Utsagnslogikk

$P, R \Rightarrow$ utsagnsvariabler

$F, G \Rightarrow$ formler

$F \wedge G$, $F \vee G$, $F \to G$

$\neg F$

Syntaksen

$v \Rightarrow$ valuasjon

$v(P) = 1$ $\quad v(R) = 0$

$1 \wedge 1 = 1$

$1 \vee 0 = 1$

$\vdots$

Førsteordens språk

x variabler $\langle a, b, \ldots ; f, g, \ldots ; P, R, \ldots \rangle$

termer

atomære formler

$\varphi \quad \psi \Rightarrow \varphi \wedge \psi, \varphi \to \psi, \ldots$

$\forall x \varphi \quad \exists x \varphi$

Syntaks $\quad \langle x \rangle = x$

$\Rightarrow$ gi en modell $\mathcal{M}$

- gi domenet D, $|\mathcal{M}| = \{a, b, c\}$
- $k^{\mathcal{M}}$, k-konstantsymbol $k^{\mathcal{M}} = a$
- $f^{\mathcal{M}}$, $f: D^n \to D$

  $\to$ aritet 2 $f: D \times D \to D$ $\quad P^{\mathcal{M}} = \{\langle x \rangle, \langle y \rangle, \ldots\}$ ($\to$ aritet 1)
- $R^{\mathcal{M}}$ $\to$ delmengde av D^n $\quad R^{\mathcal{M}} = \{\langle x, y \rangle, \ldots\}$ ($\to$ aritet 2)

aritet 1 $\leftarrow$ $P^{\mathcal{M}} = \{ a, b, c, \dots \}$

aritet 2 $\leftarrow$ $R^{\mathcal{M}} = \{ \langle a, b \rangle, \langle a, c \rangle, \dots \}$

modell

$|\mathcal{M}| = \{a, b\}$

$a^{\mathcal{M}} = a$

$b^{\mathcal{M}} = b$

$R^{\mathcal{M}} = \{\langle a, b \rangle\}$

$\langle a, b ;; R \rangle$

aritet 2

1. er Rab sann?

$\langle a, b \rangle \in R^{\mathcal{M}}$ $\Rightarrow$ sann

2. $\exists_x Rxb$ sann? $\Rightarrow$ Sann i $\mathcal{M}$

3. $\forall_x \boxed{Rxb} \Rightarrow \underbrace{Rab}_{\text{sann}} \wedge \underbrace{Rbb}_{\text{usann}}$

$\Downarrow$

usann i $\mathcal{M}$

usann

φ – oppfyllbar => gi en konkrett modell hvor φ er sann

– gyldig => gi en vilkårlig modell ~~hvo~~ og vise at φ er sann

– falsifiserbar => gi en konkrett modell hvor φ er usann

– motsigelse => vise at φ er usann i en vilkårlig modell

$\mathcal{M}$

$|\mathcal{M}| = \{1, 2\}$

$a^{\mathcal{M}} = 1$

$f^{\mathcal{M}}$ ...

La $\mathcal{M}$ være en modell

$\{1, 2, 3\}$ x

- $\forall$ lukket formel
- $\boxed{\bar{e}}$ Avtalelse: det finnes en konstant for hver elem fra domenet

$$|\mathcal{M}| = \{1, 2, 3, 4\} \quad \langle 1, 2 ;\, ; \rangle$$

$$\bar{3} \quad \bar{4}$$

sann / sann

15.2. b) $(\forall x Rxa \lor \forall x Rxb) \to \forall x (Rxa \lor Rxb)$

$a, b, c, \ldots$

$x, y, z, \ldots$

La $\mathcal{M}$ være en vilkårlig modell

$a^{\mathcal{M}} \in |\mathcal{M}|$

$b^{\mathcal{M}} \in |\mathcal{M}|$

Anta at $(\forall x Rxa \lor \forall x Rxb)$ er sann i $\mathcal{M}$

$\Rightarrow \mathcal{M} \models \forall x Rxa \lor \forall x Rxb$

$\Rightarrow \mathcal{M} \models \forall x Rxa$ eller $\mathcal{M} \models \forall x Rxb$

1) $\mathcal{M} \models \forall x Rxa$

$\Rightarrow \mathcal{M} \models R\bar{e}a$, for alle $e \in |\mathcal{M}|$

$\Rightarrow \mathcal{M} \models R\bar{e}a \lor R\bar{e}b$ | Rab, $R\bar{b}b$, Rbb

sann

sann

$\Rightarrow \mathcal{M} \models \forall x (Rxa \lor Rxb)$

2) $\mathcal{M} \models \forall x Rxb$

$\Rightarrow \mathcal{M} \models R\bar{e}b$, for alle $e \in |\mathcal{M}|$

$\Rightarrow \mathcal{M} \models R\bar{e}a \lor R\bar{e}b$

sann

sann

$\Rightarrow \mathcal{M} \models \forall x (Rxa \lor Rxb)$

15.2.d) $\forall x(Rxa \overset{\text{sann}}{\vee} Rxb) \rightarrow \exists x(Rax \overset{\text{usann}}{\wedge} Rbx) \Rightarrow$ falsifiserbar

La $|M| = \{a, b\}$

$a^M = a$

$b^M = b$

$R^M = \{\langle a, a\rangle, \langle b, b\rangle\}$

$\Rightarrow \forall x(Rxa \vee Rxb)$ sann?

sann

$x = a \Rightarrow Raa \vee Rab$

sann

sann

$x = b \Rightarrow Rba \vee Rbb$

sann

sann

usann

$\exists x(Rax \wedge Rbx)$

$x = a$

$\Rightarrow Raa \wedge Rba$

sann usann

usann

$x = b$

$\Rightarrow Rab \wedge Rbb$

usann sann

usann