and analytical solutions were developed early, both of
which were easy to apply to spatial models as GIS and computer power developed. Permafrost in mountains was recognised as an engineering and scientific topic much later (1970s), and the spatial heterogeneity of factors influencing permafrost led to the development of empirical concepts. A trend apparent today is the merging of the concepts developed in Arctic lowland regions in mountain environments. This is now possible with the increasing power of computer hardware and software, but the treatment of sub-grid heterogeneity (such as the effect of topography) over continental or hemispheric areas remains a major scientific challenge.

ACKNOWLEDGEMENTS

The authors would like to thank Caroline Duchesne and two anonymous reviewers whose comments were useful in revising the paper. Yu Zhang and Caroline Duchesne provided graphical examples of recent work for inclusion as figures. The forbearance of the journal’s editors is gratefully acknowledged.

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