IN3070/4070 – Logic – Autumn 2022

Lecture 9: Logic Programming

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18th October 2022



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University of Oslo

Today's Plan

- Motivation
- SLD Resolution
- Prolog
- Syntax
- Semantics
- Lists & Arithmetic
- Negation/Cut/If-then-else
- Summary

Outline

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The First-Order Resolution Calculus

Definition 1.1 (First-Order Resolution Calculus).

$$\frac{}{C_1, ..., \{\}, ..., C_n}\ \textit{axiom}$$

$$\frac{C_1, ..., C_i \cup \{L_1\}, ..., C_j \cup \{L_2\}, ..., C_n, C_i\sigma \cup C_j\sigma}{C_1, ..., C_i \cup \{L_1\}, ..., C_j \cup \{L_2\}, ..., C_n}\ \textit{resolution}$$

with $\sigma(L_1) = \sigma(\overline{L_2})$

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$$\frac{C_1, ..., C_i \cup \{L_1, ..., L_m\}, ..., C_n, C_i\sigma \cup \{L_1\sigma\}}{C_1, ..., C_i \cup \{L_1, ..., L_m\}, ..., C_n}\ \textit{factorization}$$

with $\sigma(L_1) = ... = \sigma(L_m)$

- a resolution proof for a set of clauses S is a derivation of S in the resolution calculus; the substitution σ is local for every rule application; variables in every clause C can be renamed

Logic Programming

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- unification is used to calculate a substitution of the variables in the given query

- in imperative programming languages, computation is explicitly constructed by the programmer (using if-then-else, while, for, ...)
- in logic programming, the program is a declarative specification and the resolution inference engine provides an implicit control

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Horn Clauses and Logic Programs

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A Horn clause is a clause that contains at most one positive literal (a positive literal is a non-negated literal). A definite clause is a Horn clause that contains a (single) positive literal.

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Definition 2.2 (Logic Program).

A logic program consists of definite clauses of the form:

- *facts:* $\{A\}$ $\qquad (A)$
- *rules:* $\{A, \neg B_1, ..., \neg B_n\}$ $\qquad (A \leftarrow B_1 \wedge ... \wedge B_n)$

where $A, B_1, ..., B_n$ *are atomic formulae.*

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Definition 2.3 (Goal or Query).

A goal/query clause has the form $\{\neg B_1, ..., \neg B_n\}$ *where* $B_1, ..., B_n$ *are atomic formulae.*

SLD Resolution

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Definition 2.4 (SLD Resolution).

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- *first step: 1st parent clause* $C_i \cup \{L_1\}$ *is the query clause*
  step $n \geq 2$*: 1st parent clauses* $C_i \cup \{L_1\}$ *is resolvent* $C_i\sigma \cup C_j\sigma$ *of step* $n-1$
- *2nd parent clauses* $C_j \cup \{L_2\}$ *is always a clause of the logic program*

An SLD Resolution Derivation

Let $\{\neg Q_1, \neg Q_2, ...\}$ be a query clause and

$\{A_1, \neg B_1, \neg B_1', ...\}$, ... $\{A_n, \neg B_n, \neg B_n', ...\}$ be a logic program.

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$$\{\neg D_1, ..., \neg D_k, ...\} \quad \{A_l, \neg B_l, \neg B'_l, ...\}$$

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$$\{\neg D_1, ..., \neg D_k, ...\} \quad \{A_l, \neg B_l, \neg B_l', ...\} \quad \text{with } \sigma(D_k) = \sigma(A_l)$$
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$\vdots$

$\{\neg F_1\}$

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- algorithm = logic + control [Kowalski 1979]

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- A Prolog program is a logic program, i.e. a set of definite clauses
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- the symbol ':-' is used to represent the implication '$\leftarrow$'

- A Prolog program is "executed" by the Prolog interpreter (control) that implements SLD resolution
- search strategy: choose leftmost literal in the first parent/goal clause (D_1) and choose second parent clause (D_2) from top to bottom among the program clauses

