

find C_{total}

$$\begin{aligned} a) C_{total} &= C_1 + C_2 + C_3 \\ &= 47pF + 10pF + 1000pF \\ &= \boxed{1057pF} \end{aligned}$$

Useful:

$$C = \frac{Q}{V}$$

parallel: $C_{total} = \sum_{n=1}^N C_n$

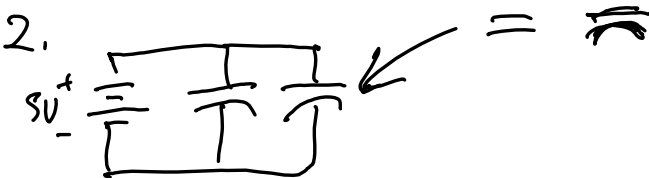
series: $\frac{1}{C_{total}} = \sum_{n=1}^N \frac{1}{C_n}$

Discharge:

$$V_c = V_0 \cdot e^{-t/\tau}$$

charge:

$$V_c = V_F (1 - e^{-t/\tau})$$



$$C_{total} = 2.2\mu F + 3.3\mu F = \boxed{5.5\mu F}$$

$$Q = C \cdot V = 5.5\mu F \cdot 5V = \boxed{27.5\mu C}$$

3. $\tau = RC$

a) $\tau = (100\Omega) \cdot (100\mu F) = \boxed{10ms}$

$$\Omega = \frac{V}{A} = \frac{V \cdot s}{C} \quad F = \frac{C}{V}$$

$\uparrow \frac{C}{s}$

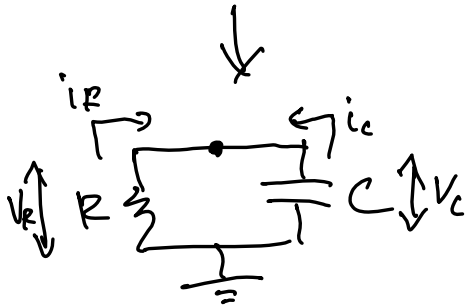
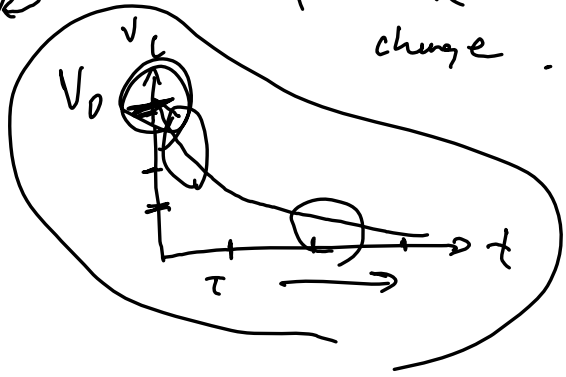
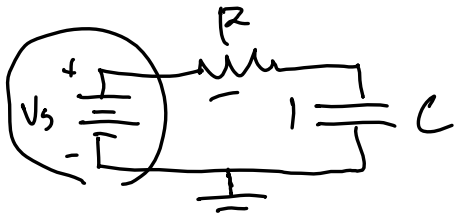
$$\Omega \cdot F = \frac{Vs}{C} \cdot \frac{C}{V} = s$$

4. fully loaded: $t = 5\tau$

$$V = V_0 (1 - e^{-5}) = \underline{0.99V_0}$$

a) $5RC = 5(47\Omega)(47\mu F) = \boxed{11ms}$

$$V_C = V_0 \cdot e^{-t/RC} \leftarrow \text{discharge } \left\{ V_C = V_R(1 - e^{-t/RC}) \right. \\ \left. \text{charge} \right.$$



KCL:

$$i_R + i_C = 0$$

$$i_R = \frac{V_R}{R} \quad i_C = C \cdot \frac{dV_C}{dt}$$

KVL:

$$V_R = V_C \quad \left(\frac{V_R}{R} + C \cdot \frac{dV_C}{dt} = 0 \right) \frac{1}{C}$$

$$\frac{dV_C}{dt} + \frac{1}{RC} \cdot \underline{V_R} = 0 \quad (1)$$

$$\left. \begin{array}{l} \frac{dx}{dt} + ax = 0 \\ x = k \cdot e^{-at} \end{array} \right\} \Rightarrow \frac{dV_C}{dt} + \frac{1}{RC} \cdot V_C = 0$$

