Exercises for the Course

Logic

Week 5



Exercise 5.1 (Substitution and Unification)

a) Let $\sigma = \{u\backslash y, y\backslash f(a), x\backslash g(u)\}$, $\theta = \{x\backslash f(g(y)), y\backslash u, z\backslash f(y)\}$ and $E = p(x, f(y), g(u), z)$. Compute $\sigma\theta$ using the Proposition on slide 11 of lecture 6. Show that $(\sigma\theta)E = \sigma(\theta(E))$.

b) Unify the following pairs of atomic formulae, i.e. try to find substitutions such that

$$\sigma_1(\, p(a, x, f(g(y)))\,) = \sigma_1(\, p(z, g(f(a)), f(z))\,),$$
$$\sigma_2(\, p(f(y), z, y)\,) = \sigma_2(\, p(z, h(z, u), f(u))\,),$$
$$\sigma_3(\, p(f(x, y), y, g(x))\,) = \sigma_3(\, p(f(v, v), z, z)\,).$$

Exercise 5.2 (DNF)

A formula is in disjunctive normal form (DNF) iff it is a disjunction of conjunctions of literals. Show that every propositional formula can be transformed to an equivalent one in DNF.

Exercise 5.3 (Propositional Clause Form)

Transform the following set of formulae to clause form:

$$\{p, p \to ((q \vee r) \wedge \neg(q \wedge r)), p \to ((s \vee t) \wedge \neg(s \wedge t)), s \to q, \neg r \to t, t \to s\}$$

Exercise 5.4 (First-order Clause Form)

Transform each of the following formulas to clausal form:

$$\forall x(p(x) \to \exists y\, q(y))$$
$$\forall x \forall y(\exists z\, p(z) \wedge \exists u(q(x, u) \to \exists v\, q(y, v)))$$
$$\exists x(\neg\exists y\, p(y) \to \exists z(q(z) \to r(x)))$$

Exercise 5.5 (Equivalence vs. Equisatisfiability)

Skolemisation does not necessarily produce a logically equivalent formula. In this exercise, you prove that the logical consequence between the formula $\forall x \exists y\, A(x, y)$ and its Skolemisation holds in one direction, but not the other.

a) $\forall x\, A(x, f(x)) \models \forall x \exists y\, A(x, y)$

b) $\forall x \exists y\, A(x, y) \not\models \forall x\, A(x, f(x))$.