CHAPTER 1

EUCLID'S GEOMETRY

The postulate on parallels . . . was in antiquity the final solution of a problem that must have preoccupied Greek mathematics for a long period before Euclid.

HANS FREUDENTHAL

THE ORIGINS OF GEOMETRY

The word "geometry" comes from the Greek *geometrein* (*geo-*, "earth," and *metrein,* "to measure"); geometry was originally the science of measuring land. The Greek historian Herodotus (5th century B.C.) credits Egyptian surveyors with having originated the subject of geometry, but other ancient civilizations (Babylonian, Hindu, Chinese) also possessed much geometric information.

Ancient geometry was actually a collection of rule-of-thumb procedures arrived at through experimentation, observation of analogies, guessing, and occasional flashes of intuition. In short, it was an empirical subject in which approximate answers were usually sufficient for practical purposes. The Babylonians of 2000 to 1600 B.C. considered the circumference of a circle to be three times the diameter; i.e., they took π to be equal to 3. This was the value given by the Roman architect Vitruvius and it is found in the Chinese literature as well. It was even considered sacred by the ancient Jews and sanctioned in scripture (I Kings 7:23) — an attempt by Rabbi Nehemiah to change

the value of π to $\frac{22}{7}$ was rejected. The Egyptians of 1800 B.C., according to the Rhind papyrus, had the approximation $\pi \sim (\frac{16}{9})^2 \sim 3.1604$.[1]

Sometimes the Egyptians guessed correctly, other times not. They found the correct formula for the volume of a frustum of a square pyramid — a remarkable accomplishment. On the other hand, they thought that a formula for area that was correct for rectangles applied to *any* quadrilateral. Egyptian geometry was not a science in the Greek sense, only a grab bag of rules for calculation without any motivation or justification.

The Babylonians were much more advanced than the Egyptians in arithmetic and algebra. Moreover, they knew the Pythagorean theorem — in a right triangle the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs — long before Pythagoras was born. Recent research by Otto Neugebauer has revealed the heretofore unknown Babylonian algebraic influence on Greek mathematics.

However, the Greeks, beginning with Thales of Miletus, insisted that geometric statements be established by deductive reasoning rather than by trial and error. Thales was familiar with the computations, partly right and partly wrong, handed down from Egyptian and Babylonian mathematics. In determining which results were correct, he developed the first logical geometry (Thales is also famous for having predicted the eclipse of the sun in 585 B.C.). The orderly development of theorems by *proof* was characteristic of Greek mathematics and entirely new.

The systematization begun by Thales was continued over the next two centuries by Pythagoras and his disciples. Pythagoras was regarded by his contemporaries as a religious prophet. He preached the immortality of the soul and reincarnation. He organized a brotherhood of believers that had its own purification and initiation rites, followed a vegetarian diet, and shared all property communally. The Pythagoreans differed from other religious sects in their belief that elevation of

[1] In recent years π has been approximated to a very large number of decimal places by computers; to five places, π is approximately 3.14159. In 1789 Johann Lambert proved that π was not equal to any fraction (rational number), and in 1882 F. Lindemann proved that π is a *transcendental number,* in the sense that it does not satisfy any algebraic equation with rational coefficients, which implies that in the Euclidean plane, it is impossible to square a circle using only straightedge and compass.

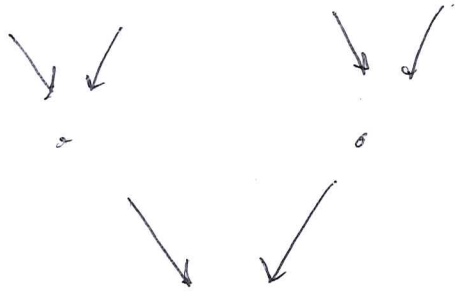
GAMLE EGYPT (før-ANTIKK)

PROBLEM

- IKKE NOE METODE
- IKKE NOE SIKKERHET

DEN AKSIOMATISKE METODE

SANNE PREMISSER: P_1 P_2 ... P_4



GYLDIGE SLUTNINGER:

SANN KONKLUSJON: K

→ GIR MATEMATIKK EN METODE

→ SIKKERHET!

MEN HVILKE SLUTNINGER ER GYLDIGE??

"Dirichlet alone, not I, nor Cauchy, nor Gauss knows what a completely rigorous mathematical proof is. Rather we learn it first from him. When Gauss says that he has proved something, it is very clear; when Cauchy says it, one can wager as much pro as con; when Dirichlet says it, it is certain..."

