

Utsagnslogikk

$P, R \Rightarrow$ utsagnsvariabler

$F, G \Rightarrow$ formler

$F \wedge G, F \vee G, F \rightarrow G$

\neg

Syntaksen

$V \Rightarrow$ valuasjon

$V(P) = 1$

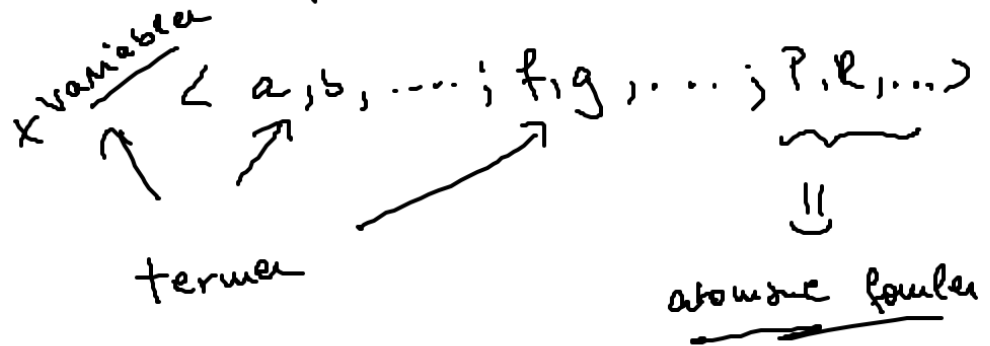
$V(R) = 0$

$1 \wedge 1 = 1$

$1 \vee 0 = 1$

\vdots

Førsteordens språk



$\varphi \psi \Rightarrow \varphi \wedge \psi, \varphi \rightarrow \psi, \dots$

$\forall x \varphi \quad \exists x \varphi$

Syntaks

$\langle x \rangle = x$

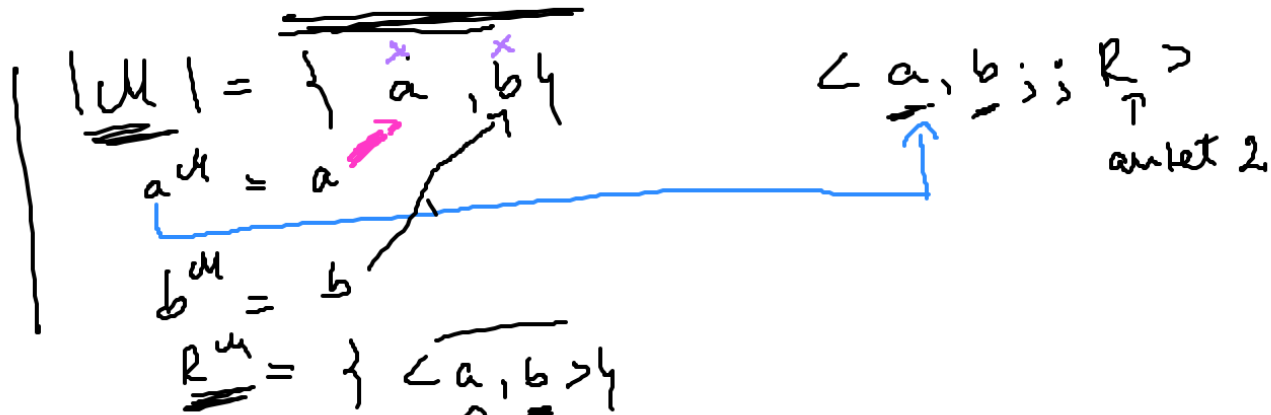
\Rightarrow gi en model \mathcal{M}

- gi domenet D , $|D| = \{a, b, c, \dots\}$
- $c^{\mathcal{M}}$, c -konstantymbol $c^{\mathcal{M}} = a$
- $f^{\mathcal{M}}$, $f: D^n \rightarrow D$
 - aritet 2 $f: D \times D \rightarrow D$ $f^{\mathcal{M}} \rightarrow$ aritet 2
 $f^{\mathcal{M}} = \{ \langle x, y \rangle, \langle y, x \rangle, \dots \}$
- $R^{\mathcal{M}}$ - delmengde av D^k $R^{\mathcal{M}} \rightarrow$ aritet 2
 $R^{\mathcal{M}} = \{ \langle x, y \rangle, \dots \}$

anitet 1 $\leftarrow \mathbb{P}^M = \{ a, b, c, \dots \}$

anitet 2 $\leftarrow \mathbb{R}^M = \{ \langle a, b \rangle, \langle a, c \rangle, \dots \}$