Prolog – An Example

- An example in Prolog (file `family.pl`)

```
male(thomas).                                    % these are facts
male(rolf).
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male(thomas).                                    % these are facts
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female(anna).
female(maria).
parent(thomas,anna).
parent(maria,anna).
parent(rolf,maria).
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parent(thomas,anna).
parent(maria,anna).
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father(X,Y) :- parent(X,Y), male(X).   % these are rules
mother(X,Y) :- parent(X,Y), female(X).
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- start Prolog and type '`[family].`' to load the program
- `Ctrl-C` stops Prolog; '`halt.`' exits Prolog

Prolog Queries – Examples

- `?- parent(maria,anna).`

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  true.
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SLD Resolution Derivation – Example

program clauses:

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male(rolf).
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father(X,Y) :- parent(X,Y), male(X).
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query: `?- grandfather(rolf,Y).`

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$\{\neg$`grandfather(rolf,Y)`$\}$ $\{$`grandfather(X,Z)`$,\neg$`father(X,U)`$,\neg$`parent(U,Z)`$\}$

$\downarrow$ $\swarrow$ with σ(`X`)=`rolf`, σ(`Y`)=`Z`

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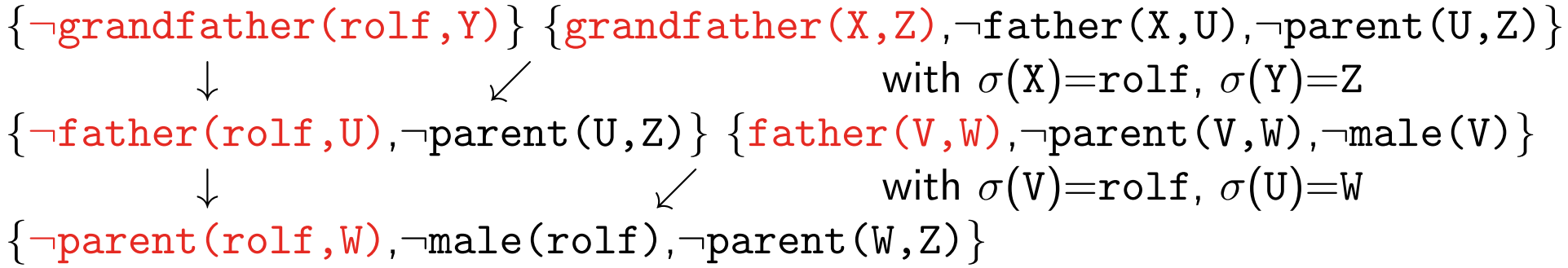
$\{\neg$`father(rolf,U)`$,\neg$`parent(U,Z)`$\}$ $\{$`father(V,W)`$,\neg$`parent(V,W)`$,\neg$`male(V)`$\}$

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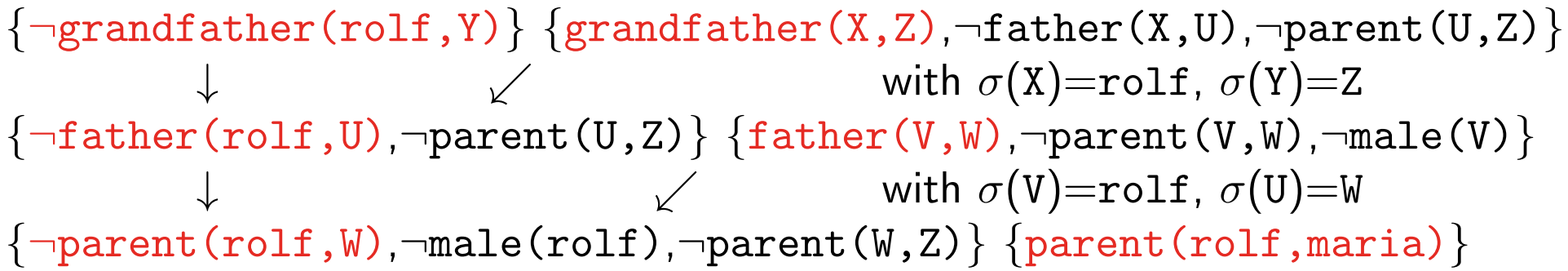


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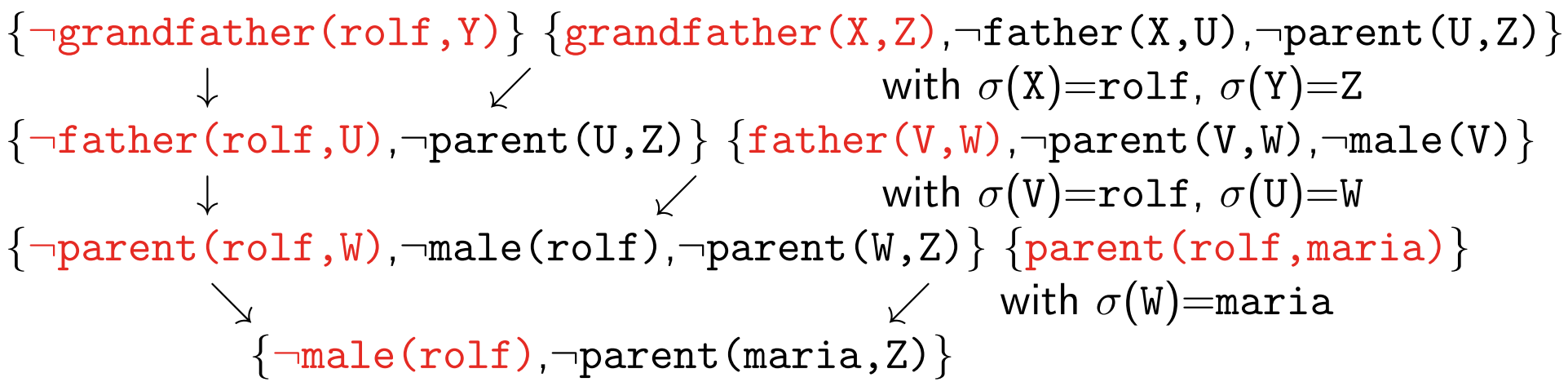


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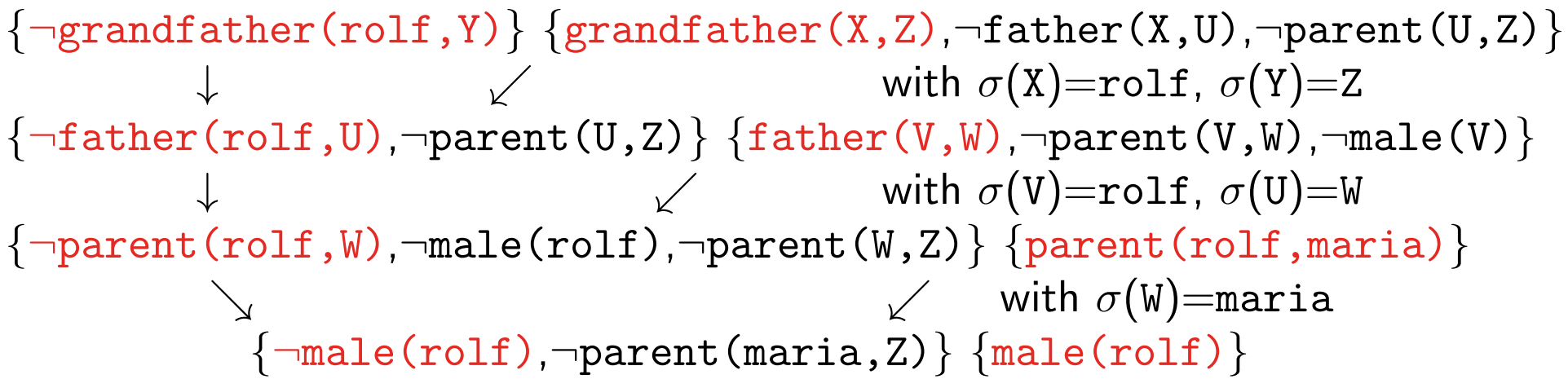


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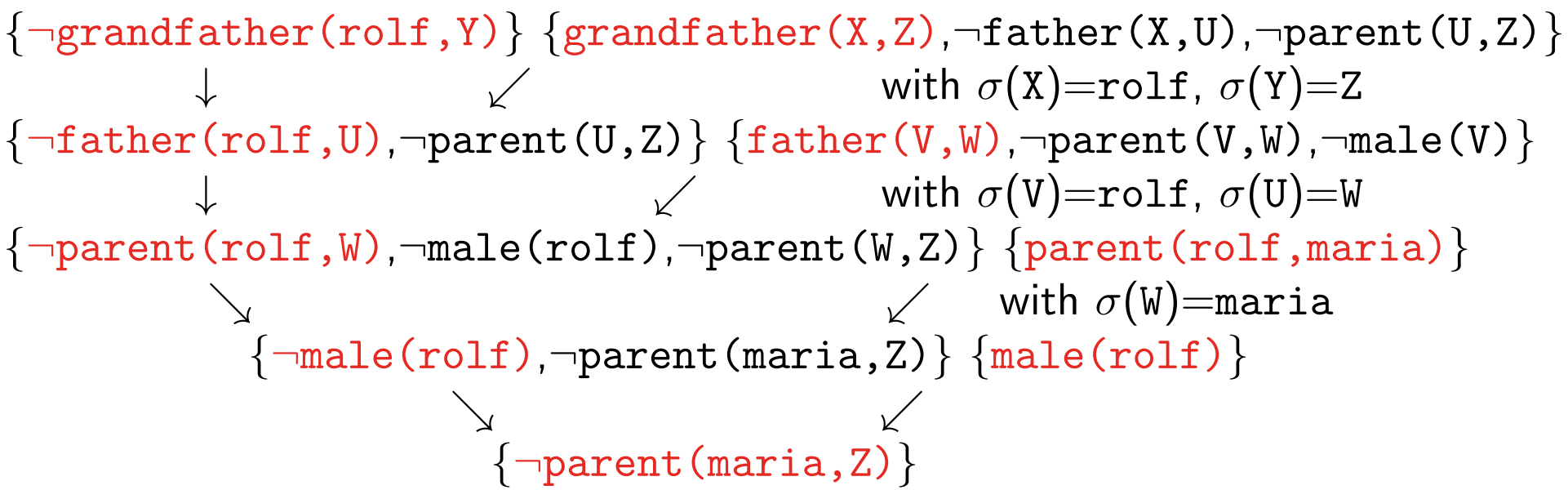


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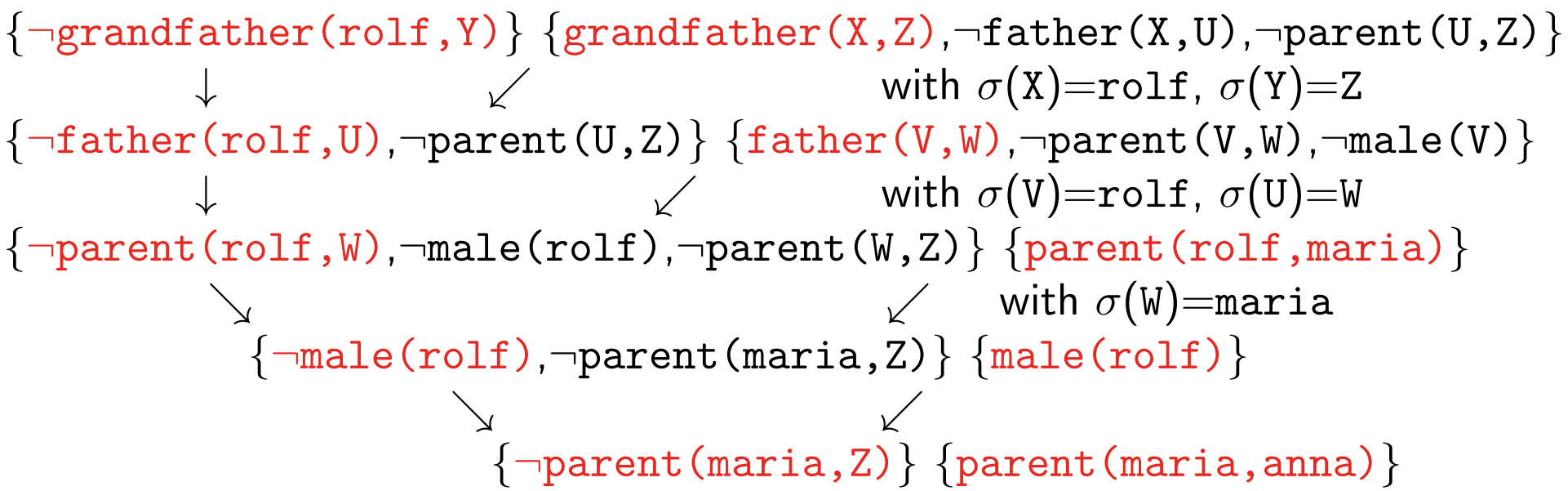


