

IN3070/4070 – Logic – Autumn 2022

Lecture 1: Introduction

Martin Giese

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DEPARTMENT OF
INFORMATICS



UNIVERSITY OF
OSLO

Today's Plan

- ▶ What is Logic?
- ▶ Logic in Computer Science
- ▶ Three Ingredients
- ▶ Applications
- ▶ Course Information

Outline

- ▶ What is Logic?
- ▶ Logic in Computer Science
- ▶ Three Ingredients
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What is Logic?

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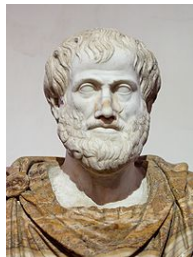
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- Abstraction!

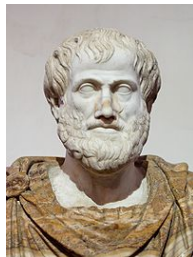
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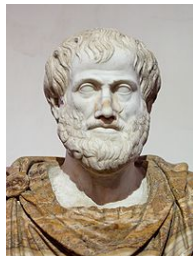
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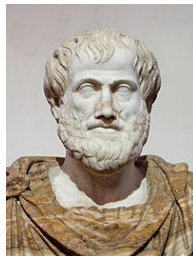
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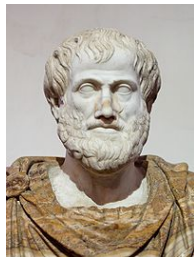
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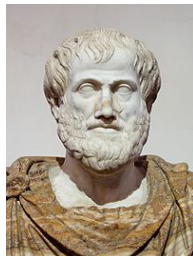
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- ▶ Also an abstraction!



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Sure, cool, but why bother?

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 - ▶ Numerical Models (Newtonian mechanics, Quantum mechanics)



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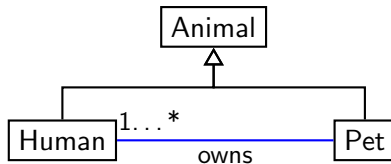
What is a Good Modelling Language

For a model that can be used by a computer. . .

- ▶ there has to be a 'language'
 - ▶ language says what is a model and what not
 - ▶ programs (and humans. . .) need to know what to expect
 - ▶ often defined by some grammar
 - ▶ UML, ER, ORM, OWL, SQL, Java, etc. are all languages
- ▶ the meaning of models should be very clear
 - ▶ otherwise, different implementations do different things
 - ▶ sometimes, 100s of pages of technical text (e.g. JLS)
 - ▶ sometimes meaning given by mathematical definitions

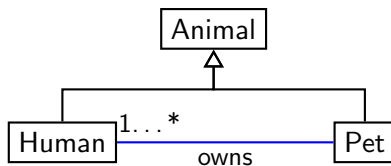
Models and Statements

Observation: much of the content of many models can be given as statements:



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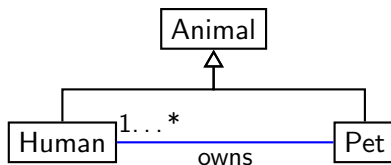
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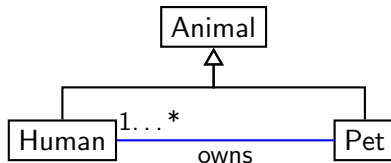
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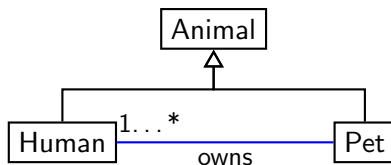
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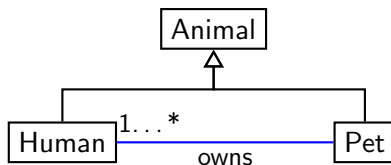
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Logics are a very expressive and precise family of modelling languages

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- ▶ What is Logic?
- ▶ Logic in Computer Science
- ▶ **Three Ingredients**
- ▶ Applications
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Three Central Ingredients

- ▶ Syntax (i.e. the language)
- ▶ Semantics (i.e. the meaning)
- ▶ Calculus (i.e. method, algorithm, usually rules)

Syntax

Most logics have some kind of *formulas*.

