Grading Guidelines
IN3070/IN4070
Autumn 2022

The maximum number of marks for the whole exam was 100.

The minimum number of marks required for each grade was as follows: For E, min. 26. For D, min. 42. For C, min. 50. For B, min. 62. For A, min. 80.

These boundaries are based on an alignment of the delivered work with the grade definitions given here:

`https://www.uio.no/studier/eksamen/karakterer/`

The final grade for each candidate is based on an evaluation of the delivered work as a whole, and may therefore in some cases deviate from the boundaries given above.

Question 1 – Sequent Calculi LK and LJ

Prove the validity of the following formulae using the given calculus. Note that the first two formulas are to be proven in the classical logic LK, while the last two are to be proven in the intuitionistic logic LJ.

In this task you can submit a hand-written answer. Use the sketching paper handed to you in the exam room for this. See instructions in the link below the task bar.

A) $(p \wedge (q \vee r)) \to ((\neg p \vee \neg q) \to (p \wedge r))$, using propositional LK [5 marks]

C) $\neg\neg(p \vee \neg p)$, using propositional LJ [6 marks]

D) $\neg\exists x\, p(x) \to \forall x\, \neg p(x)$ using first-order LJ [6 marks]

Question 2 – Intuitionistic Logic

Consider the formula: $A = \neg\neg p \vee \neg p$

A is valid in classical logic but not in intuitionistic logic.

A) Explain, by pointing to the differences between the calculi, why A can be proven using LK, but not using LJ. [5 marks]

B) Show that A is not valid in intuitionistic logic using the model semantics. I.e. construct an intuitionistic structure with a world w such that $w \not\Vdash \neg\neg p \vee \neg p$ [7 marks]

In this task you can submit a hand-written answer. Use the sketching paper handed to you in the exam room for this. See instructions in the link below the task bar.

Question 3 – The Eigenvariable Condition

The so called «eigenvariable condition» for some of the rules of the first-order sequent calculus requires to substitute a «new» constant for a quantified variable, i.e. one that does not occur in the conclusion.

You can submit a hand-written answer. Use the sketching paper handed to you in the exam room for this. See instructions in the link below the task bar.

A) What would happen without this condition, i.e. if you could substitute any constant in the rules $\exists$-left and $\forall$-right? Would the calculus become unsound? Incomplete? Neither? Both?

B) Give an example derivation that demonstrates what you wrote in A), i.e. a valid formula that cannot be proven without the eigenvariable condition and/or an invalid formula that can be proven, depending on your answer to A).

[14 marks total]

Question 5 – The Substitution Lemma

Prove the Substitution Lemma for first order terms, i.e.

for all interpretations I, assignments α for I, variables x, and terms s, t, $v_I(\alpha, t[x\backslash s]) = v_I(\alpha', t)$ where $\alpha' = \alpha\{x \leftarrow v_I(\alpha, s)\}$. [12 marks]

Hints:

- *You should prove this by structural induction over the term t.*
- *Your proof will need one base case each for constants, the variable x, and variables other than x*
- *Substitution is defined inductively as follows*

- $x[x\backslash s] = s$
- $y[x\backslash s] = y$ *for all variables* $y \neq x$
- $a[x\backslash s] = a$ *for all constants* a
- $f(t_1, \ldots, t_n)[x\backslash s] = f(t_1[x\backslash s], \ldots, t_n[x\backslash s])$

- *The modified variable assignment* $\alpha\{x \leftarrow d\}$ *is defined by*
  - $\alpha\{x \leftarrow d\}(y) = \alpha(y)$ *for all variables* $y \neq x$
  - $\alpha\{x \leftarrow d\}(x) = d$

Question 6 – Soundness and completeness for LK

In this task you can submit a hand-written answer. Use the sketching paper handed to you in the exam room for this. See instructions in the link below the task bar.

A) Is the statement "If a first-order formula has a counter model, then it is not provable in the LK calculus" equivalent to the soundness or the completeness theorem for the LK calculus? Explain. [2 marks]

B) Consider the formula $\forall x(p(x) \vee q(x)) \to \forall x p(x) \vee \forall x q(x)$.

- Construct a fair derivation of the formula in the LK calculus.
- Construct a counter model of the formula from an open branch of the derivation, using the Herbrand universe of the branch.

[10 marks]

B) The other rules of the LJ calculus also preserve validity downwards w.r.t the classical logic semantics. You do not need to show this. Explain, based on this observation, why any formula which is provable in the LJ calculus must also be provable in the LK calculus. [4 marks]

C) Show that the LJ calculus is not complete w.r.t. the classical logic semantics. (You may, in your answer, refer to other questions in this exam even if you have not answered them.) [4 marks]