$$V_C = x \quad V_C = \underline{k} \cdot e^{-\frac{t}{RC}}$$

$$a = \frac{1}{RC}$$

$$= V_0 \cdot e^{-\frac{t}{RC}}$$

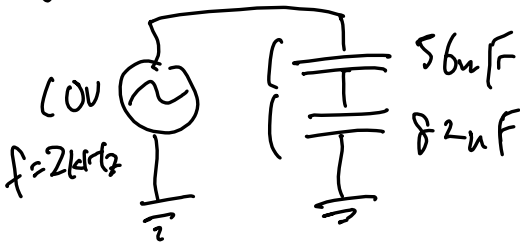
$$= V_0 \cdot e^{-t/\tau}$$

where
 $\tau = RC$

$$5. \quad X_c = \frac{1}{2\pi f \cdot C}$$

$$a) \quad X_c = \frac{1}{2\pi (10\text{Hz}) \cdot 0.047\mu\text{F}} = \boxed{339\text{k}\Omega}$$

6.



$$a) \quad X_{C_1} = \frac{1}{2\pi (2\text{kHz}) (56\mu\text{F})} = \boxed{1.42\text{k}\Omega}$$

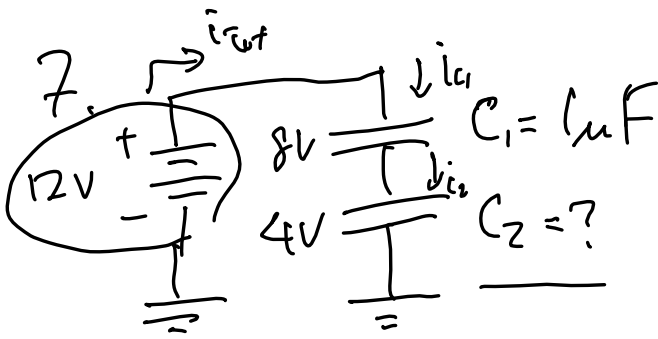
$$b) \quad X_{C_2} = \frac{1}{2\pi (2\text{kHz}) (82\mu\text{F})} = 971\Omega$$

$$c) \quad X_{C_T} = X_{C_1} + X_{C_2} = \boxed{2.39\text{k}\Omega}$$

$$d) \quad V_{C_1} = V_s \cdot \frac{X_{C_1}}{X_{C_{Total}}} = 10\text{V} \cdot \frac{1.42\text{k}\Omega}{2.39\text{k}\Omega} = \boxed{5.94\text{V}}$$

$$V_{C_2} = 10\text{V} - V_{C_1} = \boxed{4.06\text{V}}$$

$$V_{C_{Total}} = V_{C_1} + V_{C_2}$$



$$KCL: i_{total} = i_{c1} = i_{c2} \quad i = \frac{Q}{t}$$

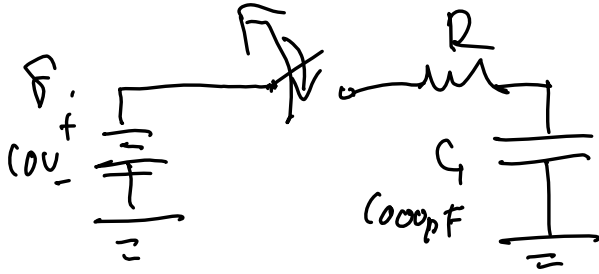
$$Q_{Total} = Q_{c1} = Q_{c2} \quad Q = CV$$

