Glacier mass balance modelling

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Why modelling?

Background

- Glaciers have retreated worldwide during the last century and are expected to continue retreating
- Socio-economic implications:
  - Sea-level rise
  - Hazard mitigation (floods)
  - Hydropower (e.g., Norway, 99% of power generation, 10% is glacier derived)
- Significant contributor to streamflow: modify runoff quantity and timing (glacier as 'water storage')

How can we investigate the glacier-climate relationship?
Reasoning

• Climate provides upper boundary conditions of the ice sheets/glaciers:
  accumulation, ablation → geometry changes

• Thermodynamics effects:
  Internal ice temperature
  Flow law coupling (cold ice is more stiff)

• Surface melt water
  Water resource, Sea level change

Glacier *surface* mass balance

- Accumulation: gain of mass (e.g. snow deposition)
- Ablation: loss of mass (e.g. snow/ice melting)
- Accumulation area: acc > abl
- Ablation area: acc < abl
- Equilibrium line altitude (ELA) in (m a.s.l.)
Methods

- Simple integrated schemes
  - Regression models
- Mass-balance models driven by specified/modelled climate:
  - Degree day methods
  - Energy balance models

Guidelines for Model Selection

1. Operation and calibration data availability
2. Expected physiographic and climatic conditions
3. Detail and type of results required.

Primary approaches to modeling

- Regression Analysis (linear or multiple)
- Temperature Index Approach
- Energy Balance Approach

Data: Engabreen, Norway, NVE
a regression model using winter precipitation and summer temperature may work well
but only for diagnostic applications!
for prognostic applications we need a deterministic model!

Regression analysis

\[ MB = 3.3962 - 0.4503^\text{T\(\text{jun-sep}\)} + 0.0019^\text{P\(\text{oct-mar}\)} \]

\[ r = 0.81 \]

Simple integrated approaches

- Analytical expressions for mass-balance
- Based on observation that accumulation occurs at higher elevation, and ablation lower down
- Simplest is a linear variation of mass-balance with height:

\[ \dot{l}(z) = \beta(z - E) \]

Mass balance (m a\(^{-1}\))
Balance gradient (a\(^{-1}\))
Equilibrium line altitude (m)

- Typically, the equilibrium line altitude will decrease with higher latitude

At ELA:
\[ T_{\text{summer}} = P_{\text{winter}} \]

But how to determine mass balance??

Regression analysis

\[ \Delta \text{ELA}(x, y) = \alpha_0 + \alpha_1 x + \alpha_2 y \]

\[ \Delta \text{ELA}(\text{ELA}) \]

Fig. 7. Temporal change in the ELA for the measured years.
Regression analysis

**Advantages:**
- Provides an estimate of total discharge from basin
- Simple
- Minimum data requirements
- Provide a good index for water resource managers

**Disadvantages:**
- Does not provide information on factors such as peak discharge.
- Threshold effects may occur.
- Assumes stationarity.
  - Climate boundary conditions can’t change.

Simple integrated approaches

**Advantages:**
- Very simple indeed
- Migration of ELA may be linked to climate change
- Attractive for use in situations where there is little data

**Disadvantages:**
- Entirely empirical
- In reality, ELA depends on local climate, aspect, etc.
- A blunt instrument for a complex problem

Energy balance

\[ 0 = Q_K + Q_H + Q_L + Q_G + Q_P + Q_M \]

\[ Q_K = S \downarrow - S \uparrow + L \downarrow - L \uparrow \]

SURFACE ENERGY BALANCE

\[ Q_0 = L_f \frac{dm}{dt} + M_i c_{pi} \frac{dT_i}{dt} \quad [\text{W m}^{-2}] \]

Energy exchange with atmosphere, melting / freezing, heating / cooling of the ice or snow

- \( Q_0 \): energy flux atmosphere to glacier
- \( L_f \): latent heat of fusion \((0.334 \times 10^6 \text{ J kg}^{-1})\)
- \( m \): amount of melt water
- \( M_i \): mass of the ice
- \( c_{pi} \): specific heat capacity of ice \((2009 \text{ J kg}^{-1} \text{ K}^{-1})\)
- \( T_i \): ice temperature
TRANSFER FORCING FROM CLIMATE STATION TO GLACIER

Some commonly used assumptions

<table>
<thead>
<tr>
<th>Variable</th>
<th>Assumption</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature</td>
<td>Constant lapse rate, i.e. $dT/dz$ constant</td>
</tr>
<tr>
<td>Wind speed</td>
<td>Constant</td>
</tr>
<tr>
<td>Humidity</td>
<td>Constant relative humidity</td>
</tr>
<tr>
<td>Cloud amount</td>
<td>Constant</td>
</tr>
<tr>
<td>Precipitation</td>
<td>Linear in elevation (used for tuning)</td>
</tr>
</tbody>
</table>

