Selected solutions week 2 - IN2080

The following are solutions to some of the weekly tasks which we discuss in the group sessions.

1.29

a.

Assume for contradiction that A_1 is regular. Let p be the pumping length, and $s = 0^p1^p2^p \in A_1$. Since $s \geq p$, for A_1 to be regular, s needs to fulfill the pumping lemma; it must be able to be split into three parts xyz, such that

1. $xy^iz \in A_1$, for all $i \in N$
2. $|y| > 0$
3. $|xy| \leq p$

To satisfy condition 3, xy can only consist of 0s, as there are p many of them. In that case, if we repeat the y part of the string, we'll only add 0s to the string, and no other symbols. As the strings in the language contains the same amount of each symbol, A_1 cannot be regular.

b.

Assume for contradiction that A_2 is regular. Let p be the pumping length, and $s = a^pba^pba^pb$. This string is in A_2, as we can let $w = a^pb$, and thus $www = s$. To satisfy condition 3, xy can only consist of as. To satisfy condition 1, $a^{p+|y|}ba^pba^pb$ must also be in A_2. This can't be the case however, as the string can't be split into three similar parts. Thus, A_2 cannot be regular.

c.

Assume for contradiction that A_3 is regular. Let p be the pumping length, and $s = a^{2^p} \in A_3$. To satisfy condition 2 and 3, $0 < |y| \leq p$. Condition 1 tells us that $a^{2^p+|y|}$ must also be in A_3. For $2^p + |y|$ to be a power of 2, $|y| \geq 2^p$. This contradicts condition 3 however, as we've stated that $0 < |y| \leq p$. As such, A_3 is not regular.

1.30

The problem here is that the pumped strings in example 1.73 must be on the form 0^n1^n, whereas the language 0^*1^* can contain any number of 0s and 1s. Strings from this language can easily be pumped, as $y = 0$ or $y = 1$ works for every string. If a string can be pumped or not is dependent on the context; a string might be able to be pumped in one language but not another.