Oblig 3

IN2080

April 28, 2020

Hand-in and deadline

Hand in a single PDF file.

We recommend LaTeX, but all major text editors allows exporting to PDF. You can get help with LaTeX at the group sessions. You can also download the LaTeX source (`.tex`) for this assignment at the assignments page.

Definitions

A *literal* is a formula on the form x or $\overline{x}$, where x is a variable. A formula φ is on *Conjunctive Normal Form* (CNF) if

$$\varphi = \left(l_1^1 \vee \cdots \vee l_{k_1}^1\right) \wedge \cdots \wedge \left(l_1^n \vee \cdots \vee l_{k_n}^n\right),$$

where l_i^j is the i-th literal of the j-th clause, and k_m is the number of literals in the m-th clause. A formula φ is on *Disjunctive Normal Form* (DNF) if

$$\varphi = \left(l_1^1 \wedge \cdots \wedge l_{k_1}^1\right) \vee \cdots \vee \left(l_1^n \wedge \cdots \wedge l_{k_n}^n\right).$$

We define the following languages:

$$\begin{aligned}
\mathsf{CNFSAT} &= \{\varphi \mid \varphi \text{ is on CNF, and } \varphi \text{ is satisfiable}\} \\
\mathsf{DNFSAT} &= \{\varphi \mid \varphi \text{ is on DNF, and } \varphi \text{ is satisfiable}\} \\
\mathsf{CNFUNSAT} &= \{\varphi \mid \varphi \text{ is on CNF, and } \varphi \text{ is unsatisfiable}\} \\
\mathsf{DNFUNSAT} &= \{\varphi \mid \varphi \text{ is on DNF, and } \varphi \text{ is unsatisfiable}\} \\
\mathsf{CNFTAUT} &= \{\varphi \mid \varphi \text{ is on CNF, and } \varphi \text{ is a tautology}\} \\
\mathsf{DNFTAUT} &= \{\varphi \mid \varphi \text{ is on DNF, and } \varphi \text{ is a tautology}\}
\end{aligned}$$

The complexity class coNP consists of complements of languages in NP. More formally, $\mathsf{coNP} = \{\overline{A} \mid A \in \mathsf{NP}\}$.

A language A is coNP-complete if

i) $A \in \mathsf{coNP}$

ii) For every language $B \in \mathsf{coNP}$, $B \leq_p A$.

You may use, without proof, that SAT and 3SAT are NP-complete.

Problem 1

At least one of the above languages is in P. Identify them, and prove that they are in P.

Problem 2

At least one of the above languages is NP-complete. Identify them, and prove that they are NP-complete.

Problem 3

a) Show that for all languages A and B, $A \leq_p B$ implies $\overline{A} \leq_p \overline{B}$.

b) Show that a language A is NP-complete if and only if $\overline{A}$ is coNP-complete.

c) At least one of the above languages is coNP-complete. Identify them, and, using what you have shown in 3a) and 3b), prove that they are coNP-complete.