

IN3020&4020 – Database Systems (2022)

Part 1: Query Processing

Lectures 5,6: Relational Algebra

31 January & 3 February

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Lecture rules (new)

Physical from now:

- nice for both you and me

Let us keep being interactive:

- ask **questions**
- I will also ask questions time to time

Lectures will be **recorded** (hopefully):

- videos will be **published** (very) soon after the lecture
- do not forget about **mandatories** (see below)

We start new part: Query Processing

Syllabus of the course:

- (Extended) Intro and SQL recap 

Part 1. Query processing in relational databases

Part 2. Transaction management in relational databases

Part 3. NoSQL DBMS

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- (Extended) Intro and SQL recap 

Part 1. Query processing in relational databases

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SQL query processing relies on two key ingredients:

Relational Algebra and Indexes

We study these ingredients **first** (almost in isolation)
and **then** put all things together

Materials to read about Relational Algebra

1. *Fundamentals of Database Systems* by R. Elmasri and S.B. Navathe (7th edition): Chapter 8 (maybe, not the best)
2. *Foundation of Databases* by S. Abiteboul, R. Hull & V. Vianu: Part B (more details)
available at <http://webdam.inria.fr/Alice/>
3. *Database Systems: the Complete Book* by H. Garcia-Molina, J. Ullman & J. Widom (2nd edition): Section 2.4
4. IN2090 – Databaser og datamodellering (Leif Harald Karlsen, Dimitru Roman): Lectures 3 (basics)
<https://www.uio.no/studier/emner/matnat/ifi/IN2090/h21/timeplan/>
5. (English) Wikipedia has a good introduction
6. etc.

Materials to Play With

- Many good places with interactive exercises:
 - **w3resource** (<https://www.w3resource.com/>)
 - **w3schools** (<https://www.w3schools.com/sql/>)
 - Web pages of **systems** (MySQL, PostgreSQL, Oracle, etc.)
- They use SQL rather than RA
- Should **not be a problem**: for simple examples
 - syntactic translation is direct
 - semantic differences are immaterial

Lecture Plan

1. Role of RA in SQL engines
2. Algebras and algebraic laws
3. Relational Algebra intro
4. Core RA operators
5. Algebraic laws for core operators
6. (Some) expressible operators

1. Role of RA in SQL engines

SQL query processor in action

```
SELECT DISTINCT title FROM StarsIn WHERE starName IN  
(SELECT name FROM MovieStar WHERE birthDate LIKE '%1960');
```

⌋ parsing

(parse tree)

⌋ preprocessing

$$\pi_{title}(\sigma_{starName=name}(\mathbf{StarsIn} \times \pi_{name}(\sigma_{birthDate \text{ LIKE } '%1960'}(\mathbf{MovieStar}))))$$

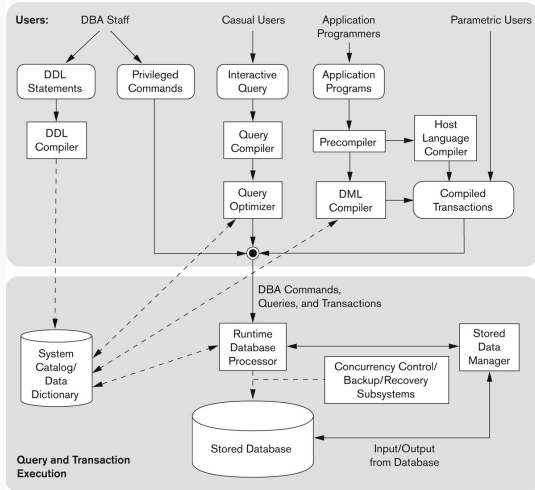
⌋ optimisation

$$\pi_{title}(\mathbf{StarsIn} \bowtie_{starName=name} \pi_{name}(\sigma_{birthDate \text{ LIKE } '%1960'}(\mathbf{MovieStar})))$$

⌋ evaluation over tables (a lot of things)

(Result)

The place of RA in DBMS architecture



2. Algebras and Algebraic Laws

Algebra (a.k.a. algebraic structure)

Definition (Algebra)

