IN3070
IN4070

Autumn 2024

Exercises for the Course

Logic

Week 11



Exercise 11.1 (DL Formalisation)

Given

- concepts Cat, Dog, $Human$, $Crazy$
- a role $hasOwner$

Write $\mathcal{ALC}$ description logic axioms that express the following:

1. Nothing is both a Cat and a Dog
2. Every Dog has at least one owner who is a $Human$
3. Anybody who owns a Cat is $Crazy$

Hint: don't invent syntax. DL has concept expressions and $\sqsubseteq$ axioms, and nothing else. In particular, no variables. But it can help to formalise in first-order logic and try to look at the correspondence between first-order and DL shown in the lecture.

Exercise 11.2 (DL Syntax and Calculus)

Consider the following set $\mathcal{K}$ of First Order logic formulas:

$$\forall x.(\mathsf{Man}(x) \leftrightarrow \mathsf{Human}(x) \wedge \mathsf{Male}(x)) \tag{1}$$
$$\forall x.(\mathsf{Parent}(x) \leftrightarrow \mathsf{Human}(x) \wedge \exists y.(\mathsf{hasChild}(x,y) \wedge \mathsf{Human}(y))) \tag{2}$$
$$\forall x.(\mathsf{Father}(x) \leftrightarrow \mathsf{Man}(x) \wedge \exists y.(\mathsf{hasChild}(x,y) \wedge \mathsf{Human}(y))) \tag{3}$$
$$\forall x.(\mathsf{GrandFather}(\mathsf{x}) \leftrightarrow \mathsf{Man}(x) \wedge \exists y.(\mathsf{hasChild}(x,y) \wedge \mathsf{Parent}(y))) \tag{4}$$

Do the following:

- Write an $\mathcal{ALC}$ TBox $\mathcal{T}$ that is logically equivalent to $\mathcal{K}$.
- Determine whether the axiom

  $$\mathsf{GrandFather} \sqsubseteq \mathsf{Parent}$$

  is a logical consequence of $\mathcal{T}$ by applying the $\mathcal{ALC}$ calculus w.r.t. a TBox.

Exercise 11.3 (Role Satisfiability)

We say that an atomic role R is satisfiable w.r.t. a TBox $\mathcal{T}$ if there exists a model $\mathcal{I}$ of $\mathcal{T}$ such that $R^{\mathcal{I}} \neq \emptyset$.

- Write a satisfiable $\mathcal{ALC}$-TBox such that role R is unsatisfiable w.r.t. $\mathcal{T}$.
- Reduce the problem of checking satisfiability of an atomic role w.r.t. an $\mathcal{ALC}$-TBox to the problem of concept satisfiability w.r.t. an $\mathcal{ALC}$-TBox.