Public Transport in Oslo

At the latest City Council meeting several resolutions were passed regarding the public transport system in Oslo. Let the public transport system be represented as a connected, undirected, weighted graph $G = (V, E)$, where $u \in V$ is a stop, an edge $\{u, v\} \in E$ is a *segment*, and the weight function w specifies the travel time (in minutes) for a segment.

(a) The graph is *connected* and *undirected*. Briefly explain what each of these properties means for someone travelling by public transport in Oslo.

(b) The City Council has decided that new stops will be built along all *segments* that exceed k minutes. During the construction work it will not be possible to travel along any of these segments. Write an algorithm that takes the graph representing the current public transport system, as well as a threshold k, and checks whether it will be possible to travel between all pairs of stops during the construction work.

(c) It turns out that the algorithm from subproblem (b) answers **`false`** for the k the City Council had in mind. Give an algorithm the City Council can use to find the smallest value of k for which the algorithm from subproblem (b) answers **`true`**.

(d) In the past year there have been two incidents where there was a total shutdown at a stop. This resulted in travellers being unable to reach their destination. The City Council has adopted a guarantee: if there is a total shutdown at a stop, it should still be possible for travellers to reach their destination via an alternative route. The

guarantee applies to all stops.

Explain how we can use an algorithm known from the syllabus to check whether the public transport system provides this guarantee.

(e) The City Council meeting dragged on for too long, so they ordered pizza, and after that new creative ideas started pouring out. The last one in this series is the following: to relieve the most heavily trafficked routes, they want to give *negative ticket prices* on the least trafficked segments.

Before initiating the project they must ensure that it is not possible to accumulate arbitrarily large amounts in negative ticket prices by travelling around and around.

The ticket price for a segment is given by a weight function c. A segment can be heavily trafficked in one direction but lightly trafficked in the other, so $c(u, v)$ may differ from $c(v, u)$ for a segment $\{u, v\} \in E$.

Explain how we can use an algorithm known from the syllabus to check whether there exist ways to accumulate arbitrarily large amounts in negative ticket prices.