Weekly exercises IN3020&4020 (2026)
Week 4: Set and Bag Relational Algebra

1 Joins and Value Invention (for sets)

Consider two relations:

$$T_1\,[\,A,\,B\,] : \{\langle 1, a\rangle, \langle 2, b\rangle, \langle 3, b\rangle\}$$
$$T_2\,[\,B,\,C\,] : \{\langle b, 2\rangle, \langle c, 4\rangle\}$$

1. Discuss the types of joins over set relations considered in the lectures: Cartesian product, natural join, theta-join, equi-join, semi-joins, anti-joins, outer joins. In particular,
   - give the brief descriptions of each operation
   - illustrate it with an example using relations T_1 and T_2 given above
   - briefly consider its expressibility in our core relational algebra
   - think about its usefulness for SQL
2. Evaluate expression $\pi_{A\times A\to Asq, B, NULL\to D, A-C\to E}(T_1 \ltimes T_2)$ over relations T_1 and T_2.

2 Core operations for sets and bags

Consider the relations

$$R\,[\,A,\,B\,] : \{\langle 1, 2\rangle, \langle 3, 4\rangle, \langle 3, 5\rangle, \langle 4, 5\rangle\}$$
$$S\,[\,A,\,B\,] : \{\langle 1, 2\rangle, \langle 3, 5\rangle, \langle 3, 6\rangle, \langle 4, 5\rangle, \langle 1, 3\rangle\}$$

and the following bag relations (which are the same as R and S except that one tuple in each is duplicated):

$$Rb\,[\,A,\,B\,] : \{\{\langle 1, 2\rangle, \langle 3, 4\rangle, \langle 1, 2\rangle, \langle 3, 5\rangle, \langle 4, 5\rangle\}\}$$
$$Sb\,[\,A,\,B\,] : \{\{\langle 1, 2\rangle, \langle 3, 5\rangle, \langle 4, 5\rangle, \langle 3, 6\rangle, \langle 4, 5\rangle, \langle 1, 3\rangle\}\}$$

Compute the following expressions:

1. $\sigma_{A=3}(Rb \cup^b Sb)$
2. $\sigma_{A=3}(R \cup S)$
3. $\sigma_{A=3}(R \cup^b S)$
4. $Sb \setminus Rb$
5. $S \setminus R$
6. $\pi^b_A(Rb)$
7. $\pi_A(Rb)$
8. $\pi^b_A(R)$
9. $\pi_A(R)$

3 Bag-specific operations

Consider the following bag relation:

T:

A	B	C
1	a	*NULL*
1	*NULL*	*NULL*
2	a	*NULL*
NULL	a	*NULL*
NULL	a	*NULL*

Evaluate the following expressions over T:

1. $\delta(T)$
2. $\delta(\pi^b_A(T))$
3. $\gamma_{\mathsf{COUNT}(B),\mathsf{COUNT_DISTINCT}(B),\mathsf{COUNT}(C),\mathsf{COUNT_DISTINCT}(C)}(T)$
4. $\gamma_{B,\mathsf{MIN}(A),\mathsf{SUM}(A),\mathsf{AVG}(A),\mathsf{SUM}(C)}(T)$

4 Expression of operations and algebraic laws

1. Express the following bag relational algebra expression using theta-join: $\pi^b_A(\sigma_{A>B \wedge B>4}(R \bowtie S))$
2. Express δ using other bag relational algebra operations.
3. (*) Full outer join of two relations R and S over bags is defined as the bag of tuples that consists of all tuples in the natural join of R and S, all dangling tuples of R extended with *NULLs* (with the same multiplicities as in R), and all dangling tuples of S extended with *NULLs* (with the

same multiplicities as in S). Give examples of R and S showing that both $R \rtimes\!\ltimes S := (R \rtimes S) \cup^b (R \ltimes S)$ and $R \rtimes\!\ltimes S := (R \rtimes S) \cup (R \ltimes S)$ are incorrect over bags, and give a correct definition of full outer join via other bag relational algebra operations.

5 Constraints

Consider relations with the following schemas (the domains and the meaning of the attributes are as expected):

$$\mathit{Product}\,[\mathit{manufacturer}, \mathit{model}, \mathit{type}]$$
$$\mathit{PC}\,[\mathit{model}, \mathit{speed}, \mathit{RAM}, \mathit{HDD}, \mathit{price}]$$

1. Express using relational algebra that *model* in *PC* is a foreign key referencing *model* in *Product* (you do not need to explicitly express that the latter is a primary key).
2. (*) Express using relational algebra the constraint that *model* in *PC* cannot be *NULL*.