SLD Resolution Derivation – Example

program clauses:

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male(rolf).
parent(maria,anna).
parent(rolf,maria).
father(X,Y) :- parent(X,Y), male(X).
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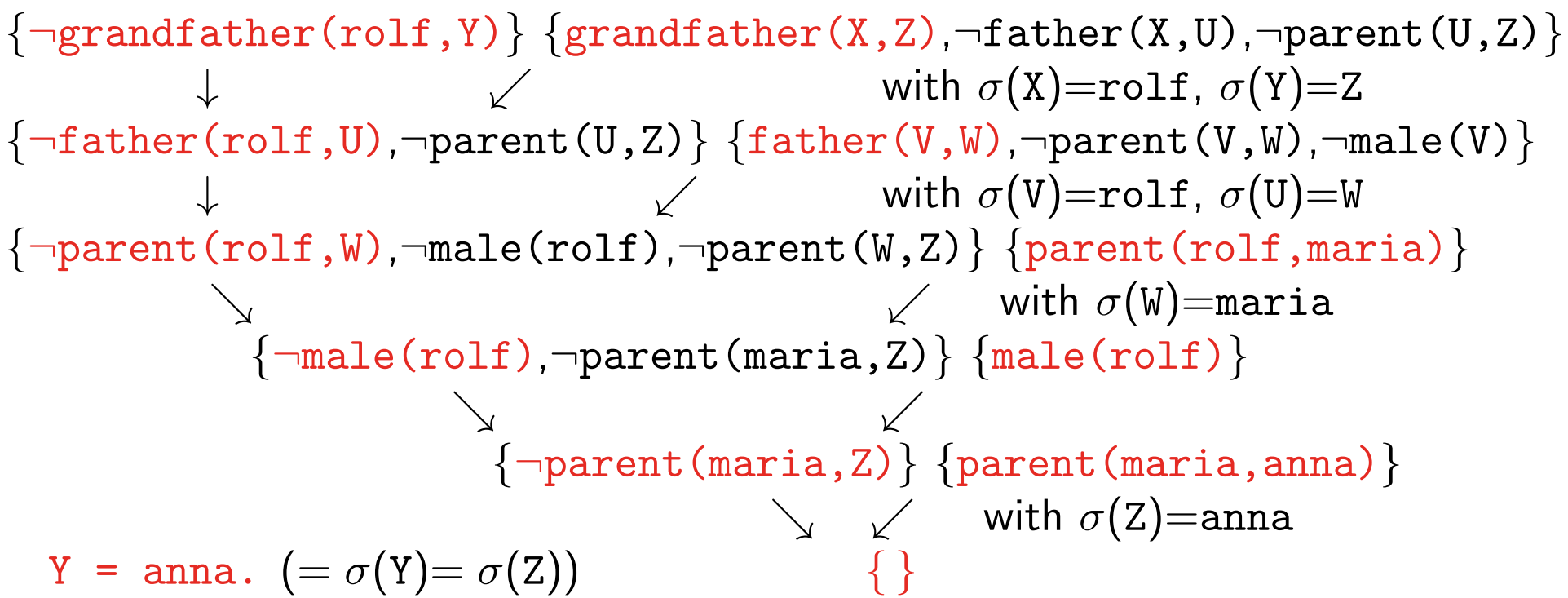


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Outline

- Motivation
- SLD Resolution
- Prolog
- Syntax
- Semantics
- Lists & Arithmetic
- Negation/Cut/If-then-else
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A Prolog program consists of clauses; a clause is either a fact or a rule. The user can query the Prolog program/database.

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- if an appropriate fact matches, then the predicate/goal succeeds
- if the head of a rule matches, then Prolog continues by trying to fulfil the predicates of the rule's body

- the database is searched top to bottom
- if more than one fact or head of a rule matches, then alternative options are considered if the search fails (via backtracking)

Operational Semantics – Example