The syntax says which strings of characters are formulas.

Syntax of Propositional Formulae

Propositional formulas are defined inductively as follows

- ▶ Every lower case letter ($p, q, r, \dots$) is a formula
- ▶ If A and B are formulae, then $\neg A$, $(A \wedge B)$, $(A \vee B)$ and $(A \rightarrow B)$ are formulae.

Inductively defined (remember IN1150): only what can be constructed using these rules is a formula.

$$p, \quad (p \wedge \neg p), \quad (p \rightarrow q) \vee (q \rightarrow p), \dots$$

But not: $((p, \neg \rightarrow q, \dots$

Model Semantics

We usually define some kind of *interpretation* or *model* or *structure*...
Always the same idea:

- ▶ We don't know what we talk about (p, q, r, x, y, z)
- ▶ We don't know what is true (p or $\neg q$?)
- ▶ So we use a mathematical object that tells us what they mean and what is true or not

Interpretation

An interpretation is a function $\mathcal{I} : \text{Letters} \rightarrow \{T, F\}$ that assigns one of the truth values T or F to every lower case letter

Model Semantics (cont.)

Truth Value

The truth value of formulas $v_{\mathcal{I}}(A)$ is defined inductively by

- ▶ $v_{\mathcal{I}}(A) = \mathcal{I}(A)$ for letters A
- ▶ $v_{\mathcal{I}}(\neg A) = T$ if $v_{\mathcal{I}}(A) = F$ and
 $v_{\mathcal{I}}(\neg A) = F$ if $v_{\mathcal{I}}(A) = T$
- ▶ $v_{\mathcal{I}}(A \wedge B) = T$ if $v_{\mathcal{I}}(A) = T$ and $v_{\mathcal{I}}(B) = T$
 $v_{\mathcal{I}}(A \wedge B) = F$ otherwise
- ▶ ...

Entailment

Formula A *entails* formula B ($A \models B$) if for every $\mathcal{I}$ with $v_{\mathcal{I}}(A) = T$ it also holds that $v_{\mathcal{I}}(B) = T$.

Model Semantics: Take Aways

Model Semantics defines the meaning of logical formulas. . .

- ▶ i.e. truth/falsity in some interpretation/model/. . .
- ▶ relations between formulas like entailment, equivalence. . .

. . . *by mathematical definitions.*

- ▶ We assume that maths, set theory, etc. “work”
- ▶ We assume that people can read formulas, understand words like “and” or “not” or “otherwise,” look up truth values in tables, etc.
- ▶ The definitions can often not be implemented directly
 - ▶ E.g. loop over infinitely many interpretations in 1st order logic

Calculi

- ▶ A calculus works on formulas, i.e. *syntax*
- ▶ Usually by *inference rules* saying how to *derive* new formulas

$$\frac{A \rightarrow B \quad A}{B}$$

- ▶ Always with some machinery that says how to use the rules
- ▶ Can be used to check entailment etc. between formulas
- ▶ Can be implemented on a computer

Natural Deduction for Propositional Logic

- rules for $\wedge$ (conjunction)

Natural Deduction for Propositional Logic

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A B

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Calculi: Take Aways

Calculi allow to determine

- ▶ *semantic properties* like equivalence, satisfiability etc.
- ▶ by *syntactic means*

...i.e. in ways that can be implemented

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Learn techniques to handle languages and their semantics

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- ▶ Foundations are not (usually) our problem :-)

Outline

- ▶ What is Logic?
- ▶ Logic in Computer Science
- ▶ Three Ingredients
- ▶ **Applications**
- ▶ Course Information

