IN3070
IN4070

Autumn 2022

Exercises for the Course

Logic

Obligatory Exercises 1

Deadline: 18 October 2022, 23:59

Exercise O1.1

(Validity and Proof Calculi)

Consider the following formulae.

F_1: $((p \to q) \land (q \to r)) \to (p \to r)$ ("Transitivity")

F_2: $\forall x\, \exists y\, (\, p(x) \land (p(y) \to q(x))\,) \to \forall z\, q(z)$ ("A modus ponens")

F_3: $((p \to r) \land (q \to r)) \to (p \to q)$ ("Permuted transitivity")

a) Prove the validity of formulae F_1 and F_2 in the *sequent calculus* (make sure you respect the Eigenvariable condition when proving F_2).

b) Prove the validity of formulae F_1 and F_2 in the *resolution calculus*. First, translate the negated formula into clausal form.

c) Show that formula F_3 is invalid by specifying a counter model using the *sequent calculus*.

Exercise O1.2

(Adding a Logical Operator)

The logical operator $\uparrow$ is defined as follows: $A \uparrow B \;\equiv\; \neg(A \land B)\,.$

Extend the LK calculus by rules for the $\uparrow$ operator, such that the operator is supported "natively", i.e. the premises/assumptions of the new rules should only contain A, B, and $A \uparrow B$ (and no additional logical operators, such as $\neg$ or $\land$ should be used or introduced).

a) Specify the two new rules $\uparrow$-left and $\uparrow$-right that have to be added to the rules of the *sequent calculus LK* as defined in Lecture 2 (slides 35 and 36).

b) There is a central part of the soundness proof for propositional LK, where a property is shown separately for each of the rules of the calculus.

   What is this property? Show that it holds for your two rules.

c) Resolution works on formulae in clause form. For a resolution-based theorem proving programme to work with full 1st-order or propositional formulae, these are tramsformed to clause form before starting resolution. What would have to be changed in such a theorem prover to make it accept formulae with a $\uparrow$ operator?