# Two-way nesting: Principles and examples

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February 3, 2016

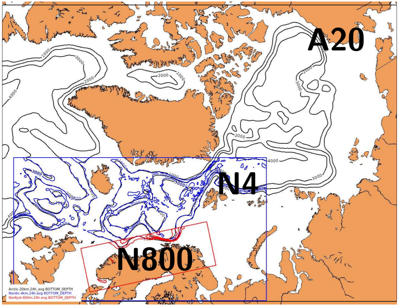
# Overview

# Literature

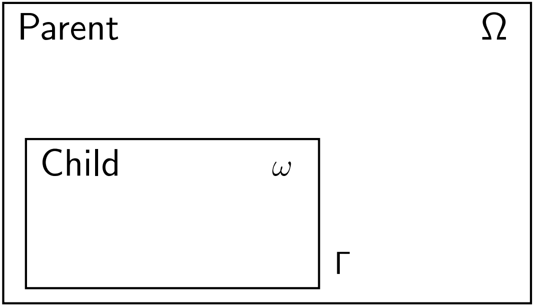
- Blayo and Debreu, Ocean Mod., 2005: OBCs based on characteristics
- Penven et al., Ocean Mod., 2006: One-way nesting,
- Blayo and Debreu, Springer, 2006: Nesting Ocean Models
- Debreu and Blayo, Ocean Dyn., 2008: Two-way embedding algorithms
- Debreu, et al., Comput. Geosci., 2008; AGRIF: Adaptive Grid Refinement In Fortran),
- Debreu, et al., Ocean Mod., 2012, Two-way nesting in ROMS-AGRIF

# Present situation at MET Norway

- Triply **one-way nested** system:
  - Running Arctic20 (A20) with FOAM at OBs
  - Feeding A20 results into Nordic4km (N4)
  - Feeding N4 result further into NorKyst800 (N800)



# Sketch of situation



# The problem



The governing equations:

$$\partial_t q = \mathcal{L}[q] \tag{1}$$

Examples are the 1-D, non-linear shallow water equations:

$$q = \begin{bmatrix} u \\ v \\ h \end{bmatrix} \quad \text{and} \quad \mathcal{L} = \begin{bmatrix} -u\partial_x & f & -g\partial_x \\ -f & -u\partial_x & 0 \\ -h\partial_x & 0 & u\partial_x \end{bmatrix}.$$

or the advection equation

$$q = \psi \quad \text{and} \quad \mathcal{L} = -u_0\partial_x,$$

or ROMS governing eqs. Note: (1) are to be solved for both domains.

# Discretization

Discretized on the parent and child domains

$$[\partial_t q]_p = \mathcal{L}_p[q_p] \quad \text{and} \quad [\partial_t q]_c = \mathcal{L}_c[q_c],$$

$\mathcal{L}_p$ and $\mathcal{L}_c$ are the same discretizations of the continuous operator $\mathcal{L}$, but at <u>*different*</u> resolutions.

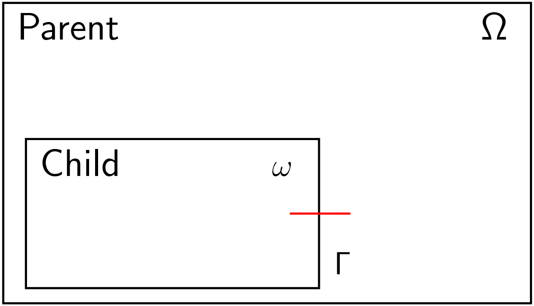
**Example:** Advection equation (upstream scheme):

$$\begin{aligned} \mathcal{L}_p &= -u_0 \frac{\tilde{\psi}^n_{j_p} - \tilde{\psi}^n_{j_p-1}}{\Delta x_p} \\ \mathcal{L}_c &= -u_0 \frac{\tilde{\psi}^n_{j_c} - \tilde{\psi}^n_{j_c-1}}{\Delta x_c} \end{aligned}$$

$\hat{\psi}_{j_p}$ and $\tilde{\psi}_{j_c}$ are $\psi$ at parent respectively child grid points $j_p, j_c$.