- **Domain** D : collection of values
- **Operators**: functions from D^k to D (k is arity of the function)
- **Expressions**:
 - **Atomic**: elements of D
 - **Complex**: operators applied to other expressions
(e.g., $f(3, g(5))$)
- Expressions **evaluate** to elements of D
- Infix notation is often used for binary ($k = 2$) operators:
 - for example, instead of $+(3, 5)$ we write $3 + 5$
 - brackets or conventions used: $(3 + 5) \times 2$ vs. $3 + 5 \times 2$

Example: Integer arithmetic

- Domain: **Integers** $\dots, -3, -2, -1, 0, 1, 2, 3, \dots$
- Operators: usual $+$, $-$, $\times$, $/$ (all binary)

- Expression examples:

$2 + 5$ evaluates to 7

$((2 - 4) \times 5) + (8 / 2)$ evaluates to -6

$8 / 3$ evaluates to

Example: Integer arithmetic

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- Expression examples:

$2 + 5$ evaluates to 7

$((2 - 4) \times 5) + (8 / 2)$ evaluates to -6

$8 / 3$ evaluates to **undefined**

Example: Regular expressions

- Domain: **Sets of strings** over an alphabet (set of symbols) Σ
- Operators:
 - binary ($k = 2$): $\circ$ (or nothing, **concatenation**), $|$ (**alternation**)
 - unary ($k = 1$): $*$ (**Kleene star**)
- Expression examples: (for $\Sigma = \{a, b\}$)
 - ab^* evaluates to $\{a, ab, abb, abbb, \dots\}$
 - $(a | b)^*ab$ evaluates to all words ending with ab
 - (Note: here, a and b are abbreviations of $\{a\}$ and $\{b\}$)
- Expressible operators:
 - $?$ ($e? := \epsilon | e$ for every domain element e)
 - $+$ ($e^+ := ee^*$ for every domain element e)

Definition (Algebra (reminder))

- **Domain** D : collection of values
- **Operators**: functions from D^k to D (k is arity of the function)

Algebraic Law: an identity (or equivalence) that holds in an algebra

Examples:

- $a + b = b + a$, $(a \times b) / b = a$ (for all integers a, b)
are laws for Integer Algebra
- $(e_1 e_2) e_3 = e_1 (e_2 e_3)$, $e^+ = e e^*$ (for all sets of strings)
are laws for RegExpr Algebra

Named Algebraic Laws

Common laws may have names:

- binary op is commutative if $e_1 \text{ op } e_2 = e_2 \text{ op } e_1$ (for all e_1, e_2)
- binary op is associative if $(e_1 \text{ op } e_2) \text{ op } e_3 = e_1 \text{ op } (e_2 \text{ op } e_3)$
- binary op is idempotent if $e \text{ op } e = e$

example: $\max(a, a) = a$ for integers

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- binary op is **associative** if $(e_1 \text{ op } e_2) \text{ op } e_3 = e_1 \text{ op } (e_2 \text{ op } e_3)$
- binary op is **idempotent** if $e \text{ op } e = e$
example: $\max(a, a) = a$ for integers
- unary op is **idempotent** if $\text{op}(\text{op}(e)) = \text{op}(e)$
(not a coincidence, but no need for you to know why)
example: $(e^*)^* = e^*$ for sets of words

Named Algebraic Laws

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- binary op is **commutative** if $e_1 \text{ op } e_2 = e_2 \text{ op } e_1$ (for all e_1, e_2)
- binary op is **associative** if $(e_1 \text{ op } e_2) \text{ op } e_3 = e_1 \text{ op } (e_2 \text{ op } e_3)$
- binary op is **idempotent** if $e \text{ op } e = e$
example: $\max(a, a) = a$ for integers
- unary op is **idempotent** if $\text{op}(\text{op}(e)) = \text{op}(e)$
(not a coincidence, but no need for you to know why)
example: $(e^*)^* = e^*$ for sets of words
- binary op is **left-distributive** over binary op' if
 $e_1 \text{ op } (e_2 \text{ op}' e_3) = (e_1 \text{ op } e_2) \text{ op}' (e_1 \text{ op } e_3)$
example: $a \times (b + c) = (a \times b) + (a \times c)$ for integers
when op is commutative, we have just **distributivity**

