IN3070
IN4070

Autumn 2024

Exercises for the Course

Logic for Computer Science

Week 1



Exercise 1.1 (Formalization)

A warm-up exercise. Not very difficult, but the other questions are more important.

"What is the secret of your long life?" a centenarian was asked. "I strictly follow my diet: If I don't drink beer for dinner, then I always have fish. Any time I have both beer and fish for dinner, then I do without ice cream. If I have ice cream or don't have beer, then I never eat fish." Formalize the answer in propositional logic. Can you simplify the answer?

Exercise 1.2 (Interpretation)

Consider the interpretation $\mathcal{I}$ where $\mathcal{I}(p) = F, \mathcal{I}(q) = T, \mathcal{I}(r) = T$.

Does $\mathcal{I}$ satisfy the following propositional formulae?

1. $(p \to \neg q) \lor \neg(r \land q)$
2. $(\neg p \lor \neg q) \to (p \lor \neg r)$
3. $\neg(\neg p \to \neg q) \land r$
4. $\neg(\neg p \to q \land \neg r)$

Exercise 1.3 (Logical Equivalence)

Show that the following formulae are logically equivalent. Describe two approaches you can use for this purpose.

1. $p \to (q \land \neg q)$ and $\neg p$
2. $(\neg p \lor q) \to q \land (q \to r) \land \neg r$ and $(q \land \neg r) \land (\neg r \to \neg q) \lor (p \land \neg q)$

Exercise 1.4 (Satisfiability and Validity)

Are the following formulae satisfiable, valid, unsatisfiable, or invalid (falsifiable)?

1. $(\neg p \lor q) \land (q \to (\neg r \land \neg p)) \land (p \lor r)$
2. $((\neg p \lor q) \to q \land (q \to r) \land \neg r) \to p$
3. $\neg p \land (\neg q \lor r) \land (\neg p \to q \land \neg r)$

Exercise 1.5 (Propositional Challenge)

A challening exercise that might take a lot of your time. The other questions are more important!

Find a formula A that contains the atoms p, q, and r such that changing any of the interpretations $\mathcal{I}(p)$, $\mathcal{I}(q)$, or $\mathcal{I}(r)$ will also change the truth value $v_{\mathcal{I}}(A)$ of A.

Exercise 1.6 **(Deduction Theorem)**

The first part is simple and a good exercise. The second is slightly more challenging.

Prove a simple version of the Deduction Theorem:

$A \models B$ if an only if $\models A \to B$.

Now let $U = \{A_1, ..., A_n\}$ be a finite (!) set of formulae. We write $\bigwedge_i A_i$ for the conjunction $A_1 \wedge \cdots \wedge A_n$ so $v_{\mathcal{I}}(\bigwedge_i A_i) = T$ iff $v_{\mathcal{I}}(A_i) = T$ for all $A_i \in U$.

You can now prove:

$U \models B$ if and only if $\models (\bigwedge_i A_i) \to B$.

Exercise 1.6 **(Sequent Calculus LK)**

Prove the following formulae in the sequent Calculus LK.

1. $\neg(p \vee q) \to (\neg p \wedge \neg q)$
2. $p \vee \neg p$
3. $(p \to (q \to r)) \to (p \wedge q \to r)$

Exercise 1.7 **(Invalid and Satisfiable in LK)**

Use the sequent calculus LK to show that the first formula is invalid and the second formula is satisfiable.

1. $(p \to (q \to r)) \to (p \vee q \to r)$
2. $\neg(p \vee q) \to (\neg p \wedge q)$