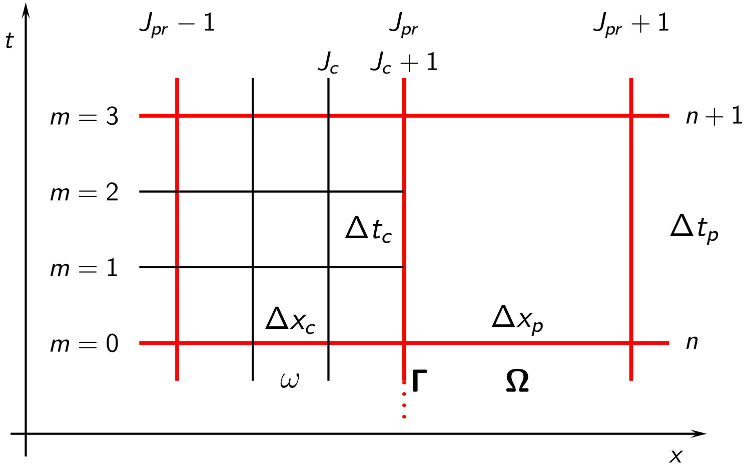
# Grid configuration

Slice through interface



# Grid configuration

Refinement factor $i_r = \frac{\Delta x_p}{\Delta x_c} = \frac{\Delta t_p}{\Delta t_c} = 3$



# Numerical problem

Solve

$$[\partial_t q]_p = \boldsymbol{\mathcal{L}}_p[q_p] \quad \text{and} \quad [\partial_t q]_c = \boldsymbol{\mathcal{L}}_c[q_c],$$

within the two domains so that

1. The parent solution impacts the child solution
2. The two solutions are nested at $\Gamma$, ideally respecting conservation of mass (volume), energy fluxes, transport, etc.
3. The child solution is allowed to impact the parent solution

# The two-way nesting algorithm (explicit schemes)

1. Advance the parent solution from level $n$ to level $n+1$

$$q_p^{n+1} = q_p^n + \Delta t_p \mathcal{L}_p[q_p^n]$$

2. Advance child on $\Gamma$ using an interpolator $\mathcal{P}$ based on the parent solution (one-way nesting)

$$q_c^{n+\frac{m}{i_r}}|_\Gamma = \mathcal{P}[q_p^n, q_p^{n+1}]; \quad m = 1, 2, \ldots, i_r$$

3. Advance child to level $n+1$ in the interior

$$q_c^{n+\frac{m}{i_r}} = q_c^{n+\frac{m-1}{i_r}} + \Delta t_c \mathcal{L}_c[q_c^{n+\frac{m-1}{i_r}}]; \quad m = 1, 2, \ldots, i_r$$

4. Update the parent within child domain using a restriction operator $\mathcal{R}$ (filter)

$$q_p^{n+1} = \mathcal{R}[q_c^{n+1}]_p$$

# Simple example: Advection of a 1-D bell function

Governing equation

$$\partial_t \psi + u_0 \partial_x \psi = 0 \quad ; \quad x \in [-L, +L]$$

Initially

$$\psi(x, 0) = \psi_0 e^{-\left(\frac{x}{\sigma}\right)^2}$$

Cyclic boundary conditions:

$$\psi(x, t) = \psi(x + 2L)$$

# Step 1: Solve for parent domain



Numerical solution using upstream scheme

$$\hat{\psi}_{j_p}^{n+1} = \hat{\psi}_{j_p}^{n} - C\left(\hat{\psi}_{j_p}^{n} - \hat{\psi}_{j_p-1}^{n}\right); \quad j_p = 2(1)J_p + 1$$

$$\hat{\psi}_{1}^{n+1} = \hat{\psi}_{J_p+1}^{n+1}; \quad \text{boundary condition}$$

where $C = u_0 \frac{\Delta t_p}{\Delta x_p} = \frac{1}{2}$ is the Courant number.

Recall that the scheme is diffusive, with diffusion coefficient

$$\kappa_p^* = \tfrac{1}{2}(1 - C)|u_0|\Delta x_p$$

$\Rightarrow$ Diffusivity depends on resolution!

# Step 2: Interpolate to get child solution at Γ

For $m = 1, 2, \ldots, i_r$

$$\begin{aligned}\tilde{\psi}_1^{n+\frac{m}{i_r}} &= (1 - \frac{m}{i_r})\hat{\psi}_{J_{pl}}^n + \frac{m}{i_r}\hat{\psi}_{J_{pl}}^{n+1}, \\ \tilde{\psi}_{J_c+1}^{n+\frac{m}{i_r}} &= (1 - \frac{m}{i_r})\hat{\psi}_{J_{pr}}^n + \frac{m}{i_r}\hat{\psi}_{J_{pr}}^{n+1}.\end{aligned}$$

where

$$J_{pl} = 1 + \frac{(1-a)L}{\Delta x_p} \quad \text{and} \quad J_{pr} = 1 + \frac{(1+b)L}{\Delta x_p}$$

$x_{J_{pl}} = -aL$ and $x_{J_{pr}} = bL$ are the left-hand respectively right-hand interface. $a \leq 1$ and $b \leq 1$ are constants.

