Exercises for the Course

Logic

Week 12



Exercise 12.1 (Intuitionistic Validity)

Prove, using the Kripke semantics and forcing relation, that the following formulae are valid in intuitionistic logic:

1. $p \to \neg\neg p$
2. $(p \wedge \neg q) \to \neg(p \to q)$
3. $(\exists x\, \neg p(x)) \to (\neg \forall x\, p(x))$

Exercise 12.2 (Intuitionistic Sequent Calculus)

Prove the validity of the three formulae in 12.1 using the intuitionistic sequent calculus LJ.

Exercise 12.3 (Intuitionistic Countermodel)

Show, using a falsifying Kripke-structure, that the following formulas are *not* valid in intutionistic logic:

1. $\neg\neg p \to p$
2. $\neg(p \wedge q) \to \neg p \vee \neg q$
3. What happens if we try to prove the formulas using LJ?

Exercise 12.4 (Properties of Kripke models)

Show the monotonicity property for Kripke models using induction on formulas. That is, show that for all Kripke models, for every formula A and for all worlds w, v, if $w \Vdash A$ and $R(w, v)$, then $v \Vdash A$.