

momentum equations

$$\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} + 2\vec{\Omega} \times \vec{u} = -\frac{1}{\rho} \nabla p - \vec{g} + \frac{1}{\rho} \nabla^2 \vec{u}$$

$\downarrow$ acceleration $\downarrow$ advection $\downarrow$ Coriolis $\downarrow$ pressure $\downarrow$ gravity $\downarrow$ stress
linear nonlinear forcing dissipation

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} - f_v = \frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{1}{\rho} \frac{\partial^2 u}{\partial x^2}$$

$$\frac{1}{\rho} = \frac{1}{\rho_0 + \rho'} \approx \frac{1}{\rho_0}$$

density equation $\rho \rho$

$$\frac{d\rho}{dt} + \vec{u} \cdot \nabla \rho + \rho (\nabla \cdot \vec{u}) = 0$$

$\downarrow$ time $\downarrow$ advection

$$\rho = \rho_0 + \rho'(x, y, z, t) \quad |\rho'| \ll \rho_0$$

Boussinesq

$$\frac{d\rho'}{dt} + \vec{u} \cdot \nabla \rho' + \rho_0 (\nabla \cdot \vec{u}) = 0$$

$$\approx \nabla \cdot \vec{u} = 0 \quad \text{incompressible}$$

$$\frac{\partial}{\partial z} \left(\frac{\tau^x}{\rho} \right) \rightarrow F^x + \nu \nabla^2 u$$

viscosity molecular $\nu \sim 10^{-5} \text{ m}^2/\text{s}$

$$\vec{u} \cdot \nabla u = u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z}$$

$$u(\nabla \cdot \vec{u}) = 0$$

$$\nabla \cdot (\vec{u} \otimes \vec{u}) = \begin{matrix} uu & uv & uw \\ uv & vv & vw \\ uw & vw & ww \end{matrix}$$

scaling

$$\frac{\partial u}{\partial t} + \vec{u} \cdot \nabla u - f_v = \frac{1}{\rho_0} \frac{\partial p}{\partial x} + F^x + \nu \nabla^2 u$$

$\frac{L^2}{\nu}$	$\frac{U}{T}$	$\frac{U^2}{L}$	f_v	$\frac{\Delta p}{\rho_0 L}$	F	$\frac{\nu U}{L^2}$
$\frac{L^2}{\nu T}$	$\frac{L U}{\nu}$	$\frac{f_v L^2}{\nu}$	$\frac{\Delta p L}{\rho_0 \nu}$	$\frac{F L^2}{\nu}$	1	

$Re = \frac{UL}{\nu}$ Reynolds number

Atmos weather $Re = \frac{10^4 \cdot 10}{10^{-5}} = 10^{12}$ Lake up $Re = \frac{(0.1)(100)}{10^{-5}} = 10^3$

$\left(\frac{L^2}{\nu T} = \frac{T_v}{T} \right) \quad T_v = \frac{L^2}{\nu} \rightarrow$ damping time

$T_v = \frac{(10^4)^2}{10^{-5}} = 10^{13} \text{ sec} \approx 3 \times 10^5 \text{ years}$

$T_v = \frac{(0.1)^2}{10^{-5}} = 10^3 \text{ sec} \approx 1 \text{ min?}$

Atmos 10^{12} Ocean 10^9 River $\frac{1(100)}{1} = 10^2$

$$\frac{\partial u}{\partial t} + \vec{u} \cdot \nabla u - f_v = \frac{1}{\rho} \frac{\partial p}{\partial x} + F^x + \nu \nabla^2 u$$

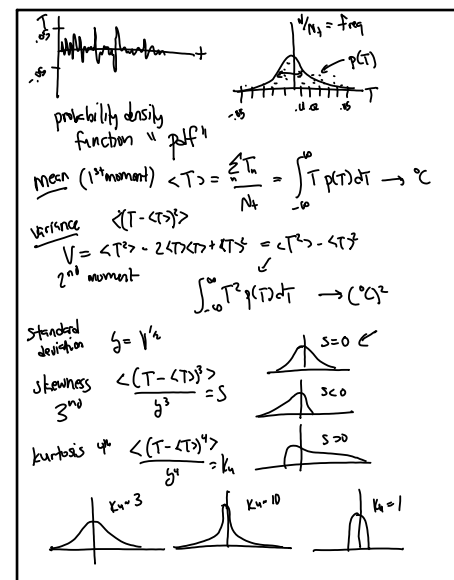
$\frac{U}{T}$	$\frac{U^2}{L}$	f_v	$\frac{\Delta p}{\rho_0 L}$	F	$\frac{\nu U}{L^2}$
$\frac{1}{T}$	$\frac{U}{L}$	1	$\frac{\Delta p}{\rho_0 L}$	$\frac{F}{f_v}$	$\frac{\nu}{L^2}$

Atmos $\frac{10}{(10^4)(10^6)}$

$R_0 \approx 0.1$

$\Omega = \frac{2\pi}{24} \sim 10^{-4} / \text{sec}$

$f = 2\Omega \sin \theta \sim 10^{-4}$



Fourier transform
 $T(t) = \sum \hat{T}_n e^{i\omega_n t} = \sum \hat{T}_n \sin(\omega_n t)$ inverse Transform

$\omega_1 = \frac{2\pi}{L}$
 $\omega_n = \frac{2\pi n}{L}$
 $\omega_{10} = \frac{2\pi(10)}{L}$
 $\omega_{100} = \frac{2\pi(100)}{L}$

$T = \hat{T}_1 \sin(\omega_1 t) + \hat{T}_3 \sin(3\omega_1 t) + \hat{T}_5 \sin(5\omega_1 t)$

$\int_0^L x \sin(\omega_1 t) dt = \int_0^L \hat{T}_1 \sin(\omega_1 t) dt + \int_0^L \hat{T}_3 \sin(3\omega_1 t) dt + \int_0^L \hat{T}_5 \sin(5\omega_1 t) dt$

$= \frac{1}{2} \hat{T}_1 + 0 + 0$

$\hat{T}_n = \int_0^L T(t) e^{-i\omega_n t} dt$ Fourier transform

real space fourier space
 $T(x) \leftrightarrow \hat{T}_n(\omega)$

$\psi(x,y) = \sum_m \hat{\psi}_m e^{ik_x x + i l_y y}$ k, l wavenumbers

$v = \frac{\partial \psi}{\partial x} = \sum_{n,m} i k \hat{\psi} e^{ik_x x + i l_y y} - \frac{\partial \psi}{\partial y} = \sum_{n,m} i l \hat{\psi} e^{ik_x x + i l_y y} = u$

$\nabla^2 \psi \rightarrow -(k^2 + l^2) \hat{\psi}$

$KE = \int \frac{u^2 + v^2}{2} dx dy \rightarrow \sum_{n,m} \frac{1}{2} |\hat{u}|^2 + \frac{1}{2} |\hat{v}|^2$
 $= \sum_{n,m} \frac{1}{2} (k^2 + l^2) |\hat{\psi}|^2$ Parseval's