# Step 3: Solve for child domain



For $m = 1, 2, \ldots, i_r$.

$$\tilde{\psi}_{j_c}^{n+\frac{m}{i_r}} = \tilde{\psi}_{j_c}^{n+\frac{m-1}{i_r}} - C(\tilde{\psi}_{j_c}^{n+\frac{m-1}{i_r}} - \tilde{\psi}_{j_c-1}^{n+\frac{m-1}{i_r}}); \quad j_c = 2(1)J_c$$

The Courant number $C = u_0 \frac{\Delta t_c}{\Delta x_c} = u_0 \frac{\Delta t_p}{\Delta x_p} = \frac{1}{2}$ is the same for the two grids.

Recall that the diffusion coefficient is

$$\kappa_c^* = \frac{1}{2} u_0 (1 - C) \Delta x_c$$

$\Rightarrow$ Child less diffusive than parent ($\Delta x_c < \Delta x_p$)

# Step 4: Update parent within child



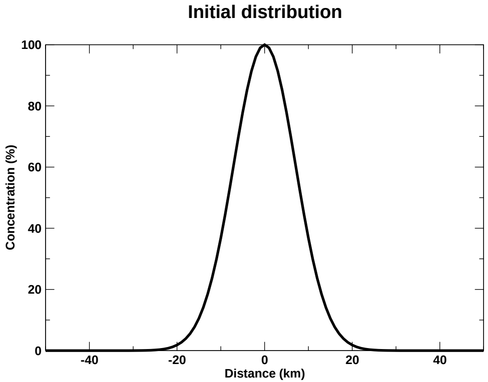
Apply the restriction operator or filter $\mathcal{R}$, for instance a simple 1-2-1 filter (Shapiro with $\mu = 0.5$)

$$\hat{\psi}_{j_p}^{n+1} = \frac{1}{4}\left(\tilde{\psi}_{j_c^*+1}^{n+1} + 2\tilde{\psi}_{j_c^*}^{n+1} + \tilde{\psi}_{j_c^*-1}^{n+1}\right); \quad j_p = J_{pl} + 1(1)J_{pr} - 1,$$

where $j_c^* = 1 + i_r(j_p - J_{pl})$ is a common child/parent grid point.

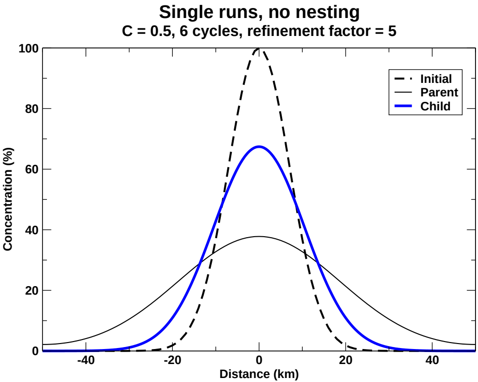
# Solutions: Initial condition

$L = 50$ km, $\sigma = L/5$, $u_0 = 1$ m/s, $C = \frac{1}{2}$, $i_r = 5$, $\Delta x_p = 1$ km, $\Delta x_p = i_r \Delta x_c$, $\Delta t_p = i_r \Delta t_c$




$\Rightarrow \Delta x_c = 0.2$ km, $\Delta t_p = 500$ s and $\Delta t_c = 100$ s.

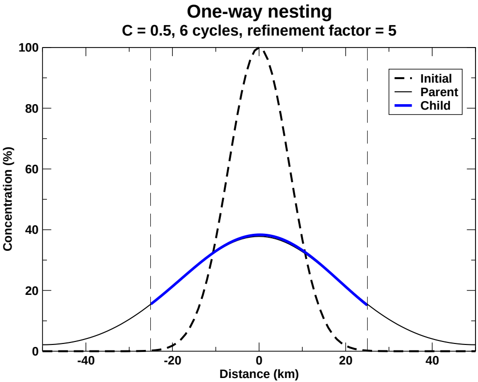
# Solution 1: Reference (no nesting)