SAT

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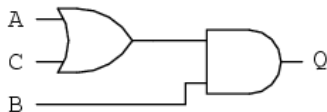
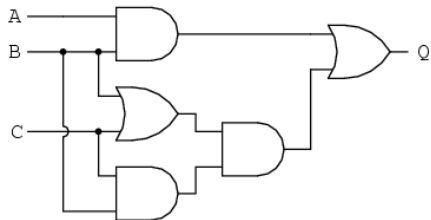
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<http://www.carstensinz.de/talks/RISC-2005.pdf>

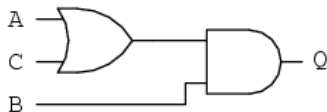
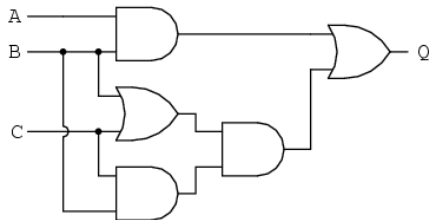
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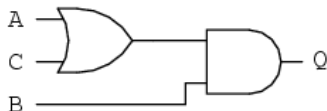
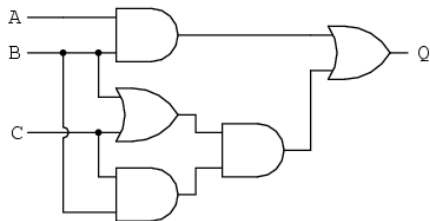


$$(A \wedge B) \vee ((B \vee C) \wedge (B \wedge C)) \quad \text{vs.} \quad (A \vee C) \wedge B$$

Logically equivalent?

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$(A \wedge B) \vee ((B \vee C) \wedge (B \wedge C))$ vs. $(A \vee C) \wedge B$

Logically equivalent?

Today, theorem provers are routinely used to check Boolean circuits

SAT applications: program verification

- ▶ A 32 bit int can be encoded as 32 boolean variables
- ▶ If SAT can handle 1M boolean variables, it can handle thousands of 32 bit words.
- ▶ Properties of programs (without loops) can be handled by SAT solvers
- ▶ Experimental, but works in many cases

SAT applications: puzzle solving

5	3			7				
6			1	9	5			
	9	8					6	
8				6				3
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SAT: problems that require finding one of a large, fixed number of combinations, but checking is easy

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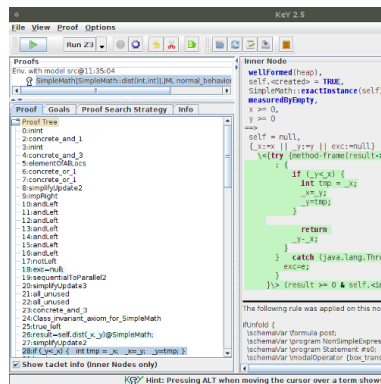
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- ▶ In combination with other techniques, first-order logic can be used to reason about programs

The KeY tool

- ▶ <https://www.key-project.org/>
- ▶ Verify behaviour of Java programs
- ▶ Based on 1st-order logic
- ▶ Extended with program operators
 - ▶ $\langle Prog \rangle p$
 Prog terminates and p holds afterwards
- ▶ Based on a Sequent Calculus



The TimSort bug

Proving that Android's, Java's and Python's sorting algorithm is broken (and showing how to fix it)

🕒 February 24, 2015 📁 Envisage ✍ Written by Stijn de Gouw. 🧑 💰

Tim Peters developed the **Timsort hybrid sorting algorithm** in 2002. It is a clever combination of ideas from merge sort and insertion sort, and designed to perform well on real world data. TimSort was first developed for Python, but later ported to Java (where it appears as `java.util.Collections.sort` and `java.util.Arrays.sort`) by **Joshua Bloch** (the designer of Java Collections who also pointed out that **most binary search algorithms were broken**). TimSort is today used as the default sorting algorithm for Android SDK, Sun's JDK and OpenJDK. Given the popularity of these platforms this means that the number of computers, cloud services and mobile phones that use TimSort for sorting is well into the billions.

