Exam
IN3070/IN4070
Autumn 2024

The maximum number of marks for the whole exam was 100.

The minimum number of marks required for each grade was as follows: For E, min. 30. For D, min. 43. For C, min. 50. For B, min. 66. For A, min. 82.

These boundaries are based on an alignment of the delivered work with the grade definitions given here:

`https://www.uio.no/studier/eksamen/karakterer/`

The final grade for each candidate is based on an evaluation of the delivered work as a whole, and may therefore in some cases deviate from the boundaries given above.

Question 1 – True or False

[1 mark for a correct answer, -1 for incorrect, 0 for missing answer. Min 0 for the whole question]

- Are the following terms unifiable? $f(x, a)$ and $f(g(y, b), y)$
- Is validity of formulas in classical first-order logic decidable?
- When a set of formulas A is satisfiable, and a subset of some larger set $A \subseteq B$, then B also has to be satisfiable.
- A resolution proof has shown unsatisfiability of a clause set when the empty set of clauses has been derived.
- A β rule splits a branch of a derivation into two branches.

Question 2 – Sequent Calculi LK and LJ

Prove the validity of the following formulae using the given calculus. Note that the first two formulas are to be proven in the classical logic LK, while the last is to be proven in the intuitionistic logic LJ.

In this task you can submit a hand-written answer. Use the sketching paper handed to you in the exam room for this. See instructions in the link below the task bar.

A) $(p \to r) \to ((q \to r) \to ((p \vee q) \to r))$ using propositional LK [5 marks]

C) $(p \to r) \to (p \to (q \vee r))$ using propositional LJ (the intuitionistic calculus) [8 marks]

Question 3 – Soundness and Completeness

In the context of a sequent calculus, which of these statements are equivalent to soundness of the calculus, or completeness, or neither.

[1 mark for a correct answer, -1 for incorrect, 0 for missing answer. Min 0 for the whole question]

(For each question, one of three options could be chosen, soundness, completeness, neither. The statements were presented to students in a randomized order.)

- Every derivable formula is valid
- Every satisfiable formula is derivable
- Every formula that has no proof is falsifiable
- There is a closed derivation for every valid formula
- A falsifiable formula cannot be derived

Question 4 – Classical first-order semantics

A) Show that the formula $\forall x(p(x) \leftrightarrow \neg p(f(x)))$ is satisfiable by constructing satisfying interpretations, namely

A1) an interpretation with a finite domain [6 marks], and

A2) an interpretation with an infinite domain (take e.g. the natural numbers) [6 marks]

B) Show that the same formula is not valid by giving a falsifying interpretation. [6 marks] An additional [2 marks] if the domain of your interpretation has only one element.

In this task you can submit a hand-written answer. Use the sketching paper handed to you in the exam room for this. See instructions in the link below the task bar.

Question 5 – Deontic Logic

We call a binary relation $R \subseteq W \times W$ serial if for all $u \in W$ there is at least one $v \in W$ with uRv. In other words, every world has an R-successor, a world that is reachable from it.

A Kripke frame (W, R) is called serial if R is a serial relation on W.

A) Show that the formula $\Box p \to \Diamond p$ is satisfied in every serial Kripke Frame, i.e. for every $w \in W$, $w \Vdash \Box p \to \Diamond p$ [8 marks]

B) Suppose that (W, R) is a Kripke frame, and $\Box p \to \Diamond p$ is forced by every world of every Kripke structure $((W, R), \{I(w)\}_{w \in W}\}$ with that frame. Show that the frame is serial.

Hint. You will need to show that an arbitrary world w has an R-successor. Use an interpretation where p is true exactly in the R-successors of w, and investigate what $\Box p \to \Diamond p$ then tells you about the world w.

[12 marks]

Question 6 – Resolution

Show that the following formula is not valid, by using the resolution calculus.

$$(\forall x \exists y.p(x, y)) \to (\exists y \forall x.p(x, y))$$

Remember that resolution is a refutation calculus, i.e. you can derive that a set of clauses is unsatisfiable. Also remember that variables should be made disjoint before applying resolution.

Transform the problem to clause form, then explain why a resolution proof is not possible from the clauses you get, and finally why this means that the formula is not valid.

- Arriving at a correct set of clauses: 5 marks
- Correct reasoning about invalidity: 5 marks

Question 7 – Propositional Substitution Lemma

We will explore a propositional logic variant of the substitution lemma. Instead of modifications to variable assignments, we will define modifications to propositional interpretations, defined as follows:

For an interpretation $\mathcal{I}$ and an atomic formula p, we define the modified interpretation $\mathcal{I}\{p\}$ resp. $\mathcal{I}\{\neg p\}$ to be identical to $\mathcal{I}$, except that p is interpreted as true in $\mathcal{I}\{p\}$ and false in $\mathcal{I}\{\neg p\}$. Formally, for two atomic formulas p and q,

$$\mathcal{I}\{p\}(q) := \begin{cases} T & \text{if } q = p \\ \mathcal{I}(q) & otherwise \end{cases}$$

and

$$\mathcal{I}\{\neg p\}(q) := \begin{cases} F & \text{if } q = p \\ \mathcal{I}(q) & otherwise \end{cases}$$

We can also define a substitution operator on formulas, that is very similar to the simplification operator defined in the DPLL lecture, but without the rewriting steps to reduce the formula. We want to define $A[p]$ as the result of replacing all occurrences of p in A by the formula $true$, and $A[\neg p]$ as the result of replacing all occurrences of p in A by the formula $false$.

A) Give a recursive definition of $A[p]$ (this will be recursive on the structure of A). You don't need to define the negative case, which is very similar. [10 marks]

B) prove the positive case of the substitution lemma: for all propositional formulae A, propositional atoms p, and interpretations $\mathcal{I}$,

$$\mathcal{I}\{p\} \models A \quad \text{iff} \quad \mathcal{I} \models A[p]$$