- Reference (blue with $\Delta x = \Delta x_c$) less diffusive than Parent
- Neither solution is perfect

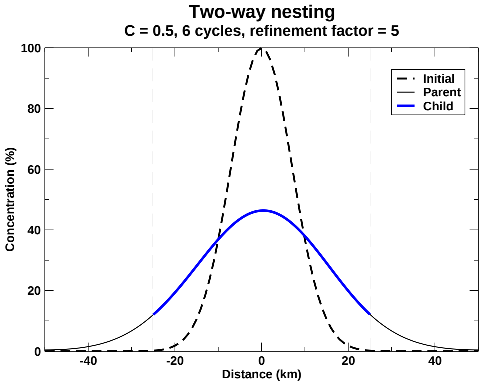
# Solution 2: One-way nesting ($a = b = \frac{1}{2}$)






- Parent (thin black) stays the same (as it should)
- Child (blue) is degraded compared to reference

# Solution 3: Two-way nesting

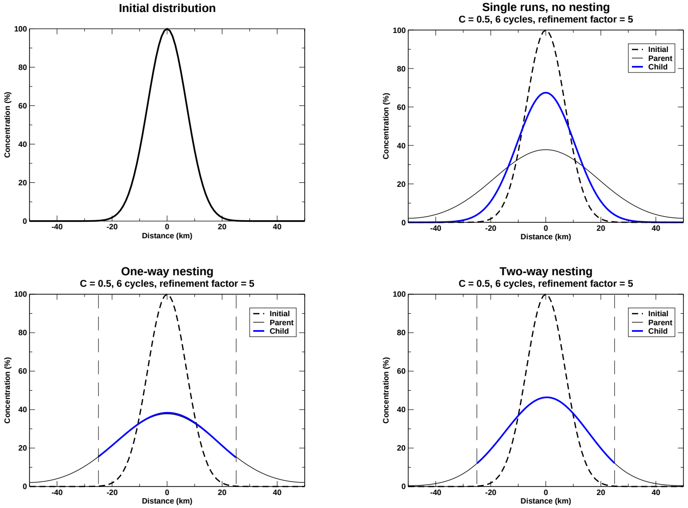



*Conclusions*:

- Both are improved
- Parent is improved both outside and inside of child domain

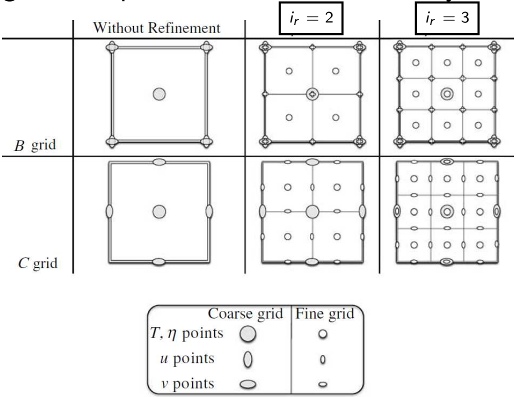
# All solutions





# Staggered grids:

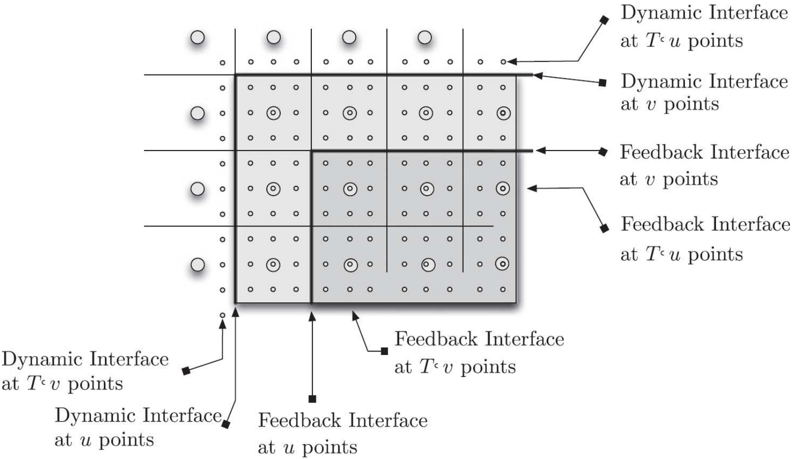
Staggered grids complicate matters considerably,




and so does time splitting (barotropic vs. baroclinic time steps)
Finally: 2-D and 3-D requires interpolation in both time and space and complicates conservation of fluxes