2 D PICTURE OF THE TEMPERATURE

In case the surface is melting

$dT/dz = \text{constant (e.g. } -0.007 \text{ K/m)}$

Free atmosphere $\downarrow$

$\begin{aligned}
\text{surface (0 °C) and free atmosphere (> 0 °C)} \\
\text{boundary layer: temperature compromise between}
\end{aligned}$

$dT/dz = ?$

Surface: temperature = 0 °C $\downarrow$

$dT/dz = 0$

MEASURED CLIMATE SENSITIVITY

46 daily means during the ablation season, Pasterze, Austria

<table>
<thead>
<tr>
<th>Temperature (°C) at glacier (2205 m a.s.l.)</th>
<th>Temperature (°C) at climate station (3106 m a.s.l.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>0, 10</td>
</tr>
<tr>
<td>5</td>
<td>1, 10</td>
</tr>
<tr>
<td>6</td>
<td>2, 10</td>
</tr>
<tr>
<td>7</td>
<td>3, 10</td>
</tr>
<tr>
<td>8</td>
<td>4, 10</td>
</tr>
<tr>
<td>9</td>
<td>5, 10</td>
</tr>
<tr>
<td>10</td>
<td>6, 10</td>
</tr>
</tbody>
</table>

Constant lapse-rate can be a bad description, because: Climate sensitivity over glacier smaller than over snow-free terrain
**SHORT-WAVE INCOMING RADIATION**

\[ S = I_o \cos(\theta_s) \left( T_{c+} - T_{c-} \right) \]

**DIRTY ICE - PASTERZE**

\[ \alpha \approx 0.2 \]

**FEEDBACK ALBEDO ⇔ SNOW AND ICE MELT**

1) Faster metamorphosis of snow
2) Ice appears earlier
3) More meltwater on top of ice
4) More water between snow grains

**SENSITIVITY INCREASES DUE TO ALBEDO FEEDBACK**

1) Faster metamorphosis of snow
2) Ice appears earlier
3) More meltwater on top of ice
4) More water between snow grains

Turbulent fluxes
Incoming long-wave radiation
Energy Balance Models

\[ 0 = Q_R + Q_H + Q_L + Q_C + Q_P + Q_M \]

- Point or spatially distributed
- Run on measured data
  - contrast to empirical models, which run on only a few measured parameters and which rely on calibration parameters at the heart of the model.
- Only as good as your measured data and understanding of the system
- Includes some empirism anyway (turbulent exchange…)
- Sacrifice simplicity for complicated measurements and algorithms.

Energy Balance problems

- Energy Balance model (parameterizations of turbulent exchange)
- Spatial distribution
- Precipitation
- Snowpack model (refreezing, metamorphism, water retention)

Temperature-Index Methods

Based on the concept that changes in air temperature provide an index of snowmelt.

T-index approach:
\[ M = C \cdot (T - T_0) \]

Air temperature
- commonly measured meteorological variable.
- secondary meteorological variable that provides an integrated measure of heat energy.
Temperature-Index Methods

Based on the concept that changes in air temperature provide an index of snowmelt.

T-index approach:

\[ M = C \times (T - T_0) \]

Air temperature
- commonly measured meteorological variable.
- secondary meteorological variable that provides an integrated measure of heat energy.

Advanced T-index models

Strategy: include the second most important energy source (global radiation)

Hock 1999:

\[ M = (C_1 + C_2 \times I) \times (T - T_0) \]

Pellicciotti et al. (2005):

\[ M = C_1 \times (T - T_0) + C_2 \times (1 - \alpha) \times I \]

\( I \) - potential clear-sky solar radiation
\( \alpha \) - albedo

Modelling: Discharge

Limitations of Degree-Day Method

Calculation of degree-day factors for various points on the Greenland ice sheet with a sophisticated atmospheric and snow model (thesis Filip Lefebre)
Importance of individual components

<table>
<thead>
<tr>
<th>Sources/Melts</th>
<th>~ 70%</th>
<th>~ 20%</th>
<th>&lt; 10%</th>
<th>~ 70%</th>
<th>~ 10 - 30%</th>
<th>&lt; 10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longwave incoming radiation</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Absorbed global radiation</td>
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<tr>
<td>Sensible heat flux</td>
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<tr>
<td>Melts</td>
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<tr>
<td>Ground heat flux</td>
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</tbody>
</table>

More trouble...

- The sensible heat flux contributes < 10%
- T-index approach:
  \[ M = C \times (T - T_0) \]

Why does the T-index approach perform that well??

LONG-WAVE INCOMING, PARAMETERISATION

\[
L \downarrow = \left[ \varepsilon_{cs}(1 - n^4) + \varepsilon_{oc}n^4 \right] \sigma T_{2m}^4
\]

Clear-sky term \((cs)\) and overcast term \((oc)\)

Emittance \((\varepsilon)\) is 1.0 for a black body.

\[
\varepsilon_{cs} = 0.23 + c_L \left( \frac{T_{2m}}{T_{2m}} \right)^{0.8}
\]

Three tunable parameters: \(a\), \(\varepsilon_{oc}\), and \(c_L\)

The longwave incoming radiation is the largest contribution to melt (~ 70%)

About 70% of the longwave incoming radiation originates from within the first 100m of the atmosphere.