3. Relational Algebra Intro

Variants of Relational Algebra

- There are **many** RAs. Two RA may be
 - **Equivalent**: some operators are **expressible** via each other (e.g., $\bowtie$ is expressible via $\ltimes$ and back in the presence of σ and ρ)
 - One **strictly more expressive** than another: one has operators **non-expressible** in the other
 - Have **different domains** (e.g., **named/unnamed**, **sets/bags**)
- Full SQL requires a **very expressive** variant of RA for **bags** (Turing complete!)
- We start with a **basic** RA for **sets** under **named** perspective (as suggested by Codd)
- Why (except historic importance)?
 - every set is a bag
 - all set operators may be simulated by bag operators
 - fundamental core, which also covers **most of real SQL queries**

(Our) Relational Algebra domain

Question: What is the domain of RA?

(Our) Relational Algebra domain

Question: What is the domain of RA?

- Domain: (Set) relations (i.e., tables)
- Relation components (reminder):
 - Relation name: just a string
 - Schema: set of attribute names with domains (i.e., datatypes)
 - Set (no repetitions!) of relation tuples (a.k.a. records, rows): assignments of values to attribute names conforming schema
- Example:

Tutorials:

<i>ID</i>	<i>site</i>	<i>tutorial</i>	<i>topic</i>
1	w3schools	SQL_2003STD	Databases
2	w3schools	HTML_5	WebDev
3	w3schools	CSS_3	WebDev
4	w3resource	SQL_2003STD	Databases
5	w3resource	MySQL	Databases

(Our) Relational Algebra operators

- Domain: Relations
- Core operators:
 - (set) union $\cup$
 - (set) difference $\setminus$
 - selection σ_C
 - projection π_{Atts}
 - Cartesian product $\times$ (a.k.a. cross product, cross join)
 - renaming ρ_N

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 - **Cartesian product** $\times$ (a.k.a. **cross product**, **cross join**)
 - **renaming** ρ_N (technical, often not mentioned)
- Some unary, some binary, usually not always applicable
- Some **parametrised**: families of operators rather than single
- Expressible operators:
 - (set) **intersection** $\cap$
 - other **joins**: **natural** $\bowtie$, **theta**- $\bowtie_\theta$, **semi**- $\ltimes$ & $\ltimes$, etc.
 - **division** $\div$
 - ...

4. Core RA operators

Union: binary, no parameters (example)

Set-theoretic union of tuples in same-structured relations:

Tutorials1:

<i>ID</i>	<i>site</i>	<i>tutorial</i>	<i>topic</i>
1	w3schools	SQL_2003STD	Databases
2	w3schools	HTML_5	WebDev
3	w3schools	CSS_3	WebDev

Tutorials2:

<i>ID</i>	<i>site</i>	<i>tutorial</i>	<i>topic</i>
3	w3schools	CSS_3	WebDev
4	w3resource	SQL_2003STD	Databases
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Tutorials1 U Tutorials2:

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=

Union: binary, no parameters (definition)

Set-theoretic union of tuples in same-structured relations

Union (formal definition)

- Let R and S be two relations
- **Union** $R \cup S$ is the relation that
 - defined when R and S have the same schema (i.e., same attribute names and their datatypes)
 - has this schema
 - contains each tuple that is either in R or in S (or both)

Set-theoretic union: **repetitions are removed!**

Difference: binary, no parameters (example)

Set-theoretic difference of tuples in same-structured relations:

Tutorials1:

<i>ID</i>	<i>site</i>	<i>tutorial</i>	<i>topic</i>
1	w3schools	SQL_2003STD	Databases
2	w3schools	HTML_5	WebDev
3	w3schools	CSS_3	WebDev

Tutorials2:

<i>ID</i>	<i>site</i>	<i>tutorial</i>	<i>topic</i>
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4	w3resource	SQL_2003STD	Databases
5	w3resource	MySQL	Databases

\

=	Tutorials1 \ Tutorials2:			
	<i>ID</i>	<i>site</i>	<i>tutorial</i>	<i>topic</i>
	1	w3schools	SQL_2003STD	Databases
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Difference: binary, no parameters (definition)

Set-theoretic difference of tuples in same-structured relations

Difference (formal definition)

- Let R and S be two relations
- Difference $R \setminus S$ is the relation that
 - defined when R and S have the same schema (i.e., attribute names and their datatypes)
 - has the same schema
 - contains each tuple that is in R but not in S