# Staggered grids: Interfaces

One also have to separate between dynamic and feedback interfaces:



# Realistic case: California Current System

Debreu et al. (2008)

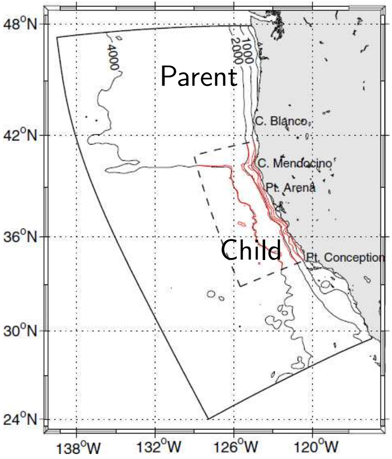
ROMS AGRIF

10 year run

$\Delta x_p = 15$ km

$i_r = 3$

$\Rightarrow \Delta x_c = 5$ km



# Realistic example: California Current System



SST on
June 8, year 6

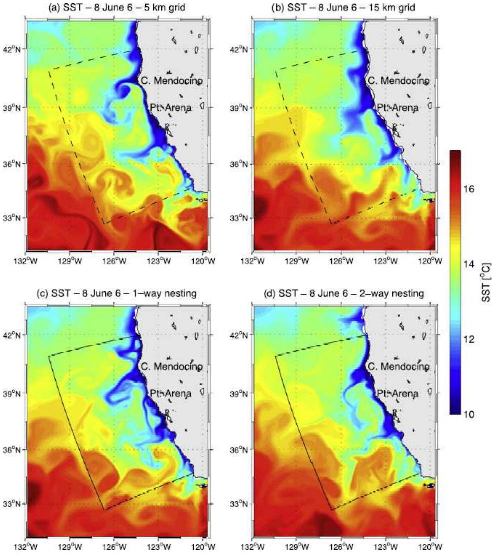



Fig. 23. Sea surface temperature [°C] for 8 June of model year 6. a: REF. b: LOW. c: 1-WAY. d: 2-WAY.

# Realistic example: California Current System



Surface vorticity
July 8, year 5

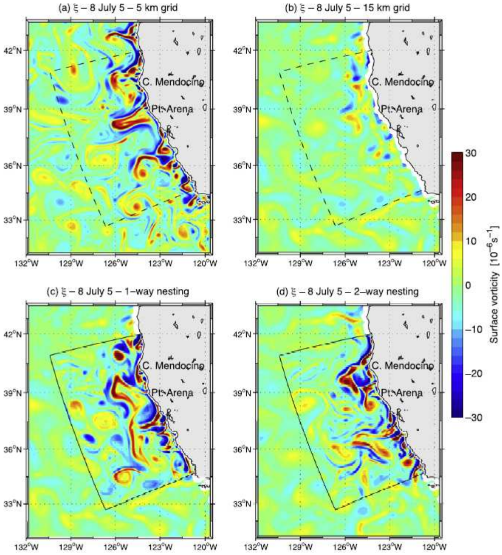



Fig. 24. Sea surface vorticity $[10^6\ s^{-1}]$ for 8 July of model year 5. a: REF. b: LOW. c: 1-WAY. d: 2-WAY.

# Conclusions

- There is no "all singing, all dancing" solutions to nesting of models. All methods are imperfect
- The principles of two-way nesting is simple and straightforward
- Impossible to perfectly conserve mass (volume), transport, fluxes, energy, etc.
- Parent solution has large influence on the child solution independent of nesting technique used

# Conclusions (cont.)

- Two-way nesting improves the parent solution both outside and inside of its domain
- Child is less detoriated using two-way nesting compared to one-way nesting
- Important that the restriction operator $\mathcal{R}$ is sufficiently strong to filter out the small scales created in child, and at the same time is sufficiently weak so as to retain energy on scales resolved by Parent
- Staggered grids and time splitting methods dramatically complicates matters

# Final remarks

In the literature many authors state that the refinement factor must be limited to say $i_r \leq 5$. Why?

- When the resolved spatial scales of parent and child become too different noise aggregates near interfaces which may lead to instabilities
- When the resolved temporal scales become too different inaccuracies due to the time interpolation at the interfaces grows
- Above all, too large differences in spatial and temporal scales leads to growing inaccurasies in the conservation of mass (volume), transports, energy etc. across interfaces

# Is the refinement factor restricted?