IN3070
IN4070

Autumn 2025

Exercises for the Course

Logic

Week 4



Exercise 4.1 (Counter-models)

a) Let p be a 1-ary predicate symbol and s a 1-ary function symbol. Construct a falsifying interpretation for the sequent

$$\forall x\,(p(x) \lor p(s(x))),\ \forall x\,(\neg p(x) \lor \neg p(s(x))) \Longrightarrow$$

by beginning an LK derivation, and constructing an interpretation from an open branch.

Hint: start by using $\forall$-left on one of the two formulae with some dummy constant o, then continue using first one then the other of them, with larger and larger terms, until you see the pattern.

b) Try to find a falsifying interpretation $\mathcal{I} = (D, \iota)$ where the domain has only two elements, $|D| = 2$. This can *not* be read off from an open branch.

c) Is there a falsifying interpretation with only one domain element?

Exercise 4.2 (Bernays-Schönfinkel)

The Bernays–Schönfinkel class of formulas, named after Paul Bernays and Moses Schönfinkel, is a fragment of first-order logic formulas where satisfiability is decidable.

It is the class of all formulae of the shape

$$B = \exists x_1 \cdots \exists x_m \forall y_1 \cdots \forall y_n\, A$$

where A

- contains no quantifiers
- contains no function symbols (i.e. all terms are variables or constants)

Give an algorithm that decides the satsfiability of this kind of formula.

Hints:

- *a formula B is satisfiable if its negation $\neg B$ is not valid.*
- *consider the first few steps an LK proof for $\neg B$ must have.*
- *remember that there are no function symbols. How does the Herbrand universe of an open branch look?*
- *do we need to have infinite branches to achieve fairness?*