```
male(thomas). male(rolf). female(anna). female(maria).
parent(thomas,anna). parent(maria,anna). parent(rolf,maria).
father(X,Y) :- parent(X,Y), male(X).
mother(X,Y) :- parent(X,Y), female(X).
grandfather(X,Z) :- father(X,Y), parent(Y,Z).
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?- grandfather(X,anna).
-> father(X,Y)
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male(thomas). male(rolf). female(anna). female(maria).
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```

```
?- grandfather(X,anna).
-> father(X,Y) -> parent(X,Y)
```

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```
?- grandfather(X,anna).
-> father(X,Y) -> parent(X,Y) -> parent(thomas, anna)
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male(thomas). male(rolf). female(anna). female(maria).
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                  male(thomas)
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                  male(thomas)
   parent(anna,anna)
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```

```
?- grandfather(X,anna).
-> father(X,Y) -> parent(X,Y) -> parent(thomas, anna)
                  male(thomas)
   parent(anna,anna) -> fail
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                               -> parent(rolf, maria)
                  male(rolf)
   parent(maria,anna)
grandfather(rolf,anna) succeeds
X = rolf.
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- variables are instantiated ("bound") during the unification of terms

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&\wedge\ \texttt{...} \\
&\wedge\ \texttt{fact_n} \\
&\wedge\ \texttt{head_1} \leftarrow \texttt{body_1} \\
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...                              ∧ ...
fact_n.                          ∧ fact_n
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...                              ∧ ...
head_m :- body_m.                ∧ head_m ← body_m )
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The `query` succeeds iff the Prolog program terminates and F is valid.

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- inference engine is a theorem prover based on SLD resolution (only Horn clauses, depth-first search (incomplete!), no occurs-check (unsound!))

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Lists are terms that are represented in the following way:

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[a, b|[c, d, e]]
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true .
?- member(X,[a,b]).
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X = a ;
X = b .
?- append([a,b],[c],Z).
```