[http:](http://www.envisage-project.eu/proving-android-java-and-python-sorting-algorithm-is-broken-and-how-to-fix-it/)

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 - ▶ Can define and reason about terminologies of up to 100.000s of concepts
 - ▶ Applications in semantic web, data integration,...
- ▶ E.g. Temporal Logic
 - ▶ “Every request is eventually followed by an acknowledgement”
 - ▶ $\Box(\text{Req} \rightarrow \Diamond \text{Ack})$
 - ▶ Can check properties of systems with hundreds of variables

The World In Between

- ▶ Many logics are stronger than propositional logic, but still decidable.
- ▶ Often efficiently in practice
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- ▶ Knowledge logics, probabilistic logics, alternating logics, belief logics, ...

Outline

- ▶ What is Logic?
- ▶ Logic in Computer Science
- ▶ Three Ingredients
- ▶ Applications
- ▶ **Course Information**

When, Where, and Who

When and Where

- ▶ Lectures
 - ▶ Tuesdays 12:15–14:00 in OJD 3438, Caml
 - ▶ Only if needed: Mondays 12:15–14:00 in Caml
- ▶ Homepage: <https://www.uio.no/studier/emner/matnat/ifi/IN3070/index-eng.html>

Lecturers



Martin Giese
(martingi@ifi.uio.no)



Henrik Forssell
(jonf@ifi.uio.no)

Exercises

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- ▶ Practical exercises every week,
- ▶ Fridays 10:15–12:00 in Datastue Chill 3433
- ▶ Exercises available on website well in advance. Come prepared!
- ▶ First session: IN1150 refresh, sets, relations, induction
- ▶ In general: part repetition of lectures, part exercises

Teacher:



Anders Sjøberg
(andersob@ifi.uio.no)

Mandatory Assignments

Assignments

- ▶ Two mandatory assignments (obliger)
- ▶ Will be in October/November
- ▶ Corrected by teacher.
- ▶ Pass/Fail
- ▶ Must have passed all assignments in order to attend exam
- ▶ For IN4070 (MSc version): more challenging oblig 2

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- ▶ Strictly for fun, no influence on grade.

Astro Discourse

- ▶ Chat (like Mattermost etc.)
- ▶ Q&A (like Piazza)
- ▶ Can use $\text{\LaTeX}$ markup in posts
- ▶ Can post anonymously

<https://in3070-discourse.uio.no/>

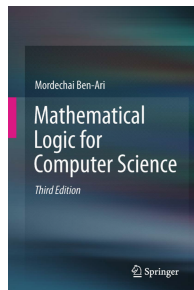
- ▶ Will post an intro PDF on the semester page

Exam

- ▶ Four hours written exam in Silurveien
 - ▶ In case of few students, might be oral exam instead
- ▶ Same exam for IN3070 and IN4070
- ▶ Grades A–F
- ▶ Probably 12 December – Check semester page!

Textbook

- ▶ Mordechai Ben-Ari
Mathematical Logic for Computer Science
3rd edition, Springer, 2012.
- ▶ only chapters 1–4 and 6–12;
not part of the curriculum:
chapter 5 (binary decision diagrams) and
chapters 12–16 (incompleteness, temporal log., prog. verification)
- ▶ download for free (within the UiO network) from Springer's website at
<http://www.springer.com/gp/book/9781447141280>



Next weeks. . .

▶ Henrik:

- ▶ Propositional Logic
- ▶ Tableaux/Sequent calculi for propositional Logic
- ▶ Soundness and Completeness
- ▶ First-order logic
- ▶ Tableaux/Sequent calculi for 1st order logic
- ▶ Soundness and Completeness

▶ Martin:

- ▶ Resolution calculus for propositional Logic
- ▶ Soundness and Completeness
- ▶ Resolution calculus for first-order Logic
- ▶ Soundness and Completeness