Dirichlet allein, nicht ich, nicht Cauchy, nicht Gauß, weiß, was ein vollkommen strenger Beweis ist, sondern wir lernen es erst von ihm. Wenn Gauß sagt, er habe etwas bewiesen, so ist es mir sehr wahrscheinlich, wenn Cauchy es sagt, ist ebensoviel pro als contra zu wetten, wenn Dirichlet es sagt, ist es *gewiß*; ich lasse mich auf diese Delikatessen lieber gar nicht ein.

Brev fra Carl Gustav Jacob Jacobi to Alexander von Humboldt, desember 21, 1846.

Gottfried Leibniz



Gottfried Wilhelm von Leibniz (1 July 1646 – 14 November 1716) was a German philosopher and mathematician.

Gottfried Leibniz is famous... for his slogan *Calculemus*, which means "Let us calculate." He envisioned a formal language to reduce reasoning to calculation, and he said that reasonable men, faced with a difficult question of philosophy or policy, would express the question in a precise language and use rules of calculation to carry out precise reasoning. This is the first reduction of reasoning to calculation ever envisioned. ...he actually designed and built a working calculating machine, the *Stepped Reckoner* ...inspired by the somewhat earlier work of Pascal, who built a machine that could add and subtract. Leibniz's machine could add, subtract, divide, and multiply, and was apparently the first machine with all four arithmetic capabilities.

- Michael J. Beeson, "The Mechanization of Mathematics," in *Alan Turing: Life and Legacy of a Great Thinker* (2004)

"[...] if controversies were to arise, there would be no more need of disputation between two philosophers than between two calculators. For it would suffice for them to take their pencils in their hands and to sit down at the abacus, and say to each other (and if they so wish also to a friend called to help): Let us calculate."

FORMALISERING

→ MAN DRØFTE OM ET SYSTEM AV REGLER OG SLUTNINGER, SÅ DET BLE HELT KLART OG TYDELIG OM ET BEVIS VAR ET BEVIS.

→ UTSAGNSLOGIKK — 1800 - tallet

→ 19/19 - 00 - tallet : FØRSTE - ORDENS LOGIKK

ALLE GREKERE ER KJEKK

SOKRATES ER GREKER

SOKRATES ER KJEKK

UTSAGNSLOGISK
↓
A
B
C
(IKKE CYCLIC)

→ I TEORIEN MEN IKKE I PRAKSIS DET LEIBNIZ DRØMTE OM.

→ JOBBES MED NÅ (SE HOMOTOPY TYPE THEORY (HoTT)

→ TO (SEPARATE) TING I DETTE KURSET:

TENKE MATEMATISK OG LOGISK

TRENE (MED OPPGAVER)

FORMALISERINGER

→ UTSAGNSLOGIKK

→ FØRSTE-ORDENS LOGIKK

↓

NOEN ULIKE TYPER BEVIS

- BEVIS VED TILFELLER

n ER GITT TALL

VIS AT $n \cdot (n+1) \cdot (n+2)$

KAN DELES PÅ TRE.

ENTEN n I 3-GANGEN

ELLER n+1 — " —

ELLER n+2 — " —

$$\frac{A \vee B \vee C \qquad A \to D \qquad B \to D \qquad C \to D}{D}$$

TILFELLE: 1) n I 3-GANGEN. DA $n \cdot (n+1) \cdot (n+2)$

2) n+1 — " — — " —

3) n+2 — " —

VISE FOR-ALLE-SETNINGER

EKS: A ER GYLDIG.

VIS AT FOR ALLE VALUASJONER V

SÅ ER A SANN UNDER V.

METODE 1: TEST ALLE (BEVIS VED TILFELLER)

METODE 2: LA EN VALUASJON VÆRE GITT

EKS:

$$\frac{\begin{array}{l} A \\ A \to B \\ B \to C \\ C \to D \\ D \to E \end{array}}{E}$$

LA VALUASJON V VÆRE GITT

KAN ANTA AT V GJØR PREMISSENE SANNE (HVORFOR?)

DA ER A SANN.

DA ER B SANN, SIDEN $A \to B$ SANN

OSV

TIL VI FÅR AT E ER SANN

SIDEN V TILFELDIG GITT ↓

SÅ V GJØR E SANN.

DA MÅ ALLE VALUASJONER SOM GJØR PREMISSENE SANNE GJØRE E SANN.

EKS FOR ALLE TALL n SÅ ER $n \cdot (n+1) \cdot (n+2)$ DELELIG PÅ 3.