No repetitions anyway (by construction)

Selection: unary, one parameter (example)

Selects tuples from a relation satisfying condition in the parameter:

$$\sigma_{ID > 2 \ \& \ topic = 'Databases'}$$

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<i>ID</i>	<i>site</i>	<i>tutorial</i>	<i>topic</i>
1	w3schools	SQL_2003STD	Databases
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$$=$$

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Selection: unary, one parameter (definition)

Selects tuples from a relation satisfying condition in the parameter

Selection (formal definition)

- Let R be a relation
- Let C be a condition—that is, Boolean (i.e., using $\&$, $\vee$, $\neg$) combination of atoms
 - $e_1 = e_2$, $e_1 \neq e_2$, where e_1 and e_2 are attribute names of R or some **values** (numbers, strings, etc.)
 - domain specific operations over attribute names and values, e.g., $e_1 > e_2$, e **LIKE** *RegExp* (no subqueries, etc.)
- **Selection** $\sigma_C(R)$ is the relation that
 - is defined when C mentions only attribute names of R and all operations in C are well-typed
 - has the same schema as R
 - contains all tuples in R satisfying C

Projection: unary, one parameter (example)

Selects columns from a relation, as specified in the parameter

$$\pi_{site, topic} \left(\begin{array}{c} \textbf{Tutorials:} \\ \hline \begin{array}{cccc} ID & site & tutorial & topic \\ \hline 1 & w3schools & SQL_2003STD & Databases \\ 2 & w3schools & HTML_5 & WebDev \\ 3 & w3schools & CSS_3 & WebDev \\ 4 & w3resource & SQL_2003STD & Databases \\ 5 & w3resource & MySQL & Databases \end{array} \end{array} \right)$$
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Projection: unary, one parameter (definition)

Selects columns from a relation, as specified in the parameter

Projection (formal definition)

- Let R be a relation
- Let $Atts$ be a list of attribute names
- **Projection** $\pi_{Atts}(R)$ is the relation that
 - is defined when $Atts$ mentions only attribute names of R
 - has schema $Atts$ (with domains from R)
 - contains projections (i.e., restrictions) of all tuples in R to $Atts$

Repetitions are removed!

Cartesian product: binary, no parameters (example)

Combines tuples from two relations in all possible ways:

Tutorials:

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1	w3schools	SQL_2003STD
2	w3schools	HTML_5
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Topics:

<i>topicID</i>	<i>topic</i>
1	Databases
2	WebDev

Tutorials × Topics:

	<i>ID</i>	<i>site</i>	<i>tutorial</i>	<i>topicID</i>	<i>topic</i>
=	1	w3schools	SQL_2003STD	1	Databases
	2	w3schools	HTML_5	1	Databases
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	4	w3resource	SQL_2003STD	1	Databases
	5	w3resource	MySQL	1	Databases
	1	w3schools	SQL_2003STD	2	WebDev
	2	w3schools	HTML_5	2	WebDev
	3	w3schools	CSS_3	2	WebDev
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Cartesian product: binary, no parameters (definition)

Combines tuples from two relations in all possible ways

Cartesian (or Cross) Product (formal definition)

- Let R and S be two relations
- Cartesian product $R \times S$ is the relation that
 - is defined when R and S have no common attribute name
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 - contains all possible combinations of tuples in a tuple in R with a tuple in S

Cartesian product: binary, no parameters (definition)

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- Note: first operator that has **more** in output than in input
- Maybe really large
- Makes little sense, not used in practice much
- Useful to present **natural join** $\bowtie$ (see below)

Renaming: unary, one parameter

Renames attributes in a relation as specified in the parameter:

$$\rho_{topic \rightarrow area} \left(\begin{array}{c} \text{ProjTutorials:} \\ \hline \begin{array}{cc} \textit{site} & \textit{topic} \\ \hline \text{w3schools} & \text{Databases} \\ \text{w3schools} & \text{WebDev} \\ \text{w3resource} & \text{Databases} \end{array} \end{array} \right) = \begin{array}{c} \rho_{topic \rightarrow area}(\text{ProjTutorials}): \\ \hline \begin{array}{cc} \textit{site} & \textit{area} \\ \hline \text{w3schools} & \text{Databases} \\ \text{w3schools} & \text{WebDev} \\ \text{w3resource} & \text{Databases} \end{array} \end{array}$$