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```
?- member(a,[a,b,c]).
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X = a ;
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?- append([a,b],[c],Z).
Z = [a, b, c].
?- append(X,Y,[a,b,c]).
```

Predefined Predicates on Lists

- `member(Element,List)` succeeds iff `Element` occurs in `List`
- `append(List1,List2,List3)` succeeds iff appending `List1` and `List2` results in `List3`
- `length(List,Length)` succeeds iff `List` has length/size `Length`
-
```
?- member(a,[a,b,c]).
true .
?- member(X,[a,b]).
X = a ;
X = b .

?- append([a,b],[c],Z).
Z = [a, b, c].
?- append(X,Y,[a,b,c]).
X = [], Y = [a, b, c] ;
X = [a], Y = [b, c] ;
X = [a, b], Y = [c] ;
X = [a, b, c], Y = [] .
```

Lists – Examples

- delete all identical elements from list

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delete([],_,[]).
delete([X1|T],X,L) :- X==X1, delete(T,X,L).
delete([X1|T],X,[X1|L]) :- X\==X1, delete(T,X,L).
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- reverse list

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reverse([],[]).
reverse([H|T],L):- reverse(T,R), append(R,[H],L).
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L = [h,e,l,l,o].
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Comparison only works if variables are instantiated to numbers.

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Outline

- Motivation
- SLD Resolution
- Prolog
- Syntax
- Semantics
- Lists & Arithmetic
- Negation/Cut/If-then-else
- Summary

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- Negation as Failure gives a non-monotonic logic
- Very different from our classical notion of logical consequence

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Example: Siblings

Disjunction and If-then-else

- `predicate :- predicate1 ; predicate2.`
  succeeds if `predicate1` succeeds or `predicate2` succeeds; backtracking over `predicate1` and `predicate2` when re-fulfilling `predicate`

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  ```
  grandparent(X,Y) :-
      grandfather(X,Y) -> true ; grandmother(X,Y).
  ```

  (no backtracking over `grandfather`)

Problems with Prolog

- No type system
- No standardized module system
- Non-declarative arithmetic
- Cut needed for efficiency
  - Cut has non-declarative semantics
  - Cut can simulate negation as failure (non-monotonic)
  - Cut can be tricky to use
  - Cut makes automated optimization hard
- IO does not play nice with backtracking

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  - Can be handy to implement theorem provers

Outline

- Motivation
- SLD Resolution
- Prolog
- Syntax
- Semantics
- Lists & Arithmetic
- Negation/Cut/If-then-else
- Summary

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- next week: DPLL (efficient SAT solving)