$$\underline{C_1 \cdot V_{c1}} = \underline{C_2 \cdot V_{c2}}$$

$$C_2 = C_1 \cdot \frac{V_{c1}}{V_{c2}}$$

$$= 1 \mu F \cdot \frac{8V}{4V}$$

$$= \boxed{2 \mu F}$$



$$t = 10 \mu s$$

$$V_C = 7.2 V$$

find R

charging: $V_C = V_F (1 - e^{-\frac{t}{RC}})$

$$1 - e^{-\frac{t}{RC}} = \frac{V_C}{V_F}$$

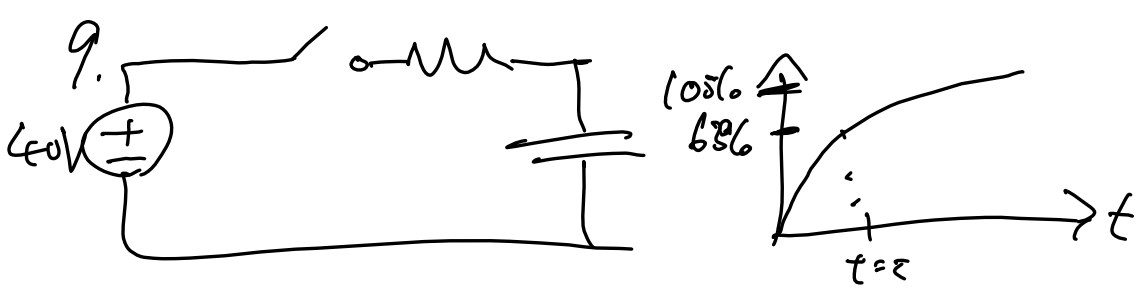
$$e^{-\frac{t}{RC}} = 1 - \frac{V_C}{V_F}$$

$$-\frac{t}{RC} = \ln\left(1 - \frac{V_C}{V_F}\right)$$

$$RC = \frac{-t}{\ln\left(1 - \frac{V_C}{V_F}\right)}$$

$$R = \frac{-t}{C \cdot \ln\left(1 - \frac{V_C}{V_F}\right)}$$

$$= \frac{-10 \mu s}{1000 pF \cdot \ln\left(1 - \frac{7.2V}{10V}\right)} = \boxed{7.86 k\Omega}$$



a) time constant

a measure how fast capacitor charges / discharges.

b) $t = \tau$

$$e^{-\frac{t}{\tau}} = e^{-1} = \frac{1}{e} = 0.37$$

37%

$$c) 1 - e^{-\frac{t}{\tau}} = 1 - e^{-1} = 1 - 0.37$$

$= 0.63$
63%

d) by convention, $t = 5\tau$ (~99%)

e) $\tau = RC$

$$C = \frac{\tau}{R} = \frac{16 \text{ ms}}{4 \text{ k}\Omega} = \text{span style="border: 1px solid black; padding: 2px;">4 \mu\text{F}}$$

$$f) \tau = RC = 8 \text{ k}\Omega \cdot 4 \mu\text{F} = \text{span style="border: 1px solid black; padding: 2px;">32 \text{ ms}$$

$$g) \underline{V_c(t)} = \text{span style="border: 1px solid black; padding: 2px;">} V_F (1 - e^{-t/\tau_c}) \text{span style="border: 1px solid black; padding: 2px;">}$$

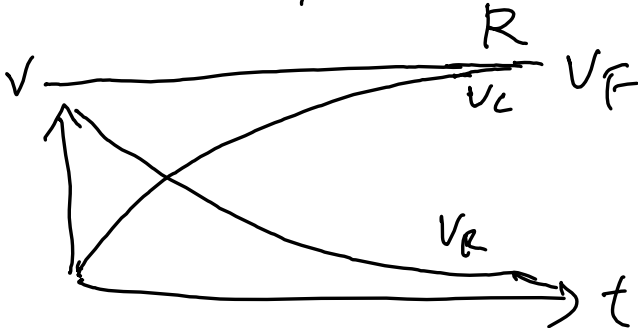
$$\underline{V_R(t)} = V_F - V_c$$

$$= V_F - V_F (1 - e^{-t/\tau_c})$$

$$= V_F (\cancel{1} - (\cancel{1} - e^{-t/\tau_c}))$$

$$= \text{span style="border: 1px solid black; padding: 2px;">} V_F \cdot e^{-t/\tau_c} \text{span style="border: 1px solid black; padding: 2px;">}$$

$$i_c(t) = i_R(t) = \frac{V_R(t)}{R}$$



$$h) V_c(20 \text{ ms}) = 40 \text{ V} (1 - e^{-\frac{20 \text{ ms}}{32 \text{ ms}}})$$

18.58 V

$$i) V_c(2\tau) = V_c(64 \text{ ms}) = 40 \text{ V} (1 - e^{-2}) = 34.6 \text{ V}$$

$$Q = VC = 34.6 \text{ V} \cdot 4 \mu\text{F} = \text{span style="border: 1px solid black; padding: 2px;">138.4 \mu\text{C}$$

$$j) V_c = 40 \text{ V} = V_F$$

$$Q = VC = 40 \text{ V} \cdot 4 \mu\text{F} = \text{span style="border: 1px solid black; padding: 2px;">160 \mu\text{C}$$