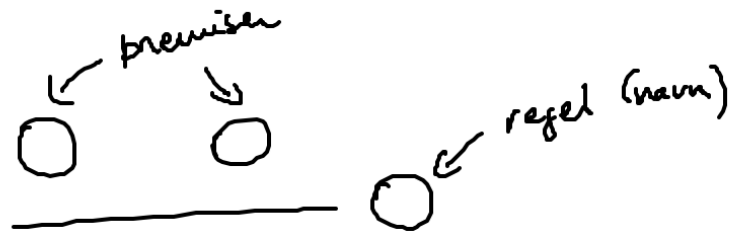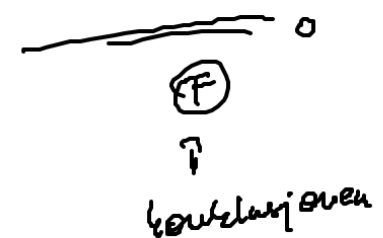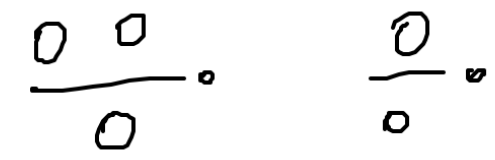2h: naturlig deduksjon

↑
Syntaktisk beskrivelse av gyldig, motsigelse

$\land, \rightarrow, \bot \Rightarrow \neg F \iff F \rightarrow \bot$

premisen

◯   ◯ ← regel (navn)
_____   ◯

◯
↑
konklusjon

◯ ◯
___ ∘
◯

↑ alle antakelser
er lukket

◯
___ ∘
◯

_____ ∘
(F)
↑
konklusjonen

F er
gyldig   ⟹   <u>bevis</u> : alle
antakelsen
lukket

$\wedge$

$$\frac{F \qquad G}{F \wedge G} \wedge i$$

$$\frac{F \wedge G}{F} \wedge E$$

$$\frac{F \wedge G}{G} \wedge E$$

c) direkte bevis

$\Rightarrow$

$$\frac{F \to G \qquad F}{G} \to E$$

$$\begin{array}{l} [\overline{F}]_1 \\ \vdots \\ G \end{array} \to i_1 \quad \to flere\ regler$$

$$\frac{}{F \to G}$$

$$\frac{}{\bot} \equiv$$

$$\frac{\bot}{F} \quad \bot$$

$$\begin{array}{c} \neg F \\ \{F \to \bot\}_1 \\ \vdots \\ \bot \\ \hline F \end{array} \quad \boxed{RAA}$$

$$\Rightarrow \text{motrigelsesbevis}$$

$$\Rightarrow \quad 0 \text{ eller flere} \\ \text{regler}$$

ex: $P \wedge Q \to Q \wedge P$

$$\frac{\{P \wedge Q\}_1}{Q} \wedge E \qquad \frac{\{P \wedge Q\}_1}{P} \wedge E$$

$$\frac{Q \wedge P}{} \wedge i$$

$$\frac{}{P \wedge Q \to Q \wedge P} \to i_1$$

$$\frac{\{P \wedge Q\}_1}{Q} \wedge E \quad \frac{\{P \wedge Q\}_1}{P} \wedge E$$

$$\frac{Q \wedge P}{} \wedge i$$

$$\frac{}{P \wedge Q \to Q \wedge P} \to i_1$$

24.5

a) $(A \to B) \to (\neg B \to \neg A)$

$B \to \bot$

$A \to \bot$

$$\frac{\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{[A \to B]_3 \ [A]_2}{B} \quad [\neg B]_1}{\bot} \to_i 2}{A \to \bot} \to_i 1}{\neg B \to \neg A}}{(A \to B) \to (\neg B \to \neg A)} \to_i 3}$$

$\cfrac{\bot}{\not{\;}} \bot \qquad \cfrac{\cfrac{}{\neg F}}{\not F} \; RAA$

RAA er med

$\Downarrow$

ikke
intuisjonistisk

24.10

c)

$\neg\neg\neg p \rightarrow \neg p$

$\boxed{P \rightarrow \bot \rightarrow \bot}$ $\Rightarrow$ $\boxed{P \rightarrow \bot \rightarrow \bot}$

$$\frac{[\neg\neg\neg q]_2 \quad [\neg\neg p]_1}{\frac{\bot}{\neg p} \; RAA_1} \rightarrow E$$

$$\frac{\bot}{\neg\neg\neg p \rightarrow \neg p} \rightarrow i_2$$

$$\frac{F \rightarrow G \qquad F}{G} \rightarrow E$$

$\downarrow$ ... $\uparrow$

$P \rightarrow \bot \rightarrow \bot$ $\qquad$ $\{P \rightarrow \bot \rightarrow \bot\}$

$$\frac{[\neg\neg p]_1 \quad \neg\neg p}{\frac{\bot}{\neg p}} \rightarrow E \rightarrow i$$

$$\frac{}{\neg\neg\neg p \rightarrow \neg p} \rightarrow i_1$$

$\neg\neg P$

P

$$\frac{\boxed{P \rightarrow \bot \rightarrow \bot} \quad \{P \rightarrow \bot \rightarrow \bot\}}{\bot} \rightarrow E$$

$$\frac{\bot}{\boxed{P \rightarrow \bot}} \rightarrow i$$