Variations of screen-level temperatures can be regarded as representative of this boundary layer.
Temperature-index models:
+ low data demand
+ applicable in all scales
- trouble with quality control

Accumulation
• Treated in a very simple way:
  • Precipitation = snow for $T < 2\, ^\circ C$
  • Precipitation = rain for $T \geq 2\, ^\circ C$
  • Linear distribution with elevation

Snow radar:
GPR mapping of snow thickness

$S = F(x,y,z)$
$F$ is a multiple regression function
Accumulation model

- **Spatial distribution** using an index map
- **Temporal distribution** using a scaled precipitation series from Ny-Ålesund

\[
\text{acc} = C_3 \left( \frac{\text{P}_{\text{rad}}}{\sum \text{P}_{\text{rad}}} \right) C_4
\]

Scales & snow distribution

- **micro**
  - Surface roughness
  - Surface cover (vegetation)
  - Wind exposure
  - Microclimate

- **meso**
  - Large-scale topography (mountain range)
  - Distance to moisture source
  - Geogr. latitude

- **macro**

Transition

roughness

precipitation
SURFACE ENERGY BALANCE

\[ Q_0 = L_f \frac{dm}{dt} + M_i c_{pi} \frac{dT_i}{dt} \quad [\text{Wm}^{-2}] \]

- **Energy exchange with atmosphere**
- **Melting / Freezing**
- **Heating / Cooling of the ice or snow**

- **Energy flux atmosphere to glacier:** \( Q_0 \)
- **Latent heat of fusion:** \( L_f \)
- **Amount of melt water:** \( m \)
- **Mass of the ice:** \( M_i \)
- **Specific heat capacity of ice:** \( c_{pi} \)
- **Ice temperature:** \( T_i \)

**Melt physics**

- To **melt** 1 kg snow/ice requires 334 000 J kg\(^{-1}\)
- Latent heat of fusion
- To **sublimate** 1 kg of snow requires 2 600 000 J kg\(^{-1}\)
- Latent heat of sublimation (8x \( L_f \)!!!)
- To **warm** 1 kg of snow 1 K requires 2009 J kg\(^{-1}\) K\(^{-1}\);
  - Ice: 2097 J kg\(^{-1}\) K\(^{-1}\)

**Specific heat capacity**

- Refreezing of 1 g water \( \rightarrow \) warms 160 g snow by 1 K

**Removing cold content**

- Melt-water
- Snow

- Condition for melt: snow must be at melting temperature, otherwise refreezing will occur

- Cold content = energy needed to bring the snow / ice to 0 °C.

- In the given example, refreezing of 2.5 l melt-water is needed to compensate for the cold content of the snow pack (snow density, \( \rho_s = 400 \text{ kg m}^{-3} \)).

**Stake data vs model**

- We measure mass balance, but model melt
refreezing

- Using a simple model:
  \[
  p_{\text{max}} = 0.6
  \]
  the proportion of the winter snow pack that refreezes within one year

  \[
  \text{if (si+melt-p_{\text{max}}*ini\_snow)} \rightarrow \\
  \text{si=si+melt & abl=0}
  \]
  & snow = snow-melt

- Alternative:
  Woodward model
  (pmax is a function of MAAT)

Summary

- predictive power, assessability
- Interpolation
  Regression models
  T-index model
  Enhanced T-index
  Energy balance models

Fig. 16. Effects of an increase in equilibrium line altitude on a small ice cap and a valley glacier. Observe the large sensitivity of the ice cap (Strodder & John 1984, p. 185).
Climate sensitivity

Static sensitivity:
Apply a change to the climate data set, the glacier geometry does not change

Dynamic sensitivity:
Changes of the glacier geometry are considered as well

Uniform temperature or precipitation changes to the input data:
Temp increase $\rightarrow -1.06$ m a$^{-1}$ K$^{-1}$ (-0.99)
Prec increase $\rightarrow +0.35$ m a$^{-1}$ (10%)$^{-1}$ (+0.35)

Seasonal sensitivity characteristic
Sensitivity varies with season

SSC (Oerlemans & Reichert, 2000):
Monthly perturbations in precipitation or temperature

Model input: meteo data

Continuous time series from 1974 to date

DNMI station Glomfjord, ca. 20 km N of Engabreen at 39 m a.s.l.
Potential clear-sky solar radiation

DEM of 25m resolution

Model output: glacier mass

Engabreen
Norway

Mass balance measurements

Mass balance data (NVE) available since 1970

Model performance

\[ r^2 = 0.885 \]
\[ n = 174 \]
Model performance

Winter balance
$r^2 = 0.91$

Net balance
$r^2 = 0.90$

Summer balance
$r^2 = 0.90$

- measured - modelled

Model performance

$r^2 = 0.74$

Calibration period

- measured - modelled