IN3070
IN4070

Autumn 2024

Exercises for the Course

Logic

Week 6



Exercise 6.1 (Resolution Calculus)

Show that the following set of clauses is unsatisfiable, using resolution.

$$\{\{p, q\},\ \{\neg p, r\},\ \{\neg p, \neg r\},\ \{p, \neg q\}\}$$

Exercise 6.2 (CNF and Resolution Calculus)

Show that the first two formulae are valid and the third formula is unsatisfiable by using the resolution calculus. Remember that you may have to negate some of the formulae before transforming them to clause form.

a) $(p \to (q \to r)) \to (p \wedge q \to r)$

b) $(((\neg p \vee q) \to q) \wedge (q \to r) \wedge \neg r) \to p$

c) $\neg p \wedge (\neg q \vee r) \wedge (\neg p \to (q \wedge \neg r))$

Exercise 6.2 (Positive and Negative clauses)

A *positive clause* is one where none of the atoms is negated, e.g. $\{p, q\}$. A *negative clause* is one where all of the atoms are negated, e.g. $\{\neg p, \neg q, \neg r\}$.

(The empty clause counts as both positive and negative.)

Show that every unsatisfiable clause set S contains at least one negative clause and at least one positive clause.

Hint: there is a very 'simple' interpretation that satisfies all clauses that are not positive. And another, also very 'simple' interpretation that satisfies all clauses that are not negative.