IN3070
IN4070

Autumn 2024

Exercises for the Course

Logic

Obligatory Exercises 1



Deadline: 4 October 2024, 23:59

Exercise O1.1

(Validity and Proof Calculi)

Consider the following formulae.

F_1: $((p \to q) \wedge (q \to r)) \to (p \to r)$ ("Transitivity")

F_2: $\forall x \, \exists y \, (\, p(x) \wedge (p(y) \to q(x)) \,) \to \forall z \, q(z)$ ("A modus ponens")

F_3: $((p \to r) \wedge (q \to r)) \to (p \to q)$ ("Permuted transitivity")

a) Prove the validity of formulae F_1 and F_2 in the *sequent calculus* (make sure you respect the Eigenvariable condition when proving F_2).

b) Show that formula F_3 is invalid by specifying a counter model using the *sequent calculus*.

Exercise O1.2

(Adding a Logical Operator)

The logical operator $\uparrow$ is defined as follows: $A \uparrow B \equiv \neg(A \wedge B)$.

Extend the LK calculus by rules for the $\uparrow$ operator, such that the operator is supported "natively", i.e. the premises/assumptions of the new rules should only contain A, B, and $A \uparrow B$ (and no additional logical operators, such as $\neg$ or $\wedge$ should be used or introduced).

a) Specify the two new rules $\uparrow$-left and $\uparrow$-right that have to be added to the rules of the *sequent calculus LK* as defined in Lecture 2 (slides 35 and 36).

b) There is a central part of the soundness proof for propositional LK, where a property is shown separately for each of the rules of the calculus.

What is this property? Show that it holds for your two rules.