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Renaming (formal definition)

- Let R be a relation
- Let N be a list $A_1 \rightarrow A'_1, \dots, A_n \rightarrow A'_n$, where $A_1, \dots, A_n, A'_1, \dots, A'_n$ are all different attribute names
- **Renaming** $\rho_N(R)$ is the relation that
 - is defined when $A_1, \dots, A_n$ are attribute names of R and $A'_1, \dots, A'_n$ are not
 - is the same as R except that $A_1, \dots, A_n$ renamed to $A'_1, \dots, A'_n$

Independent set of operators

Core Set RA

*Selection σ_C , Projection π_L , Cartesian Product $\times$,
Union $\cup$, Difference $\setminus$, Renaming ρ_N*

- Can express **everything** in Codd's algebra (but not all SQL!)
- These are **independent** operators: dropping any of them makes the language weaker
- There are other useful **expressible** operators: intersection $\cap$, natural join $\bowtie$, division $\div$, etc.
 - there are **effective** algorithms for them
 - it is often **simpler** to formulate queries using them

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 - there are **effective** algorithms for them
 - it is often **simpler** to formulate queries using them
- We discuss them after algebraic laws for core operators

5. Algebraic laws for core RA operators

RA Algebraic Laws: initial observations

The power of RA is largely in its rich set of **algebraic laws**

- allows to convert complex queries to **equivalent but simpler** ones (before evaluating them)
- ‘**simple**’ here means not ‘short’ or ‘neat-looking’, but ‘**expected to be evaluated quicker**’

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- we will discuss how to decide what is simpler (in this sense) than other **later on**

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- ‘**simple**’ here means not ‘short’ or ‘neat-looking’, but ‘**expected to be evaluated quicker**’
- we will discuss how to decide what is simpler (in this sense) than other **later on**
- there are (too) **many** laws, and we consider only **most prominent examples**
- start now with **core operators**, later touch expressible ones

Laws for set-theoretic operations

Union is **commutative**, **associative**, and **idempotent**:

$$R \cup S = S \cup R \quad (R \cup S) \cup T = R \cup (S \cup T) \quad R \cup R = R$$

(for all relations R, S, T)

Difference is **right-distributive** over union:

$$(R \cup S) \setminus T = (R \setminus T) \cup (S \setminus T)$$

Laws for set-theoretic operations

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Question: Is it **left-distributive**: $R \setminus (S \cup T) \neq (R \setminus S) \cup (R \setminus T)$?

Laws for set-theoretic operations

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Answer: No (come to counter-example by yourself)

Laws for set-theoretic operations

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Question: Is it **left-distributive**: $R \setminus (S \cup T) \neq (R \setminus S) \cup (R \setminus T)$?

Answer: No (come to counter-example by yourself)

Other (unnamed) laws:

$$R \setminus (S \cup T) = (R \setminus S) \setminus T = (R \setminus T) \setminus S$$

...

Laws for Cartesian product

Cartesian product is **commutative** and **associative**:

$$R \times S = S \times R \quad (R \times S) \times T = R \times (S \times T)$$

(for all relations R, S, T)

Laws for Cartesian product

Cartesian product is **commutative** and **associative**:

$$R \times S = S \times R \quad (R \times S) \times T = R \times (S \times T)$$

(for all relations R, S, T)

Question: is it idempotent ($R \times R \not\equiv R$)?

Laws for Cartesian product

Cartesian product is **commutative** and **associative**:

$$R \times S = S \times R \quad (R \times S) \times T = R \times (S \times T)$$

(for all relations R, S, T)

Question: is it idempotent ($R \times R \stackrel{?}{=} R$)?

Answer: Of course **no**: it is never defined

Laws for Cartesian product

Cartesian product is **commutative** and **associative**:

$$R \times S = S \times R \quad (R \times S) \times T = R \times (S \times T)$$

(for all relations R, S, T)

Question: is it idempotent ($R \times R \not\equiv R$)?