Modell



1. $\exists x \in R \text{ a } b$ Sann? \Rightarrow Sann
 $\langle a, b \rangle \in R^U$

2. $\exists x \in R \times b$ sann? \Rightarrow Sann iU

3. $\forall x \in R \times b \Rightarrow R \text{ a } b \wedge R \text{ b } b$
Sann \wedge Sann
Sann
Sann iU

- φ - oppfyltbar \Rightarrow gi en konkrett modell hvor φ er sann
- gyldig \Rightarrow gi en vilkårlig modell ~~hvor~~ og vise at φ er sann
- falsifiserbar \Rightarrow gi en konkret modell hvor φ er usann
- motsigelse \Rightarrow vise at φ er usann i en vilkårlig modell

\mathcal{A}
 $\{ \mathcal{A} \} = \{ 1, 2 \}$
 $a^{\mathcal{A}} = 1$
 $f^{\mathcal{A}} \dots$

\mathcal{A} \mathcal{A} vise en modell
 $\{ 1, 2, 3 \}$ ✗
1

• γ Kubitet formel

• \bar{e} Antakelse: det finnes en constant for
hver elem for domenet

$$|d| = \{ (1, 2, \overset{\downarrow}{3}, \overset{\downarrow}{4}) \} \quad \langle \overset{\cdot}{1}, \overset{\cdot}{2}, \overset{\cdot}{3}, \overset{\cdot}{4} \rangle$$

$\bar{3} \quad \bar{4}$

15.2. b) $(\forall x R_{x a} \vee \forall x R_{x b}) \rightarrow \forall x (R_{x a} \vee R_{x b})$

sann (sann)

La \mathcal{M} være en vilkårlig modell

$a, b, c \dots$
 x, y, z, \dots

$$\begin{cases} a^{\mathcal{M}} \in |M| \\ b^{\mathcal{M}} \in |M| \end{cases}$$

Anta at $(\forall x R_{x a} \vee \forall x R_{x b})$ er sann i \mathcal{M}

$$\Rightarrow \mathcal{M} \models \forall x R_{x a} \vee \forall x R_{x b}$$

$$\Rightarrow \mathcal{M} \models \forall x R_{x a} \text{ eller } \mathcal{M} \models \forall x R_{x b}$$

1) $\mathcal{M} \models \forall x R_{x a}$

$$\Rightarrow \mathcal{M} \models R_{\bar{c} a}, \text{ for alle } \bar{c} \in |M|$$

$$\Rightarrow \mathcal{M} \models \underbrace{R_{\bar{c} a}}_{\text{sann}} \vee \underbrace{R_{\bar{c} b}}_{\text{sann}} \quad \begin{matrix} R_{ab} \\ R_{\bar{c} b} \\ R_{bb} \end{matrix}$$

$$\Rightarrow \mathcal{M} \models \forall x (R_{x a} \vee R_{x b})$$

2) $\mathcal{M} \models \forall x R_{x b}$

$$\Rightarrow \mathcal{M} \models R_{\bar{c} b}, \text{ for alle } \bar{c} \in |M|$$

$$\Rightarrow \mathcal{M} \models R_{\bar{c} a} \vee \underbrace{R_{\bar{c} b}}_{\text{sann}}$$

$$\Rightarrow \mathcal{M} \models \forall x (R_{x a} \vee R_{x b})$$

15.2.d) $\forall x (R_{xa} \vee R_{xb}) \rightarrow \exists x (R_{ax} \wedge R_{bx}) = 1$ falsifizierbar

La $\{a, b\}$

$a^u = a$

$b^u = b$

$R^u = \{ \langle a, a \rangle, \langle b, b \rangle \}$

$\Rightarrow \forall x (R_{xa} \vee R_{xb})$ Sann?
 $x = a \Rightarrow R_{aa} \vee R_{ab}$
Sann
Sann

$x = b \Rightarrow R_{ba} \vee R_{bb}$
Sann
Sann

usann

$\exists x (R_{ax} \wedge R_{bx})$

$x = a$
 $\Rightarrow R_{aa} \wedge R_{ba}$
Sann usann
usann

$x = b$
 $\Rightarrow R_{ab} \wedge R_{bb}$
usann Sann
usann