LA n VÆRE GITT. DA (ARGUMENTET PÅ FORRIGE SIDE)

VISE "DET FINNES" - UTSAGN

EX VIS AT EN SETNING ER FALSIFISERBAR.

VIS AT $A \vee B \vee C \vee D \vee E$ ER FALSIFISERBAR.

BEVIS: TA VALUASJONEN: A, B, C, D OG E FALSK.

DA ER $A \vee B \vee C \vee D \vee E$ FALSK.

SÅ — " — ER FALSIFISERBAR.

MOTSIGELSES BEVIS

ANTAR DET MOTSATTE AV DET DU SKAL VISE,

OG VISER AT DET ER UMULIG.

SKAL VISE AT A ER SANN.

ANTA AT A ER FALSK

UTLED MOTSIGELSE – DET ER IKKE MULIG AT A ER FALSK.

KONKLUDER AT A ER SANN.

NYTT TEMA

RELASJONER

HVA ER EN RELASJON?

VI BEGYNNER MED EN MENGDE, F. EKS D.

ANGI HVILKE TING SOM ER RELATERTE.

VI GJØR DET VED Å SPESIFISERE

EN DELMENGDE AV $D \times D$

EX: ETTERFØLGER-RELASJONEN:

$$\{ \langle n, n+1 \rangle \mid n \in \mathbb{N} \} \subseteq \mathbb{N} \times \mathbb{N}$$

DEFINISJON:

1) EN (BINÆR) RELASJON PÅ EN MENGDE D

ER EN DELMENGDE AV $D \times D$.

$$R \subseteq D \times D$$

2) EN n-ÆR RELASJON PÅ EN MENGDE D

ER EN DELMENGDE AV $D^n = \underbrace{D \times \ldots \times D}_{n \text{ GANGER}}$

$$R \subseteq D^n$$

BINÆRE RELASJONER

– NOEN TYPER

REFLEKSIV (BINÆR) RELASJON

EN RELASJON $R \subseteq D \times D$ ER REFLEKSIV

HVIS $\langle d, d \rangle \in R$, FOR ALLE $d \in D$.

→ SKRIVER OGSÅ $R(d,d)$ I STEDET FOR $\langle d, d \rangle \in R$

→ KAN OGSÅ TEGNE: 

(OG $R(a,b)$, DVS $\langle a, b \rangle \in R$, TEGNE $a \to b$)

Eks:

$$R = \{\langle n, n\rangle \mid n \in \mathbb{N}\} \subseteq \mathbb{N} \times \mathbb{N}$$

1)

$$= \{\langle m, n\rangle \mid m, n \in \mathbb{N} \text{ og } m = n\} \subseteq \mathbb{N} \times \mathbb{N}$$

2) $$R = \mathbb{N} \times \mathbb{N} \subseteq \mathbb{N} \times \mathbb{N}$$

$$\| $$

$$\{\langle m, n\rangle \mid m, n \in \mathbb{N}\}$$

Som graf: R er refleksiv

Hvis det for alle elementer i mengden



går en pil fra elementet til seg selv.

ANTI-REFLEKSIV ELLER IRREFLEKSIV

IRREFLEKSIV /

R ER ANTI-REFLEKSIV HVIS INGEN ELEMENTER

ER RELATERT TIL SEG SELV.

EX:

$$\emptyset \subseteq D \times D$$

SYMMETRISK

EN RELASJON $R \subseteq D \times D$ ER SYMMETRISK

HVIS DET GJELDER FOR ALLE $a, b \in D$

AT

HVIS $R(a,b)$ SÅ $R(b,a)$



ANTI - SYMMETRISK

$R \subseteq D \times D$

EN RELASJON ER ANTI-SYMMETRISK

HVIS DET GJELDER FOR ALLE $a, b \in D$

AT HVIS $R(a,b)$ OG $R(b,a)$

SÅ $a = b$.

TRANSITIV

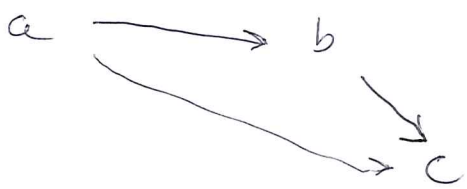
$R \subseteq D \times D$

EN RELASJON ER TRANSITIV

HVIS DET GJELDER FOR ALLE $a, b, c \in D$ AT

HVIS $R(a,b)$ OG $R(b,c)$

SÅ $R(a,c)$



MANDAG → PARTIELT ORDNEDE MENGDER

EKVIVALENS RELASJONER