Answer: Of course **no**: it is never defined

Cartesian product is **distributive** over both union and difference:

$$(R \cup S) \times T = (R \times T) \cup (S \times T)$$
$$(R \setminus S) \times T = (R \times T) \setminus (S \times T)$$

Other (unnamed) laws:

...

Laws for selection

Selection is **idempotent**:

$$\sigma_C(\sigma_C(R)) = \sigma_C(R) \quad (\text{for every relation } R \text{ and condition } C)$$

Conditions can be **reshuffled** and **split**:

$$\sigma_{C_1 \& C_2}(R) = \sigma_{C_1}(\sigma_{C_2}(R)) = \sigma_{C_2}(\sigma_{C_1}(R))$$

$$\sigma_{C_1 \vee C_2}(R) = \sigma_{C_1}(R) \cup \sigma_{C_2}(R)$$

Selection can be **pushed** through union, difference, and Cartesian product (**not always!**):

$$\sigma_C(R \cup S) = \sigma_C(R) \cup \sigma_C(S)$$

$$\sigma_C(R \setminus S) = \sigma_C(R) \setminus \sigma_C(S)$$

$$\sigma_C(R \times S) = \sigma_C(R) \times S \quad \text{if } C \text{ mentions only attributes of } R$$

... (other laws)

Laws for projection

Projection is **idempotent**:

$$\pi_{Atts}(\pi_{Atts}(R)) = \pi_{Atts}(R) \text{ (for relation } R \text{ and attributes } Atts)$$

Projected attributes can be **reshuffled** and **split**:

$$\pi_{Atts_1, Atts_2}(R) = \pi_{Atts_1}(\pi_{Atts_2}(R)) = \pi_{Atts_2}(\pi_{Atts_1}(R))$$

Projection can be **pushed** through union and Cartesian product:

$$\pi_{Atts}(R \cup S) = \pi_{Atts}(R) \cup \pi_{Atts}(S)$$

$$\pi_{Atts_R, Atts_S}(R \times S) = \pi_{Atts_R}(R) \times \pi_{Atts_S}(S)$$

if $Atts_R$ are attributes of R and $Atts_S$ are attributes of S

... (other laws)

Laws for renaming

...

...

Homework: find a couple by yourself

6. (Some) expressible operators

Intersection: binary, no parameters

Set-theoretic intersection of tuples in same-structured relations:

Tutorials1:

<i>ID</i>	<i>site</i>	<i>tutorial</i>	<i>topic</i>
1	w3schools	SQL_2003STD	Databases
2	w3schools	HTML_5	WebDev
3	w3schools	CSS_3	WebDev

Tutorials2:

<i>ID</i>	<i>site</i>	<i>tutorial</i>	<i>topic</i>
3	w3schools	CSS_3	WebDev
4	w3resource	SQL_2003STD	Databases
5	w3resource	MySQL	Databases

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Tutorials1 $\cap$ Tutorials2:

<i>ID</i>	<i>site</i>	<i>tutorial</i>	<i>topic</i>
3	w3schools	CSS_3	WebDev

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=	Tutorials1 $\cap$ Tutorials2:			
	<i>ID</i>	<i>site</i>	<i>tutorial</i>	<i>topic</i>
	3	w3schools	CSS_3	WebDev

Intersection (formal definition)

Intersection: $R \cap S := R \setminus (R \setminus S)$ for every R and S
(including the undefined cases)

Natural Join: binary, no parameters (example)

Combines tuples from two relations in all possible ways that agree on common attributes:

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<i>ID</i>	<i>site</i>	<i>tutorial</i>	<i>topicID</i>
1	w3schools	SQL_2003STD	1
2	w3schools	HTML_5	2
3	w3schools	CSS_3	2
4	w3resource	SQL_2003STD	1
5	w3resource	MySQL	1

Topics:

<i>topicID</i>	<i>topic</i>
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<i>ID</i>	<i>site</i>	<i>tutorial</i>	<i>topicID</i>	<i>topic</i>
1	w3schools	SQL_2003STD	1	Databases
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3	w3schools	CSS_3	2	WebDev
4	w3resource	SQL_2003STD	1	Databases
5	w3resource	MySQL	1	Databases

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Natural Join: binary, no parameters (definition)

Natural Join (formal definition)

Let R and S be two relations:

