Selected solutions week 11 - IN2080

The following are solutions to some of the weekly tasks which we discuss in the group sessions, which are not present in the book.

7.45

We can prove a more general statement: for any two languages $A, B \in P$, $A \leq_P B$, given that $B \neq \emptyset$ and $B \neq \Sigma^*$. If we prove this, then obviously A and B will be NP-complete. Let $w \in B$, $w' \notin B$, and M decide A in polynomial time. The reduction f is defined as follows: $f(x) = w$, if M accepts x; otherwise, $f(x) = w'$. This reduction is computed by a polynomial time TM, as simulating M, clearing the tape and writing either w or w' is done in polynomial time.

7.47

If $PATH$ is NP-complete, then $P = NP$, as $PATH \in P$. This is generally not believed to be the case in the scientific community.

To show that if $PATH$ is not NP-complete, then $P \neq NP$, assume the opposite; $PATH$ is not NP-complete, but $P = NP$. This directly contradicts the previous task, as we proved that every non-trivial language in P would be NP-complete if $P = NP$. As such, $PATH$ is not NP- complete iff $P \neq NP$.