

UNIVERSITETET I OSLO

Det matematisk-naturvitenskapelige fakultet

Examination in: INF1100 — Introduction to programming with scientific applications

Day of examination: Friday, December 17, 2010

Examination hours: 09.00 – 13.00

This examination set consists of 6 pages.

Appendices: None.

Permitted aids: None.

Make sure that your copy of the examination set is complete before you start solving the problems.

- Read through the complete exercise set before you start solving the individual exercises. If you miss information in an exercise, you can provide your own reasonable assumptions as long as you explain them in detail.
- Most of the exercises result in short code where there is little need for comments, unless you do something complicated or non-standard. In that case, comments should convey the idea behind the program constructions such that it becomes easy to evaluate the solution.
- An exercise may ask you to “write a function”. A main program calling the function is then not required, unless it is explicitly stated. In the function you write, you can assume that necessary modules are already imported outside the function. On the other hand, if you are asked to write a complete program, explicit import of modules is an important part of the solution.
- The maximum possible score on the exam is 75 points. There are 8 exercises, and the number of points for each exercise is given in the heading.

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Exercise 1 (5 points)

What is printed by this program?

```
L = [(0,0), (1,0), (1,1), (0,2), (0,0)]
for p in L[2:-1]:
    print p[1]
```

Exercise 2 (5 points)

What is printed by this program:

```
from math import sqrt

def f(x):
    return x**4

def g(x):
    return sqrt(sqrt(x))

q = 2
v = g(f(q))
print v
```

Exercise 3 (10 points)

Implement the mathematical function

$$T(x, t) = e^{-qx} \cos(t - qx)$$

as a Python function `T(x, t)`. Also, write a main program that plots $T(x, t)$ as a function of x when $q = 1$, $x \in [0, 3]$, and $t = 1$.

Exercise 4 (10 points)

Write a complete program for making a movie (animation) of the function $T(x, t)$ from Exercise 3. Each frame in the movie corresponds to a certain time t and shows a graph of T versus x . Let $q = 1$, $x \in [0, 3]$, and $t \in [0, 6\pi]$. Use a spacing between t values of $\pi/16$.

Hint: Let the frames of the movie be stored in files with names `tmp_0000.png`, `tmp_0001.png`, `tmp_0002.png`, and so forth, created by `'tmp_%04d.png' %`

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c , where c is an integer counter $(0, 1, 2, \dots)$. The movie can then be created by

```
from scitools.std import movie
movie('tmp_*.png', encoder='convert', fps=2,
      output_file='movie.gif')
```

To remove old frame files from a previous run of the program, you can use the code

```
import glob, os
for name in glob.glob('tmp_*.png'):
    os.remove(name)
```

Exercise 5 (10 points)

A file with name `density.dat` contains information on how the density of air varies with temperature. The file may look as follows:

```
# Density of air versus temperature (1 atm pressure)
# Column 1: temperature in Celsius degrees
# Column 2: density in kg/m^3
-10    1.341
-5     1.316
 0     1.293
 5     1.269
10     1.247
15     1.225
20     1.204
25     1.184
30     1.164
# Source: Wikipedia
```

Lines starting with `#` are comment lines. Blank lines are not allowed in the file. The first column contains temperatures in Celsius degrees while the second column contains the corresponding densities. Your task is to read this file in a Python program and write out a similar file where the temperature is given in Fahrenheit instead of Celsius degrees. The relation between Fahrenheit (F) and Celsius (C) degrees reads $F = 1.8C + 32$.

Exercise 6 (15 points)

A flip-coin game costs 1 NOK to play. You flip a coin five times. If heads come up three times or more, you get paid 3 NOK. Make a program that determines if you, in the long run, will earn money by playing this game.

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Exercise 7 (10 points)

Systems of ordinary differential equations of the form

$$u' = f(u, t)$$

can be solved by classes in the ODESolver class hierarchy shown below (this is a slightly simplified version of ODESolver.py from the course material).

```
class ODESolver:
    """
    Superclass for numerical methods solving ODEs

    du/dt = f(u, t)

    Attributes:
    t: array of time values
    u: array of solution values (at time points t)
    k: step number of the most recently computed solution
    f: callable object implementing f(u, t)
    dt: time step (assumed constant)
    """
    def __init__(self, f, dt):
        self.f = lambda u, t: numpy.asarray(f(u, t), float)
        self.dt = dt

    def set_initial_condition(self, u0, t0=0):
        self.u = []      # u[k] is solution at time t[k]
        self.t = []      # time levels in the solution process

        self.u.append(numpy.asarray(u0, float))
        self.t.append(float(t0))
        self.k = 0 # time level counter

    def solve(self, T):
        """
        Advance solution from t = t0 to t = T, in steps of dt.
        """
        self.k = 0
        tnew = 0
        while tnew < T:
            unew = self.advance()

            self.u.append(unew)
            tnew = self.t[-1] + self.dt
            self.t.append(tnew)
            self.k += 1
```

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```

        return numpy.array(self.u), numpy.array(self.t)

class ForwardEuler(ODESolver):
    def advance(self):
        u, dt, f, k, t = \
            self.u, self.dt, self.f, self.k, self.t[-1]

        unew = u[k] + dt*f(u[k], t)
        return unew

class RungeKutta4(ODESolver):
    def advance(self):
        u, dt, f, k, t = \
            self.u, self.dt, self.f, self.k, self.t[-1]
        dt2 = dt/2.0
        K1 = dt*f(u[k], t)
        K2 = dt*f(u[k] + 0.5*K1, t + dt2)
        K3 = dt*f(u[k] + 0.5*K2, t + dt2)
        K4 = dt*f(u[k] + K3, t + dt)
        unew = u[k] + (1/6.0)*(K1 + 2*K2 + 2*K3 + K4)
        return unew

```

The task now is to extend the `ODESolver` hierarchy with a new subclass implementing the following numerical method for $u' = f(u, t)$:

$$\begin{aligned}
 u_* &= u_k + \frac{1}{2}\Delta t (f(u_k, t_{k+1}) + f(u_k, t_k)), \\
 u_{k+1} &= u_k + \frac{1}{2}\Delta t (f(u_*, t_{k+1}) + f(u_k, t_k))
 \end{aligned} \tag{1}$$

Here, u_k denotes $u(t)$ at the k -th time level, where $t = t_k = k\Delta t$. u_* is a help variable. Implement this numerical method in a subclass `RK2` of class `ODESolver`. The following minimalistic demonstration code for solving $u' = -u$, $u(0) = 1$, should work:

```

from ODESolver import RK2
def f(u, t):
    return -u

dt = 0.1
method = RK2(f, dt)
method.set_initial_condition(1)
u, t = method.solve(T=3)

```

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Exercise 8 (10 points)

We have the following system of ordinary differential equations for two functions $x(t)$ and $y(t)$:

$$x'(t) = y(t)/Y - x(t)/X, \quad (2)$$

$$y'(t) = x(t)/X - y(t)/Y, \quad (3)$$

with initial conditions $x(0) = y(0) = 1$. Here, X and Y are two known parameters. Use a class in the `ODESolver` hierarchy from Exercise 7 to compute approximations to $x(t)$ and $y(t)$ for $t \in [0, 3000]$ when $X = 480$ and $Y = 2400$. Use a time step $\Delta t = 10$. Store the discrete values of x and y in arrays `x` and `y`. Describe how you can verify that the program works.

END