- $A_1, \dots, A_n$ and $A'_1, \dots, A'_m$ their attributes
- $\{B_1, \dots, B_k\} = \{A_1, \dots, A_n\} \cap \{A'_1, \dots, A'_m\}$: common

Natural Join: $R \bowtie S := \rho_N(\pi_L(\sigma_C(\rho_{N_R}(R) \times \rho_{N_S}(S))))$ where

- N_R and N_S rename attributes of R and S apart:

$$N_R = \{A_1 \rightarrow R.A_1, \dots, A_n \rightarrow R.A_n\}$$

$$N_S = \{A'_1 \rightarrow S.A'_1, \dots, A'_m \rightarrow S.A'_m\}$$

- C is the condition matching common attributes:

$$C = (R.B_1 = S.B_1 \ \& \ \dots \ \& \ R.B_k = S.B_k)$$

- L are all attributes of R and S except the repetitions:

$$L = R.A_1, \dots, R.A_n, S.B'_1, \dots, S.B'_\ell$$

$$\text{where } \{B'_1, \dots, B'_\ell\} = \{A'_1, \dots, A'_m\} \setminus \{A_1, \dots, A_n\}$$

- N renames attributes back:

$$N = \{R.A_1 \rightarrow A_1, \dots, R.A_n \rightarrow A_n, S.B'_1 \rightarrow B'_1, \dots, S.B'_\ell \rightarrow B'_\ell\}$$

Natural Join (observations)

- Product $\times$ is **expressible** in the presence of natural join $\bowtie$
 - **Question:** Any ideas how?

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when $R \times S$ is defined
(the difference is that $\bowtie$ is defined in more cases than $\times$)

Natural Join (observations)

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when $R \times S$ is defined
(the difference is that $\bowtie$ is defined in more cases than $\times$)
- **Another core set RA:** selection σ_C , projection π_L ,
natural join $\bowtie$, union $\cup$, difference $\setminus$, renaming ρ_N
- Natural join is one of the **most used** operations in practice
- Implementing $\bowtie$ via $\times$ is **extremely non-effective**
- Native algorithms for $\bowtie$ are **common** (we will see)

Theta Join: binary, one parameter (example)

Normal join + selection:

Tutorials:

<i>ID</i>	<i>site</i>	<i>tutorial</i>	<i>topicID</i>
1	w3schools	SQL_2003STD	1
2	w3schools	HTML_5	2
3	w3schools	CSS_3	2
4	w3resource	SQL_2003STD	1
5	w3resource	MySQL	1

$\bowtie_{ID > topicID}$

Topics:

<i>topicID</i>	<i>topic</i>
1	Databases
2	WebDev

=	Tutorials $\bowtie_{ID > topicID}$		Topics:		
	<i>ID</i>	<i>site</i>	<i>tutorial</i>	<i>topicID</i>	<i>topic</i>
	3	w3schools	CSS_3	2	WebDev
	4	w3resource	SQL_2003STD	1	Databases
	5	w3resource	MySQL	1	Databases

(This is not very meaningful query,
but the operation is very useful in practice)

Theta Join: binary, one parameter (definition)

Theta Join (formal definition)

Theta Join: $R \bowtie_{\theta} S := \sigma_{\theta}(R \bowtie S)$ for relations R and S ,
and condition $\theta = RAtt_1 \text{ op}_1 SAtt_1 \ \& \ \dots \ \& \ RAtt_k \text{ op}_k SAtt_k$
where $RAtt_i$ and $SAtt_i$ are attributes of R and S , and
 $\text{op}_i \in \{=, \neq, >, <, \geq, \leq\}$
(including the undefined cases)

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(including the undefined cases)

- **Question:** Why to bother? Why only these conditions?
- **Answer:**
 - one the one hand, very common in practice
 - one the other, we can design specialised efficient algorithms
- **Equi-Join:** theta join with $=$ as all op_i (e.g., $R \bowtie_{RAtt=SAtt} S$)
 - essentially, naturally join with incorporated renaming
 - even more common and own algorithms

Semi-joins, anti-joins: binary, no parameters

Semi-Joins (formal definition)

Left Semi-Join: $R \ltimes S := \pi_{R\text{Atts}}(R \bowtie S)$ for relations R and S where $R\text{Atts}$ are attributes of R

Right Semi-Join: $R \rtimes S := \pi_{S\text{Atts}}(R \bowtie S)$ for relations R and S where $S\text{Atts}$ are attributes of S (symmetric: $R \rtimes S = S \rtimes R$)

- **Left Semi-Join:** all tuples in R that **join** with tuples in S

Anti-Joins (formal definition)

Left Anti-Join: $R \rhd S := R \setminus (R \ltimes S)$ for relations R and S

Right Anti-Join: $R \lhd S := \dots$ (symmetric: $R \lhd S = S \rhd R$)

- **Left Anti-Join:** all tuples in R that **do not join** with tuples in S

Semi-joins, anti-joins: binary, no parameters

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Question: Where is Outer Joins?

Semi-joins, anti-joins: binary, no parameters

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- **Left Anti-Join:** all tuples in R that **do not join** with tuples in S

Question: Where is Outer Joins? **Answer:** Next time: not in classical relational algebra, need NULLs

Division: binary, no parameters (example 1)

'Inverse' of Cartesian Product:

TutTimesTop:

<i>ID</i>	<i>site</i>	<i>tutorial</i>	<i>topicID</i>	<i>topic</i>
1	w3schools	SQL_2003STD	1	Databases
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3	w3schools	CSS_3	2	WebDev
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TutTimesTop $\div$ Topics:

<i>ID</i>	<i>site</i>	<i>tutorial</i>
1	w3schools	SQL_2003STD
2	w3schools	HTML_5
3	w3schools	CSS_3
4	w3resource	SQL_2003STD
5	w3resource	MySQL

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Division: binary, no parameters (example 2)

'Inverse' of Cartesian Product:

TutTimesTop:

<i>ID</i>	<i>site</i>	<i>tutorial</i>	<i>topicID</i>	<i>topic</i>
1	w3schools	SQL_2003STD	1	Databases
3	w3schools	CSS_3	1	Databases
4	w3resource	SQL_2003STD	1	Databases
5	w3resource	MySQL	1	Databases
1	w3schools	SQL_2003STD	2	WebDev
2	w3schools	HTML_5	2	WebDev
4	w3resource	PostgreSQL	2	WebDev
5	w3resource	MySQL	2	WebDev
5	w3resource	PostgreSQL	2	WebDev

Topics:

<i>topicID</i>	<i>topic</i>
1	Databases
2	WebDev

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TutTimesTop ÷ Topics:

<i>ID</i>	<i>site</i>	<i>tutorial</i>
1	w3schools	SQL_2003STD
5	w3resource	MySQL

Division: binary, no parameters (definition)

Division (formal definition)

Division: $R \div S := \pi_{Atts}(R) \setminus \pi_{Atts}((\pi_{Atts}(R) \times S) \setminus R)$ for relations R and S , where $Atts$ is the attributes of R not mentioned in S (including the undefined cases)

- Defined only when each attribute of S is an attribute of R (follows from the definition)
- Equivalently, can be defined as:
 $R \div S$ is all tuples t from $\pi_{Atts}(R)$ such that $\{t\} \times S \subseteq R$
- As expected, $(R \times S) \div S = R$ for every R and S
- However, $(R \div S) \times S$ may be different from R
- (cf. **Euclidian division**, a.k.a. **division with remainder** for integers)

(Some) algebraic laws for expressible operators

Our definitions of the expressible operators are essentially laws:

$$\text{e.g., } R \div S = \pi_{Atts}(R) \setminus \pi_{Atts}((\pi_{Atts}(R) \times S) \setminus R)$$

Intersection $\cap$ is **commutative**, **associative**, and **idempotent**, as well as **distribute over union** $\cup$

Natural join is **commutative**, **associative**, and **idempotent**:

$$R \bowtie S = S \bowtie R \quad (R \bowtie S) \bowtie T = R \bowtie (S \bowtie T) \quad R \bowtie R = R$$

(the most important laws in practice)

Selection can be **pushed** through join (**not always!**):

$$\sigma_C(R \bowtie S) = \sigma_C(R) \bowtie S \quad \text{if } C \text{ mentions only attributes of } R$$

(also very important)

... (other laws)

What have we learned?

Classic Relational Algebra:

core and expressible operators, algebraic laws